

# Research on the Nonlinear Governor of Diesel Engine with Variable Structure Control Theory

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**Abstract**—Diesel engine speed control system is one typical nonlinear system. In order to control speed of the single input nonlinear engine, the nonlinear model of the 4135 engine governor is established and the sliding mode variable structure controller is designed in the paper. Simulation in MATLAB has demonstrated that the sliding mode variable structure control is able to respond quickly, robust against system parameters and external disturbances, and able to keep the system stable, obtain strong robustness. This paper solves the problem that for the conventional PID controller, the overshoot is too big, settling and rise times are too long.

**Keywords**—diesel engine speed control system, sliding mode variable structure controller, nonlinear

## I. INTRODUCTION

Variable structure control(VSC) is a synthesis method which belongs to the nonlinear control theory on the basis of phase-plane. Its basis principle is that system state variables reaches some switching surface firstly and then slide to the origin more and more on this surface. When the sliding movement has good quality, the purpose of control is accomplished[1]. The main characteristic of this kind of control is extremely strong robustness. That is, it is insensitive to the model error, system parameter variation and external disturbance[2].

For the general control system, the equation is given as

$$\dot{x} = f(x, u, t) \quad x \in R^n, \quad u \in R^m, \quad t \in R$$

Sliding mode controller is designed by determining switching function vector

$$s(x), \quad s \in R^m,$$

and seeking variable structure control

$$u(x) = \begin{cases} u^+(x), & s(x) > 0 \\ u^-(x), & s(x) < 0 \end{cases} \quad u^+(x) \neq u^-(x)$$

The following descriptions are required

- sliding mode exists, that is, meets  $\lim_{s \rightarrow 0} \frac{ds}{dt} \leq 0$ ;
- if the reaching condition is fulfilled, the phase-contour beyond the switching surface  $s$  will reach switching surface in limit time;

- switching surface is sliding mode region, whose sliding movement is more and more steady, and the system has good dynamic characteristic.

Therefore, main tasks of designing sliding mode controller are determining switching surface and designing control law. Above all, the structure of variable structure controller or the figure of switching surfaces and the form of switching functions should be confirmed.

Generally, we choose linear switching functions, whose figure means the control variables count [3], i.e.,

$$s(x) = cx, \quad s \in R^m, \quad c \in R^{m \times n}, \quad x \in R^n$$

and choose uniform velocity convergence law as the following form:

$$\dot{s} = -\varepsilon \operatorname{sgn}(s), \quad (\varepsilon > 0, \quad \varepsilon \in R^m)$$

or proportional convergence law

$$\dot{S} = -\varepsilon \operatorname{sgn}(S) - kS \quad \varepsilon, k > 0$$

or accelerating convergence law

$$\dot{S} = -k|S|^\alpha \operatorname{sgn}(S) \quad k > 0 \quad 0 < \alpha < 1$$

where  $\operatorname{sgn}(s) = \begin{cases} +1, & s > 0 \\ -1, & s < 0 \end{cases}$

Obviously, for a VSC system, the existence and the reaching conditions are stated as  $s\dot{s} < 0$ .

The fundamental difference between VSC control and the conventional control lies in the discontinuity of VSC control which is a kind of switching characteristic of making system structure change at any time[4]. The control characteristic can force system structure change purposefully by the way of jumping in the process of sudden changes according to system state (deviation and all differentials)at that time, and to do the upper and lower movements(sliding mode or sliding mode movement) of low range, high frequency along the fixed state orbit under certain condition. This kind of sliding mode can be designed, and have nothing to do with system parameters and disturbances. The characteristic of control is to make the system respond fast and to be robust extremely, i.e., to be not sensitive extremely to mode error, change of target parameters and perturbations outside.

## II. DIESEL ENGINE SPEED CONTROL SYSTEM MODEL

### A. Electronically Controlled Fuel System of the Engine

The speed of diesel engine depends on the fuel-injection amount. Development of the electronically controlled fuel system went through three generations. For the first generation of position control system, the electronic governor is used to replace mechanical or hydraulic governor on the basis of injection device that already exists. Usually, we get the linearized model of diesel engine at first. Then PID algorithm is employed which could be adjusted on line as long as the control parameter is confirmed. Especially under the steady-state, when meeting external strong disturbances, the control system overshoot is too big, settling and rise times are too long, which influences the whole dynamic and static performance of control system to raise. In addition, the saturation phenomenon of the integrator takes place in the process of starting up or the great dynamic regulation, at the same time, the overshoot of the control system is too big, recover time of steady-state lengthens. For the second generation of time controlled system, injection timing is determined by energize time of electromagnetic valve and the fuel-injection amount is determined by energize duration time of electromagnetic valve. Electronically controlled fuel system employs the supplying fuel principle of the traditional array and the assign pumping plunger which is that fuel is compressed by fuel pumping plunger driven by the camshaft of fuel injected pump which is driven by the diesel crankshaft. Then, the high-pressure pulse is produced which is conducted to the nozzle in the form of pressure wave to raise the needle valve. Thus, this kind of time controlled type injection system of the automatically controlled diesel engine, no matter which pump (electronically controlled single pump, or united pump-nozzle), their injection pressures depend on speed of the engine excessively. Then, when speed and load are low, its injection pressure is very low, and injecting many times is very difficult. The third generation of common rail system gets rid of constraint of cam, whose injection pressure is a kind of flexible control according to different diesel engine working mode. The pressure-time common rail diesel engine realizes not only precise control and regulation of the fuel-injection amount and the injection timing, but also precise regulation of injection pressure and flexible control of injection rule. However, no matter which generation production, its speed control algorithms are all PID and fuzzy- PID.

