A Delay-Dependent Stability Criterion for Neural Networks with Interval Time-varying Delays

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Abstract—**This paper presents a new result of stability analysis for neural networks with interval time-varying delays. A less conservative stability criterion is established by constructing a new Lyapunov-Krasovskii functional and introducing some free weighting matrices. Numerical examples show that the proposed criterion is effective and is an improvement over some existing results in the literature.**

Keywords—**stability, interval time varying delays, linear matrix inequality (LMI), neural networks**

I. INTRODUCTION

In recent years, the study of neural networks has attracted considerable attention since it plays an important role in applications such as classification of patterns, associative memories and optimization. However, in neural processing and signal transmission, significant time delays as a source of instability and bad performance may occur. Therefore, there has been a growing research interest on the stability analysis problems for delayed neural networks, and a large amount of literature has been available, see [1-8,10-14] for some recent results.

Stability criteria for delay neural networks can be classified into two categories: delay-independent [1,2,4] and delay-dependent criteria [3,5-8,10-13].Delay-independent criteria do not employ any information on the size of the delay; while delay-dependent criteria make use of such information at different levels. Delay-dependent stability conditions are generally less conservative than delay-independent ones especially when the delay is small.

Recently, a special type of time delay in practical engineering systems, i.e., interval time-varying delay, is identified and investigated [11]. Interval time-varying delay is a time delay that varies in an interval in which the lower bound is not restricted to be 0. It is worth noting that these stability criteria in [11] leave much room for improvement. A significant source of conservativeness that could be further reduced lies in the calculation of the time-derivative of the Lyapunov–Krasovskii functional. To the best of our knowledge, very few papers investigate the stability problem of neural networks with interval time-varying delays, which remains open but challenging. Therefore, it is of great significance to consider the stability of neural networks with interval time-varying delays.

In this paper, we study the stability problem for neural networks with interval time-varying delays by choosing an appropriate Lyapunov functional. A delay-dependent stability criterion is derived based on the new Lyapunov functional and the consideration of range for the time-delay. The resulting criterion is applicable to both fast and slow time-varying delays. Finally, numerical examples are given to demonstrate the effectiveness and the merit of the proposed method.

Notations: The notations used throughout the paper are fairly standard. The superscript " T " stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $P>0$ means that P is real symmetric and positive definite; 0 represents zero matrix. In symmetric block matrices or long matrix expressions, we use an asterisk ($\star$) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. PROBLEM FORMULATION

Consider the following neural networks with interval time-varying delays:

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-d(t))) + u, \qquad (1)$$

where $x(t)=[x_1(t), x_2(t), ..., x_n(t)]^T$ is the neural state vector, $g(x(t))=[g_1(x_1(t)), g_2(x_2(t)), ..., g_n(x_n(t))]^T$ denotes the bounded neuron activation function with $g(0)=0$, $u=[u_1, u_2, \ldots, u_n]^T$ is a constant input vector. $C=diag[c_1, c_2, ..., c_n]>0$, $A=(a_{ij})_{n\times n}, B=(b_{ij})_{n\times n}$ are the inter connection matrices representing the weight coefficients of the neurons,

The time-varying delay $d(t)$ satisfies

$$0 \le h_1 \le d(t) \le h_2, \quad \dot{d}(t) \le \mu, \tag{2}$$

where h_1, h_2, μ are constants.

Remark 1. When, $\mu = 0$, $h_1 = h_2$ then $d(t)$ denotes a constant delay; the case when $h_1 = 0$, it implies that $0 \le d(t) \le h_2$ which is investigated in almost all the reported literature.

