Internal Model Control for Pseudo-Hammerstein Systems with Backlash

Ruili Dong
A0503221, Department of Automation
Shanghai Jiaotong University
Shanghai 200040, China
e-mail: npu_dongruili@hotmail.com

Yonghong Tan+
College of Mechanical and Electronic Engineering,
Shanghai Normal University,
Shanghai 201418, China
e-mail: tanyongh@yahoo.com.ce-mail:
tanyongh@yahoo.com.cn

+ Corresponding author

***Abstract*—In this paper, a discrete-time internal model control (IMC) approach to control the nonlinear systems described by pseudo-Hammerstein model with backlash is presented. In this method, the design strategy for discrete-time control is implemented by introducing the inverse function of pseudo-Hammerstein model with backlash. Moreover, the design of the filters to guarantee the robustness of the control system will be discussed. Finally, the simulation results are presented to illustrate the efficiency of the method.**

***Keywords*—Internal model control, backlash, robust, inverse model**

I. INTRODUCTION

Backlash is a kind of non-smooth nonlinearity with multi-valued mapping. It often exists in a wide range of physical systems and devices, such as mechanical actuators, electronic relay circuits and gear transmission devices [1]. Usually, backlash may lead to undesirable oscillations, even instability for closed-loop control systems [1-2]. Thus, the control of the systems with backlash is an important and challenging problem.

There have been some approaches to control the systems with backlash [1-9]. Reference [2] designed a feedforward compensator to eliminate the effect of backlash in valves. Reference [3] proposed a hybrid model based predictive control method to handle the compensation for the saturation and backlash in actuators. Moreover, [4, 5] developed adaptive control based on a smooth inverse model used to approximately cancel the effect of backlash in control systems. On the other hand, some other alternatives for control of the systems with backlash have been developed. Reference [6] used a variable structure control method to control the systems preceded with backlash-like hysteresis. Reference [7] employed a fuzzy control method to control the system with output backlash. Reference [8] used recurrent neural control strategy to handle the backlash existing in mechanical systems. Also, [9] applied the combination of PID strategy and feedforward compensator to the control of a system with backlash.

As backlash is a non-smooth nonlinear phenomenon with multi-valued mapping, it is quite difficult to obtain an accurate model to describe its operating behavior. In this case, it may cause some uncertainty in the modeling procedure. Thus, a robust control strategy has to be considered to tackle the problem of model uncertainty.

It is known that internal model control (IMC) is one of the effective robust control techniques [10-15]. It has been widely applied to industrial process control systems due to its simple structure and easy tuning property. Recently, IMC has been extended to control the smooth nonlinear systems [10, 11, 13-15]. However, up till now, there have been very few literatures concerning the application of the IMC method to the control of systems with backlash.

In this paper, a pseudo-Hammerstein model with backlash based on the key term separation principle [16] is proposed. Based on the proposed model, the corresponding inverse model is obtained. Then, the discrete time internal model control strategy based on the proposed model is developed. Considering the characteristic of the systems involved with backlash, an IMC strategy to guarantee the robust stability of the system is developed. After that, the paper will demonstrate a simulation example based on the proposed control method.

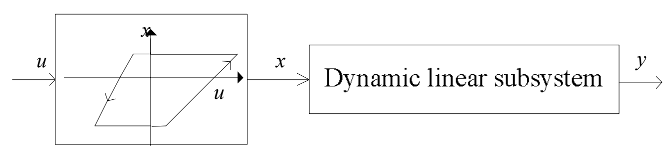

Figure 1. Architecture of pseudo-Hammerstein systems with backlash

II. MODELING PSEUDO-HAMMERSTEIN SYSTEMS WITH BACKLASH

The pseudo-Hammerstein system with backlash is that the linear dynamic subsystem with preceded backlash. The corresponding structure of this system in discrete time is shown in Fig 1. It is noted that input u and output y can be measured directly. However, the embedded internal variable x is unmeasurable.