### B. The Nonlinear Model of the Speed Control System

The main function of the diesel engine speed control system is to regulate speed. Deviation of the speed of the diesel engine depends on the difference of its power torque and load torque directly. According to the power mechanical equation

$$J \frac{d\omega}{dt} = M - M_l$$

Where  $J$  is equivalent moment of inertia of total running parts including the diesel engine itself and mechanical parts driven by the engine.  $M$  and  $M_l$  are changeable complicated function.

The characteristic of the load torque  $M_l$  is either a function related to speed such as propeller characteristic in the stable rivers, vehicle characteristic on the steady road, etc., or a kind

of quantity with very strong random factor such as propeller load in heavy stormy waves, vehicle load on complicated road, etc. In order to make the controller meet the application in general condition, load can be regarded as a random disturbances. So, we can concentrate on discussing the characteristic of the output torque  $M$  of the diesel engine in the controller design.

The output torque  $M$  is torque of diesel engine in dynamic process. Considering that the output torque of dynamic process could be approximate to a series of quasi-steady state, we can omit details in working process of the diesel engine when studying the control strategy. Then through substituting the torque characteristic of the dynamic process with torque characteristic under the steady state of the diesel engine, we get the function form of the power characteristic:

$$p_{me} = f(n, q)$$

where  $p_{me}$  is mean effective pressure (M.E.P.),  $n$  is the speed of diesel engine,  $q$  is the fuel-injection amount every cycle. So, the output torque  $M$  can be expressed as:

$$M = M(n, q) = \frac{\partial M}{\partial n} dn + \frac{\partial M}{\partial q} dq + M_0 \quad (1)$$

If we linearize dynamic characteristics of the diesel engine near a certain balanced working point, i.e.,  $\partial M / \partial n$  and  $\partial M / \partial q$  are both constants, (1) approximates good near some point. But if we linearize in a large scope, the error at the moment is quite big. Thus, the nonlinear characteristic of the diesel engine's dynamic characteristic should be sufficiently considered. Omitting internal details of the diesel engine's working process, dynamic characteristic of diesel engine can employ nonlinear fitting, i.e.,  $\partial M / \partial n$  and  $\partial M / \partial q$  are both considered variables, and are functions of  $n$  and  $q$ .

Generally speaking, indicated thermal efficiency of the diesel engine  $\eta_i$  is a function which varies according to the speed of engine and load. But under the situation that speed is a constant (load characteristic), indicated thermal efficiency changes very tiny. Thus, we suppose that indicative thermal efficiency is a function of the diesel engine speed which is a constant when speed doesn't vary and establish:

$$p_{mi} = kq + p_{m0} \Big|_{n=const}$$

where  $k$  is a proportional constant which relates to indicated efficiency,  $p_{m0}$  is a constant. When the speed of diesel engine doesn't change, mechanical lost mean pressure can be regarded as a constant. Thus, M.E.P. is :

$$p_{me} = p_{mi} - p_m = kq + p_{m0} - p_m = kq + c \Big|_{n=const} \quad (2)$$

That is to say, relation between M.E.P. and the fuel injected every cycle can be considered to be linear. Fig. 1 lists the relation between fuel injected every cycle and M.E.P. for several speeds. However, for every speed, the slope of the fitting straight line is different, and interception between the fitting with straight line and column is different, too, which are because that  $k$  and  $c$  in (2) are both parameters relate to the

diesel engine speed. And we find that  $k$  and  $C$  can expressed in degree-2 polynomial form of speed, i.e., the method is polynomial approximation. Then its dynamic characteristic can be written in a polynomial of degree two as follows:

$$p_{me} = b_1 n^2 q + b_2 n^2 + b_3 n q + b_4 n + b_5 q + b_6$$

where  $p_{me}$  is M.E.P. (KPa),  $n$  is the speed of diesel engine(r/min),  $q$  is the fuel-injection amount every cycle(g),  $b_i$  is a constant.