In the following, we always shift the equilibrium point x^* to the origin by transformation $z(\cdot) = x(\cdot) - x^*$ puts system (1) into the following form:

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t - d(t))), \tag{3}$$

where $z(t) = [z_1(t), z_2(t), ..., z_t(t)]^T$ is the state vector of the transformed system. With

$$f(z(t)) = f_1(z_1(t)), f_2(z_2(t)), ..., f_n(z_n(t))]^T,$$

and

$$f_j(z_j((t)) = g_j(z_j(t) + x_j^*) - g_j(x_j^*), j = 1, 2, \ldots, n.$$

Assumption.1 There exist constants F_j^+ and F_j^- such that

$$F_j^- \le \frac{f_j(z_j(t))}{z_j(t)} \le F_j^+, \quad j = 1, 2, ..., n \tag{4}$$

Remark 2. The constants in F_j^-, F_j^+, Assumption 1 are allowed to be positive, negative or zero. This type of activation function is clearly more general than the usual sigmoid activation functions and the recently commonly used Lipschitz conditions. Note that with such a milder assumption, the analysis methods developed in [11] cannot be applied directly.

The following lemma is useful in deriving our LMI-based stability criterion.

Lemma 1. [Schur complement]Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_{2,}^T$ then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, \quad or \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0. \tag{5}$$

III. MAIN RESULTS

For the convenience of presentation, we denote

$$F_1 = diag\left\{F_1^- F_1^+, F_2^- F_2^+, \ldots, F_n^- F_n^+\right\},$$

$$F_2 = diag\left\{-\frac{F_1^- + F_1^+}{2}, -\frac{F_2^- + F_2^+}{2}, \ldots, -\frac{F_n^- + F_n^+}{2}\right\}.$$

Then we are in the position to give the main result.

Theorem 1. For given scalars $0 \le h_1 < h_2$ and μ, the neural network (3) is asymptotically stable, if there exist matrices $P > 0, Q_r = Q_r^T \ge 0, r = 1, 2, 3, 4$, $Z_j = Z_j^T > 0, \ j = 1, 2,$ $N = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T \end{bmatrix}^T$, $S = \begin{bmatrix} S_1^T & S_2^T & S_3^T & S_4^T & S_5^T & S_6^T \end{bmatrix}^T$, $M = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T \end{bmatrix}^T$, $K = diag(k_1, k_2, \cdots k_i) > 0,$ $i = 1, 2, \cdots, n,$ such that the following LMI holds:

$$\begin{bmatrix} \Upsilon_1 + \Upsilon_2 + \Upsilon_2^T & h_2 N & \delta S & \delta M & W^T U \\ \star & -h_2 Z_1 & 0 & 0 & 0 \\ \star & \star & -\delta(Z_1 + Z_2) & 0 & 0 \\ \star & \star & \star & -\delta Z_2 & 0 \\ \star & \star & \star & \star & -U \end{bmatrix} < 0, \tag{6}$$

where

$$\Upsilon_1 = \begin{bmatrix} \Upsilon_{11} & 0 & 0 & 0 & \Upsilon_{15} & PB \\ \star & \Upsilon_{22} & 0 & 0 & 0 & -F_2 H \\ \star & \star & -Q_1 & 0 & 0 & 0 \\ \star & \star & \star & -Q_2 & 0 & 0 \\ \star & \star & \star & \star & \Upsilon_{55} & KB \\ \star & \star & \star & \star & \star & \Upsilon_{66} \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} N_1 & -N_1 + S_1 - M_1 & M_1 & -S_1 & 0 & 0 \\ N_2 & -N_2 + S_2 - M_2 & M_2 & -S_2 & 0 & 0 \\ N_3 & -N_3 + S_3 - M_3 & M_3 & -S_3 & 0 & 0 \\ N_4 & -N_4 + S_4 - M_4 & M_4 & -S_4 & 0 & 0 \\ N_5 & -N_5 + S_5 - M_5 & M_5 & -S_5 & 0 & 0 \\ N_6 & -N_6 + S_6 - M_6 & M_6 & -S_6 & 0 & 0 \end{bmatrix},$$

and

$$\Upsilon_{11} = -PC - C^T P + Q_1 + Q_2 + Q_3 - F_1 D,$$
$$\Upsilon_{15} = PA - C^T K - F_2 D,$$
$$\Upsilon_{22} = -(1 - \mu) Q_3 - F_1 H,$$
$$\Upsilon_{55} = Q_4 + KA + A^T K - D,$$
$$\Upsilon_{66} = -(1 - \mu) Q_4 - H,$$
$$U = h_2 Z_1 + \delta Z_2,$$
$$\delta = h_2 - h_1.$$