It is assumed that the linear dynamic subsystem of the system is asymptotically stable. Also, the steady gain of the linear dynamic subsystem is not zero. Moreover, coefficient b0 in the linear part of the system is assumed to be equal to unity, which is normalized for unique representation. Therefore, the corresponding dynamic linear subsystem in discrete time is described by

$$y(k) = -\sum_{i=1}^{n_a} a_i y(k-i) + \sum_{j=0}^{n_b} b_j x(k-j-d) \tag{1}$$

where d is the time-delay; n_a and n_b are the orders of the linear dynamic subsystem, $a_1,\cdots,a_{n_a}$ and $b_1,\cdots,b_{n_b}$ are the coefficients.

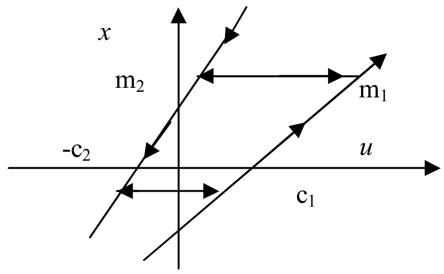

Figure 2. The description of backlash

Generally, the backlash shown in Fig.2 can be considered as a kind of non-symmetric function. Parameters $\infty > m_1 > 0$, $\infty > m_2 > 0$, $\infty > c_1 > 0$ and $\infty > c_2 > 0$, are the constants to specify the characteristic of the backlash, where u and x are respectively the input and output of the backlash.

Based on the characteristic of the backlash and the key term separation principle, the corresponding discrete-time mathematical model is represented by

$$m(k) = m_1 + (m_2 - m_1) g(\Delta u(k)) \tag{2}$$

$$\begin{aligned} x1(k) &= m_1 u(k) + (m_2 - m_1) g(\Delta u(k)) u(k) \\ &\quad - 0.5 c_1 m(k)[\mathrm{sgn}(\Delta u(k)) + 1] g_1(k) \\ &\quad + 0.5 c_2 m(k)[-\mathrm{sgn}(\Delta u(k)) + 1] g_2(k) \end{aligned} \tag{3}$$

and

$$x(k) = x1(k) + [x(k-1) - x1(k)](g_1(k) - 1)(g_2(k) - 1) \tag{4}$$

where *sgn(·)* is the sign function, and the switching function *g(x)* is defined as:

$g(x) = \begin{cases} 0, & x \geq 0 \\ 1, & x < 0 \end{cases}$, moreover, $\triangle u(k) = u(k) - u(k-1)$,

$g_1(k) = g\left(\dfrac{x(k-1)}{m_1} + c_1 - u(k)\right)$, and

$g_2(k) = g\left(u(k) - \dfrac{x(k-1)}{m_2} + c_2\right)$.

Substitute (4) and (3) into (1), it leads to

$$\begin{aligned} y(k) = &-\sum_{i=1}^{n_a} a_i y(k-i) + m_1 u(k-d) + (m_2 - m_1) g(\Delta u(k-d)) u(k-d) \\ &- 0.5 c_1 m(k-d)[\mathrm{sgn}(\Delta u(k-d)) + 1] g_1(k-d) + \\ &0.5 c_2 m(k-d)[-\mathrm{sgn}(\Delta u(k-d)) + 1] g_2(k-d) + \\ &[x(k-1-d) - x1(k-d)](g_1(k-d) - 1)(g_2(k-d) - 1) \\ &+ \sum_{j=1}^{n_b} b_j x(k-j-d) \end{aligned} \quad .(5)$$

Let

$$\begin{aligned} y_c(k) = y(k) - [x(k-1-d) - x1(k-d)](g_1(k-d) - 1) \\ (g_2(k-d) - 1) \end{aligned} \quad . \tag{6}$$

Eqs.(2)-(6)constitutes the novel special form of the pseudo-Hammerstein model with backlash.