According to relation between M.E.P. and the output torque, we can get:

$$M = a_1 n^2 q + a_2 n^2 + a_3 n q + a_4 n + a_5 q + a_6 \quad (3)$$

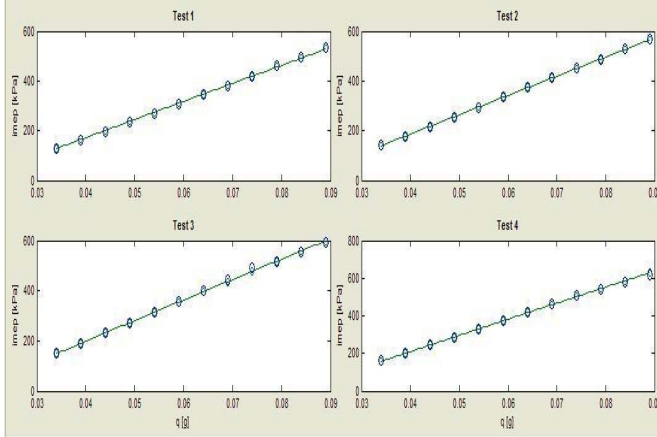


Figure1. Relation between M.E.P. and the fuel-injection amount

Compared (1) with (3), it shows that  $\partial M / \partial n$  and  $\partial M / \partial q$  are functions of speed  $n$  in (3).

$$\frac{\partial M}{\partial n} = a_2 n + a_4, \quad \frac{\partial M}{\partial q} = a_1 n^2 + a_3 n + a_5$$

Then the diesel engine dynamic characteristic formula is as follows when working condition varies in large scope.

$$2\pi J \frac{dn}{dt} = a_1 n^2 q + a_2 n^2 + a_3 n q + a_4 n + a_5 q + a_6 - M_l \quad (4)$$

Equation(4) is not only a typical polynomial nonlinear differentiating equation, but also a typical single input imitative nonlinear system(relation between the state equation and  $u$  is linear).

$$\frac{d}{dt} x = f(x) + g(x)u \quad x \in R^n \quad u \in R$$

Generally, we employ the mean value model which is the average of every cycle to describe engine working process when studying engine. Then regard the engine speed control system as a single input nonlinear system which is the same as that the fuel injected is input and speed  $n$  is state variable. Thus, we can choose reasonable switching surface and control law of VSC to get VSC control model of nonlinear engine speed control system.

### III. SIMULATION MODEL

In this paper, the vehicle's road characteristic is chose as the characteristic of  $M_l$

$$M_l = (1+r)kn^2$$

Where  $k$  is a constant,  $r$  is a random disturbances.

If the boundaries of uncertainties [5]  $r$  is known,

letting

$$r = 0.1 \sin(2\pi t),$$

we can get the following mathematical model as (5) according to (4):

$$\dot{n} = \{d_2 - [1 + 0.1 \sin(2\pi t)]k'\}n^2 + d_4 n + d_6 + (d_1 n^2 + d_3 n + d_5)q \quad (5)$$

Choosing the state variables as follows:

$$x_1 = \int_0^t e dt, \quad x_2 = e = n_0 - n,$$

where  $n_0$  is the target speed of diesel engine,  $n$  is the actual speed,  $e$  is the error of speed.

then the state equation of the original system can be written as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\{d_2 - [1 + 0.1 \sin(2\pi t)]k'\}n^2 - d_4 n - d_6 - (d_1 n^2 + d_3 n + d_5)u \end{cases} \quad (6)$$

Choosing the following switching function for (6):

$$s = cx_1 + x_2$$

then it should meet the following sliding mode equation as long as the system enters the sliding mode:

$$s = cx_1 + x_2 = \dot{x}_1 + cx_1 = 0$$

which is solved by :

$$x_1(t) = x_1(0)e^{-ct}$$

Obviously, only when  $c > 0$ , the aforesaid is steady. Thus, the VSC system is steady too. At the same time, we can get the following equivalent control according to  $ds/dt = 0$ .

$$u_{eq}(x) = \frac{-\{d_2 - [1 + 0.1 \sin(2\pi t)]k'\}n^2 - d_4 n - d_6 + cx_2}{d_1 n^2 + d_3 n + d_5}$$

Choosing proportional convergence law:

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - ks, \quad \varepsilon, k > 0$$

we can confirm the structure of the sliding mode VSC controller:

$$\begin{cases} u^+ = \frac{-\{d_2 - [1 + 0.1 \sin(2\pi t)]k'\}n^2 - d_4 n - d_6 + cx_2 + \varepsilon + ks}{d_1 n^2 + d_3 n + d_5} & s > 0 \\ u^0 = \frac{-\{d_2 - [1 + 0.1 \sin(2\pi t)]k'\}n^2 - d_4 n - d_6 + cx_2}{d_1 n^2 + d_3 n + d_5} & s = 0 \\ u^- = \frac{-\{d_2 - [1 + 0.1 \sin(2\pi t)]k'\}n^2 - d_4 n - d_6 + cx_2 - \varepsilon + ks}{d_1 n^2 + d_3 n + d_5} & s < 0 \end{cases}$$