Proof. The Lyapunov functional of system (3) is defined by:

$$V(z(t)) = \sum_{i=1}^{4} V_i(x(t))$$

$$V_1(z(t)) = z^T(t) P z(t) + 2\sum_{i=1}^{n} k_i \int_0^{z_i} f_i(s) ds$$

$$V_2(z(t)) = \int_{t-h_1}^{t} z^T(s) Q_1 z(s) ds + \int_{t-h_2}^{t} z^T(s) Q_2 z(s) ds$$

$$V_3(z(t)) = \int_{t-d(t)}^{t} \left[z^T(s) Q_3 z(s) + f^T(z(s)) Q_4 f(z(s)) \right] ds$$

$$V_4(z(t)) = \int_{-h_2}^{0} \int_{t+\theta}^{t} \dot{z}^T(s) Z_1 \dot{z}(s) ds d\theta + \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \dot{z}^T(s) Z_2 \dot{z}(s) ds d\theta.$$

where $P > 0, Q_r = Q_r^T \ge 0, r = 1, 2, 3, 4$ and $Z_j = Z_j^T > 0, \ j = 1, 2,$ are to be determined. From the Leibniz–Newton formula, the following equations are true for any matrices N, S and M with appropriate dimensions,

$$\Lambda_1 = 2\xi^T(t)N\left[z(t) - z(t-d(t)) - \int_{t-d(t)}^{t} \dot{z}(s)ds\right] = 0, \quad (7)$$

$$\Lambda_2 = 2\xi^T(t)S\left[z(t-d(t)) - z(t-h_2) - \int_{t-h_2}^{t-d(t)} \dot{z}(s)ds\right] = 0, \quad (8)$$

$$\Lambda_3 = 2\xi^T(t)M\left[z(t-h_1) - z(t-d(t)) - \int_{t-d(t)}^{t-h_1} \dot{z}(s)ds\right] = 0, \quad (9)$$

It can be derived from Assumption 1 that

$$(f_j(z_j(t)) - F_j^- z_j(t))(f_j(z_j(t)) - F_j^+ z_j(t)) \le 0, \quad (10)$$

$$\begin{aligned}&(f_j(z_j(t-d(t))) - F_j^- z_j(t-d(t)))\\ &\times(f_j(z_j(t-d(t))) - F_j^+ z_j(t-d(t)))) \le 0,\end{aligned} \quad (11)$$

which are, respectively, equivalent to

$$\begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix}^T \begin{bmatrix} F_j^- F_j^+ e_j e_j^T & -\frac{F_j^+ + F_j^-}{2} e_j e_j^T \\ -\frac{F_j^+ + F_j^-}{2} e_j e_j^T & e_j e_j^T \end{bmatrix} \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix} \le 0, \quad (12)$$

$$\begin{aligned}&\begin{bmatrix} z(t-d(t)) \\ f(z(t-d(t))) \end{bmatrix}^T \begin{bmatrix} F_j^- F_j^+ e_j e_j^T & -\frac{F_j^+ + F_j^-}{2} e_j e_j^T \\ -\frac{F_j^+ + F_j^-}{2} e_j e_j^T & e_j e_j^T \end{bmatrix}\\ &\times \begin{bmatrix} z(t-d(t)) \\ f(z(t-d(t))) \end{bmatrix} \le 0,\end{aligned} \quad (13)$$

where e_r the unit column vector having one element on its r th row and zeros elsewhere. Here we denote $D = diag\{d_1, d_2, \ldots, d_n\}, H = diag\{h_1, h_2, \ldots, h_n\}$.