III. BRIEF DESCRIPTION OF INTERNAL MODEL CONTROL

Based on the pseudo-Hammerstein model with backlash, an internal model control strategy is developed in this section. Fig. 3 illustrates the architecture of the IMC. In Fig. 3, P is the controlled plant involved with backlash, while G represents the above-mentioned pseudo-Hammerstein model with backlash; the controller in the structure consists of two filters, i.e. F_1 and F_2 respectively as well as G^{-1}, i.e. the inverse of G. Q is an available operation. r is the reference trajectory, u is the output of the controller, and $\hat{u}$ is the pseudo-output of the controller.

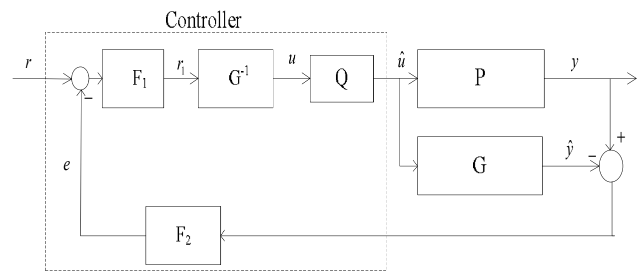

Figure 3. The architecture of the IMC

Define the model mismatch as $\eta(z^{-1})$ where $|\eta(z^{-1})| \leq \eta_M$, $\eta_M > 0$. z^{-1} is the unit back-shift operator, i.e. $z^{-1}x(k) = x(k-1)$ .Then, the controlled plant can be described by:

$$P(z^{-1}) = (1 + \eta(z^{-1})) G(z^{-1}) . \tag{7}$$

From Fig.3, it can be obtained that

$$e(k) = F_2(z^{-1})(y(k) - \hat{y}(k)) \tag{8}$$

$$\hat{u} = Q(u) = u + D(c) \tag{9}$$

where

$$D(c) = \begin{cases} c_1, & g_3(k) = 1 or (g_3(k) + g_4(k) = 0 and g_3(k-d_1) = 1) \\ -c_2, & g_4(k) = 1 or (g_3(k) + g_4(k) = 0 and g_4(k-d_1) = 1) \end{cases};$$

$g_3(k)$ and $g_4(k)$ will be explained in section 4; d_1 which is variable is transient delay from the linear zones to memory zones.

$$\hat{u}(k) = F_1(z^{-1}) G^{-1}(z^{-1})(r(k) - e(k)) . \tag{10}$$

$$y(k) = P(z^{-1}) \hat{u}(k) . \tag{11}$$

Based on (8)-(11), it has:

$$y(k) = F_1(z^{-1}) G^{-1}(z^{-1}) P(z^{-1})[\mathrm{r(k)} - \mathrm{e(k)}] . \tag{12}$$

According to (7), (11) can be rewritten as:

$$\begin{aligned} y(k) = [1 + F_2(z^{-1}) F_1(z^{-1}) \eta(z^{-1})]^{-1} \\ F_1(z^{-1}) G^{-1}(z^{-1}) P(z^{-1}) r(k) \end{aligned} . \tag{13}$$

If $G(z^{-1})$ is non-minimum phase, then (9) will be unstable due to $G(z^{-1})$ is un-invertible. Thus, the obtained controller will not be implemented. Thus, $G(z^{-1})$ can be divided into $G_+(z^{-1})$ and $G_-(z^{-1})$, i.e.

$$G(z^{-1})=G_+(z^{-1})G_-(z^{-1}) \tag{14}$$

where $G_+(z^{-1})$ represents the un-invertible part of the model where all the zeros located outside the unit circle, and $G_-(z^{-1})$ is the remained stable zeros and poles of the model. Therefore, the control strategy can be approximately designed as

$$\hat{u}(k)=F_1(z)G_-^{-1}(z)G_+^{-1}(1)(r(k)-e(k)) \tag{15}$$

where $G_+^{-1}(1)=\lim_{z\to 1}G_+^{-1}(z^{-1})$.

In this case the corresponding model mismatch can be approximately represented by

$$\eta_m=[P(z^{-1})-G(z^{-1})]G_-^{-1}(z^{-1})G_+^{-1}(1). \tag{16}$$

In order to guarantee the robust stability of the control system, the filters, i.e. F_1 and F_2 should be selected to satisfy the following conditions [18]:

$$\| F_2(e^{-j\omega T})F_1(e^{-j\omega T})\eta_m(e^{-j\omega T})\|<1,\quad \omega\in(-\pi,\pi). \tag{17}$$

As the controller contains the inverse model of the controlled plant, the inverse model of the pseudo-Hammerstein model with backlash will be discussed in the following section.

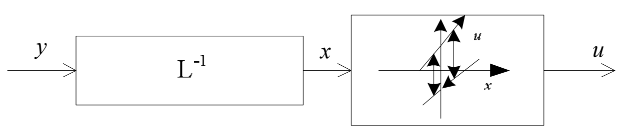

Figure 4. The architecture of the inverse pseudo-Hammerstein model with backlash

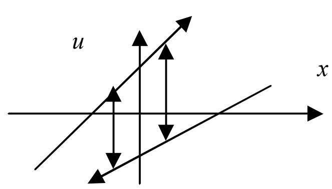

Figure 5. The inverse backlash

IV. THE INVERSE MODEL OF THE PSEUDO-HAMMERSTEIN SYSTEMS WITH BACKLASH

Suppose the inverse model of the linear subsystem in the pseudo-Hammerstein model to be denoted by L^{-1}, i.e. the linear subsystem in the inverse model, while the inverse model of the backlash subsystem in the pseudo-Hammerstein model to be denoted by N^{-1}, i.e. the nonlinear subsystem in the inverse model. Assume both L^{-1} and N^{-1} exit. Then, $(NL)^{-1}=L^{-1}N^{-1}$. The architecture of the inverse model is shown in Fig.4. The inverse model of the backlash is shown in Fig. 5 [3].

Based on (1), L^{-1} can be written as

$$x(k-d)=-\sum_{j=1}^{n_b}b_j x(k-j-d)+y(k)+\sum_{i=1}^{n_a}a_i y(k-i). \tag{18}$$

Considering the case where $d=1$ and (14) and (15), (18) can be rewritten as:

$$x(k)=-\sum_{j=1}^{n_b}b_j x(k-j)+y(k)+\sum_{i=1}^{n_a}a_i y(k-i). \tag{19}$$

Based on (2)-(4) and the key term separation principle, the discrete-time model of the inverse backlash is rewritten as

$$\begin{aligned} q1(k)&=(m_1)^{-1}x(k)+((m_2)^{-1}-(m_1)^{-1})g(\Delta x(k))x(k)\\ &\quad +c_1g_3(k)-c_2g_4(k) \end{aligned} \tag{20}$$

and

$$u(k)=q1(k)+[u(k-1)-q1(k)](g_3(k)-1)(g_4(k)-1) \tag{21}$$

where $\Delta x(k)=x(k)-x(k-1)$; $g_3(k)=\begin{cases}1, & x(k)>x(k-1)\\ 0, & else\end{cases}$ and $g_4(k)=\begin{cases}1, & x(k)<x(k-1)\\ 0, & else\end{cases}$.

Substituting (19) into (20) based on the key term separation principle, it leads to:

$$\begin{aligned} q1(k)&=(m_1)^{-1}[-\sum_{j=1}^{n_b}b_j x(k-j)+y(k)+\sum_{i=1}^{n_a}a_i y(k-i)]\\ &\quad+[(m_2)^{-1}-(m_1)^{-1}]g(\Delta x(k))x(k)+c_1g_3(k)-c_2g_4(k) \end{aligned}. \tag{22}$$

Re-substituting (22) into (21), it results in

$$\begin{aligned} u(k)&=(m_1)^{-1}[-\sum_{j=1}^{n_b}b_j x(k-j)+y(k)+\sum_{i=1}^{n_a}a_i y(k-i)]\\ &\quad+[(m_2)^{-1}-(m_1)^{-1}]g(\Delta x(k))x(k)+c_1g_3(k)-c_2g_4(k)\\ &\quad+[u(k-1)-q1(k)](g_3(k)-1)(g_4(k)-1) \end{aligned}. \tag{23}$$

Eqs.(19)-(23) constitute the inverse model of the pseudo-Hammerstein system with backlash.