Calculating the derivative of $V(z(t))$ along the solutions of system (3) and Combining (7)–(9) and adding the terms on the left side of (12)-(13) into it

$$\begin{aligned}
\dot{V}(x(t)) \le& -2z^T(t)PCz(t) + 2z^T(t)PAf(z(t)) \\
&+ 2z^T(t)PBf(z(t-d(t))) \\
&- 2f^T(z(t))KCz(t) + 2f^T(z(t))KAf(z(t)) \\
&+ 2f^T(z(t))KBf(z(t-(t))) + z^T(t)(Q_1+Q_2)z(t) \\
&- z^T(t-h_1)Q_1 z(t-h_1) - z^T(t-h_2)Q_2 z(t-h_2) \\
&+ z^T(t)Q_3 z(t) + f^T(z(t))Q_4 f(z(t)) \\
&- (1-\mu)f^T(z(t-d(t)))Q_4 f(z(t-d(t))) \\
&- (1-\mu)z^T(t-d(t))Q_3 z(t-d(t)) \\
&+ \dot{z}^T(t)(h_2 Z_1 + \delta Z_2)\dot{z}(t) - \int_{t-d(t)}^{t} \dot{z}^T(s)Z_1\dot{z}(s)ds \\
&- \int_{t-h_2}^{t-d(t)} \dot{z}^T(s)(Z_1+Z_2)\dot{z}(s)ds - \int_{t-d(t)}^{t-h_1} \dot{z}^T(s)Z_2\dot{z}(s)ds + \sum_{i=1}^{3}\Lambda_i \\
&- \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix}^T \begin{bmatrix} F_1 D & F_2 D \\ F_2 D & D \end{bmatrix} \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix} \\
&- \begin{bmatrix} z(t-d(t)) \\ f(z(t-d(t))) \end{bmatrix}^T \begin{bmatrix} F_1 H & F_2 H \\ F_2 H & H \end{bmatrix} \begin{bmatrix} z(t-d(t)) \\ f(z(t-d(t))) \end{bmatrix} \\
\le& \xi^T(t)\left[\Upsilon_1 + \Upsilon_2 + \Upsilon_2^T + W^T(h_2 Z_1 + \delta Z_2)W + h_2 N Z_1^{-1} N^T \right.\\
&\left. + \delta M Z_2^{-1} M^T + \delta S(Z_1+Z_2)^{-1} S^T\right]\xi(t) \\
&- \int_{t-d(t)}^{t} [\xi^T(t)N + \dot{z}^T(s)Z_1]Z_1^{-1}[N^T\xi(t) + Z_1\dot{z}(s)]ds \\
&- \int_{t-d(t)}^{t-h_1} [\xi^T(t)M + \dot{z}^T(s)Z_2]Z_2^{-1}[M^T\xi(t) + Z_2\dot{z}(s)]ds \\
&- \int_{t-h_2}^{t-d(t)} [\xi^T(t)S + \dot{z}^T(s)(Z_1+Z_2)](Z_1+Z_2)^{-1} \\
&\times [S^T\xi(t) + (Z_1+Z_2)\dot{z}(s)]ds
\end{aligned} \quad (14)$$

where

$$\xi(t) = \begin{bmatrix} z(t) \\ z(t-d(t)) \\ z(t-h_1) \\ z(t-h_2) \\ f(z(t)) \\ f(z(t-d(t))) \end{bmatrix}, N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{bmatrix}, S = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix}, J = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix},$$

$W = [-C \quad 0 \quad 0 \quad 0 \quad A \quad B], \delta = h_2 - h_1$, and Υ_1, Υ_2 are defined in Theorem 1.

Since $Z_j > 0, j = 1, 2$, then the last three parts in (14) are all less than 0. So, if

$$\Upsilon_1 + \Upsilon_2 + \Upsilon_2^T + W^T(h_2 Z_1 + \delta Z_2)W + h_2 N Z_1^{-1} N^T + \delta M Z_2^{-1} M^T + \delta S(Z_1+Z_2)^{-1} S^T < 0$$

which is equivalent to (6) by Schur complements, then $\dot{V}(z(t)) < -\varepsilon\|z(t)\|^2$ for a sufficiently small $\varepsilon > 0$ and $z(t) \ne 0$, which ensures the asymptotic stability of system (3), see e.g. [9]. The proof is completed.