Note that switch functions $g_1(k)$, $g_2(k)$, $g_3(k)$ and $g_4(k)$ in the model and the inverse model of the pseudo-Hammerstein system with backlash cannot be calculated directly, the internal variables, i.e. $m(k)$, $x1(k)$, $x(k)$ and $q1(k)$ are actually un-measurable. However, they can be predicted based on the previously estimated results. In this paper, the recursive general identification algorithm (RGIA) [17] can be applied to the identification of both the model and the corresponding inverse model of the pseudo-Hammerstein system with backlash. As the inverse model of the pseudo-Hammerstein model with backlash can be described as (19)-(23), the inverse model can also be obtained based on the pseudo-Hammerstein model. Fig.6 illustrates that the obtained inverse model derives the satisfactory compensation result.

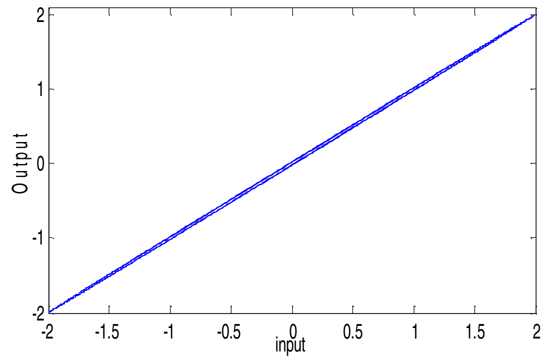

Figure 6. The validation of the inverse model of the pseudo-Hammerstein system with backlash

V. INTERNAL MODEL CONTROL BASED ON PSEUDO-HAMMERSTEIN MODEL WITH BACKLASH

From Fig. 3 and the model as well as the inverse model of the pseudo-Hammerstein system with backlash, the corresponding internal model control strategy can be obtained as follows:

$$\mathrm{y(k) = P(\hat{u}(k))} \tag{24}$$

$$\mathrm{\hat{y}(k) = LN(\hat{u}(k))} \tag{25}$$

$$\mathrm{e(k) = F_2[y(k) - LN(\hat{u}(k))]} \tag{26}$$

and

$$\mathrm{u(k) = N^{-1}L^{1}F_1[r(k) - e(k)]}\,. \tag{27}$$

The above-mentioned switch functions, i.e. $g_1(k)$, $g_2(k)$, $g_3(k)$ and $g_4(k)$ are implemented to switch the model between the linear zones and memory zones.

Both filters, F_1 and F_2 can be chosen as

$$\mathrm{F_1 = \frac{1-f_1}{1-f_1 z^{-1}}},\quad \mathrm{0 < f_1 < 1} \tag{28}$$

and

$$\mathrm{F_1 = \frac{1-f_2}{1-f_2 z^{-1}}},\quad \mathrm{0 < f_2 < 1}\quad . \tag{29}$$

Base on (17), the proper selected filtering parameters f_1 and f_2 can ensure the robust stability and the performance of the control system when the model mismatch exists. If the filtering parameters approach 1, the control system will demonstrate the robustness to tolerate the existence of larger model mismatch. However, it may also result in sluggish response suppose the filtering parameters to be closed to one. On the hand, if the filtering parameters are chosen to be far away from one, the control system may illustrate the fast response with serious oscillation. Therefore, a proper combination of the filtering parameters, i.e. f_1 and f_2 can ensure the control system to obtain both robust stability and the good response performance.

VI. SIMULATION

In this section, the proposed method is used for controlling a system described by the pseudo-Hammerstein model with backlash. Suppose that the parameters of the backlash in the controlled system are: $m_1 = 1$, $m_2 = 1.5$, $c_1 = 0.5$ and $c_2 = 1$; the linear dynamic subsystem is represented as:

$$\begin{aligned} y(k) = &-0.5y(k-1) - 0.35y(k-2) \\ &+ x(k-1) + 0.4x(k-2) \end{aligned}.$$

Suppose the obtained the parameters of the backlash model are $m_1 = 0.9799$, $m_2 = 1.4786$, $c_1 = 0.47934$ and $c_2 = 0.9696$. The corresponding the linear dynamic sub-model is:

$$\begin{aligned} \hat{y}(k) = &-0.4792\hat{y}(k-1) - 0.3688\hat{y}(k-2) \\ &+ \hat{x}(k-1) + 0.4188\hat{x}(k-2) \end{aligned}.$$

Then, the corresponding filters are respectively chosen as:

$$F_1(z) = \frac{(1-0.457)}{1-0.457z^{-1}}$$

and

$$F_2(z) = \frac{(1-0.996)}{1-0.996z^{-1}}\,.$$

Then, the corresponding IMC strategy is obtained. In order to make comparison, the PID control strategy is also applied to this system. The parameters of the PID controller are:

$K_p = 0.455$; $K_I = 0.1778$; and $K_d = 0.1838$.

When the reference trajectory is a sin-wave, both system responses respectively controlled by the proposed IMC and the PI strategies are shown in Fig.7 (a). The corresponding control signals are shown in Fig.7 (b). Moreover, the controlled system errors are illustrated in Fig.7(c). From Fig. 7, it is noted that both control strategies can handle the system with backlash. However, the proposed IMC method has obtained better control performance. From Fig. 7(c), we know that the PID control approach leads to larger dynamic error comparing with the proposed IMC method.

Also, the above-mentioned two control strategies are applied to the system when the reference trajectory is a square-wave. The corresponding control responses are shown in Fig.8 (a). The corresponding control signals are shown in Fig.8 (b). The simulation results show that the proposed IMC method has derived faster and better control performance than that of the PID control method.

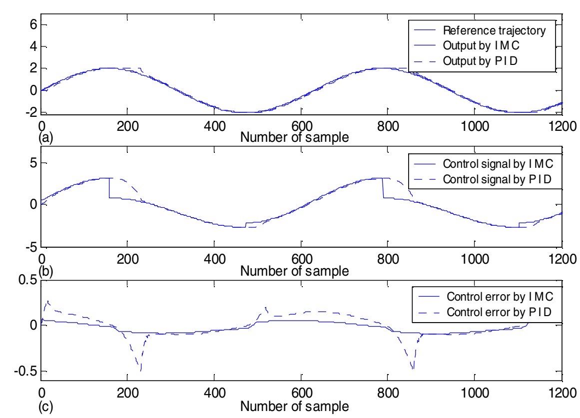

Figure 7. Simulation results; (a) closed-loop responses; (b) control signal ; (c) control errors

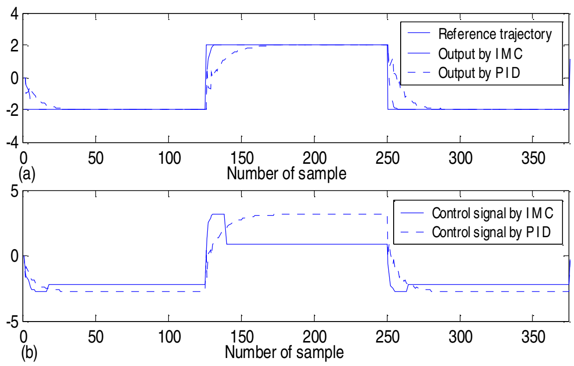

Figure 8. Simulation results; (a) closed-loop responses; (b) control signal

VII. CONCLUSION

An IMC scheme based on the pseudo-Hammerstein model with backlash is proposed in this paper. The novel inverse model of the pseudo-Hammerstein model is obtained. Moreover, choosing proper filters of the controller can ensure the robustness of the control system and make the system accurately track the expected reference signals. Finally the simulation results have illustrated the efficiency of the proposed method.

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