Remark 3. Ref.[11] has proposed some delay-dependent stability criteria for neural networks with fast time-varying interval delay. However, when estimating the upper bound of the time-derivative of the Lyapunov-Krasovskii functional, the terms $-\int_{t-\tau_0-\delta}^{t-\tau(t)} \dot{x}^T(s)R_2 x(s)ds - \int_{t-\tau_0}^{t-\tau_0+\delta} \dot{x}^T(s)R_2 x(s)ds$ and $-\int_{t-\tau_0-\delta}^{t-\tau_0} \dot{x}^T(s)R_2 x(s)ds - \int_{t-\tau(t)}^{t-\tau_0+\delta} \dot{x}^T(s)R_2 x(s)ds$ in the derivative of $V_3(t)$ are ignored. This may bring conservativeness. In our Theorem 1, none of useful terms is ignored. In addition, the results in [11] are only applicable to neural networks with fast time-varying delay. In fact, in many cases, the derivative of time-varying delays is known and may be small. Thus, the results in [11] may have limited use. In our Theorem 1, μ can be any value or unknown. Therefore, Theorem 1 is applicable to both cases of fast and slow time-varying delays.

IV. NUMERICAL EXAMPLES

In this section, two examples are given to show the effectiveness and less conservativeness of our results.

Example 1. Consider the following neural networks with interval time-varying delays, borrowed from [11]:

$$C=\begin{bmatrix}0.7 & 0\\ 0 & 0.7\end{bmatrix}, A=\begin{bmatrix}-0.3 & 0.3\\ 0.1 & -0.1\end{bmatrix}, B=\begin{bmatrix}0.1 & 0.1\\ 0.3 & 0.3\end{bmatrix},$$

μ is unknown, and the activation functions are as follows:

$$f_1(x)=\left[|x+1|-|x-1|\right]/2,\ f_2(x)=f_1(x),$$

which means that

$$F_1^-=0, F_1^+=1, F_2^-=0, F_2^+=1.$$

The calculation results obtained by Theorem 1 in this paper, Theorem 1 in [10] and Theorem I in [11] for different cases of h_1 and unknown μ are listed in Table 1, in which "—"means that the results are not applicable to the corresponding cases. When $h_1=0$, it is clear that our results are improvement over those in [10] and [11]. On the other hand, for neural networks with interval time-varying delays in a range, Table 1 also lists the comparison between our results and those in [11] and shows the merit of Theorem 1.

Example 2. Consider the following neural networks with interval time-varying delays:

$$C=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}, A=\begin{bmatrix}0.7 & 0.8\\ -0.5 & 0.3\end{bmatrix}, B=\begin{bmatrix}0.2 & 0.2\\ -0.6 & 0.1\end{bmatrix},\ \mu=1.5$$

and the activation functions are as follows:

$$f_1(x)=\tanh(0.6x)-0.2\sin x, f_2(x)=\tanh(-0.4x),$$

which means that

$$F_1^-=-0.2, F_1^+=0.8, F_2^-=-0.4, F_2^+=0,$$

By Theorem 1, the maximum allowable upper delay bounds for different levels of lower delay bounds h_1 are listed in Table II, in which "—"means that the results are not applicable to the corresponding cases. Therefore, our method is less conservative in some degree than that in [11].

TABLE I. CALCULATED DELAY BOUNDS h_2 FOR GIVEN h_1

	Calculated delay bounds h_2 for given h_1 and unknown μ		
	[10]	**[11]**	**Theorem 1**
$h_1=0$	0.2916[a]	2.3297	2.7104
$h_1=0.1$	—	2.4297	2.5297
$h_1=0.2$	—	2.7315	2.7537
$h_1=0.3$	—	2.6297	2.7772
$h_1=0.4$	—	2.7297	2.8071
$h_1=0.5$	—	2.8297	2.8603

TABLE II. THE MAXIMUM ALLOWABLE UPPER BOUND OF h_2

	The maximum allowable upper bound of h_2			
	$h_1=0$	$h_1=0.1$	$h_1=0.3$	$h_1=0.5$
[11]	—	—	—	—
Theorem 1	4.6214	4.6473	4.8024	5.0011

V. CONCLUSIONS

The stability problem for neural networks with interval time-varying delays is considered. Based on the Lyapunov-Krasovskii functional approach, a delay-dependent stability criterion is derived by introducing free weighting matrices, which are used to reduce the conservatism of the obtained criterion. Numerical examples are given to show the effectiveness of the method.

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