If parameters are chose as follows for 4135 diesel engine, simulating (6) as Fig. 2 and Fig. 3, and choosing disturbances as  $r = \sin(2\pi t)$ , we can get simulation result as shown in Fig. 4:

$$d_1 = -0.0037 \quad d_2 = -0.00014 \quad d_3 = 10.4864$$

$$d_4 = -0.1482 \quad d_5 = 7406.3 \quad d_6 = -146.5449$$

$$c = 24 \quad \varepsilon = 3 \quad k = 1.28 \quad k' = 0.00007$$

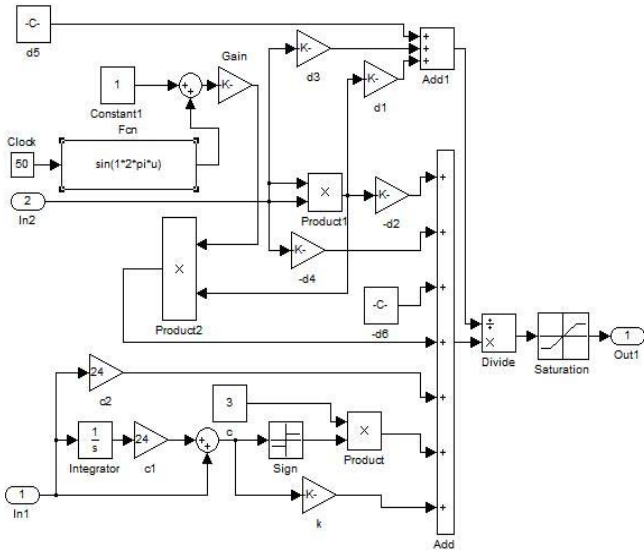


Figure 2. VSC controller structure of nonlinear speed control system

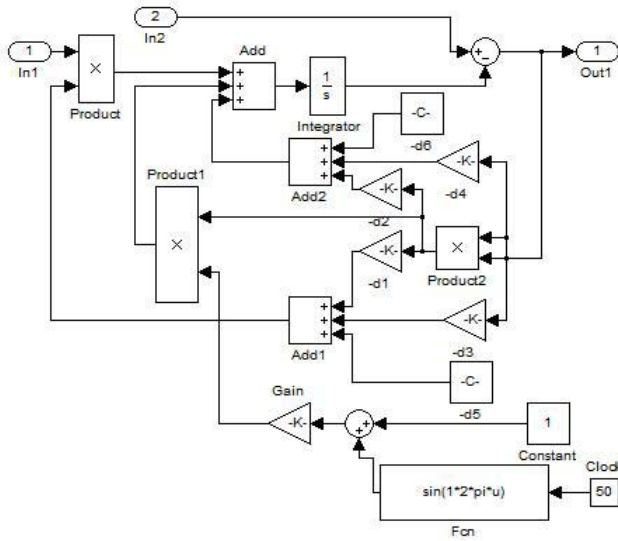


Figure 3. Model of nonlinear speed control system

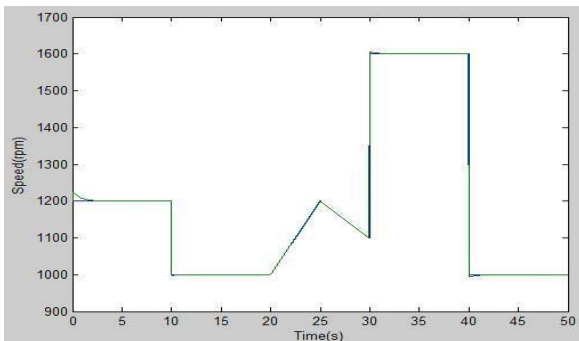


Figure 4. The simulation result of the VSC

According to the simulation result, it is clear that the controller's characteristic of VSC algorithm for the nonlinear system includes: good tracing performance, quick response, tiny overshoot. Especially adding or subtracting 100% of the load, the control system is robust.

#### IV. SIMULATION RESULTS

To prove the controller and the model to be correct, the nonlinear model is replaced by the actual diesel engine. So, we should obtain the working MAP of the diesel engine at first.

Carrying out vast experiments, we get data of the fuel-injection amount and M.E.P. for different speed. Then, on the basis of supposing the injection fuel ahead angle to be not changing, perform inserting value operation by using Model-Based Calibration Toolbox and CAGE in MATLAB software, and obtain property data of the diesel engine on other working points. Lastly, establish MAP of the real-time simulation and log them in working module of the diesel engine.

As shown in Fig.5, when simulating in-ring, the controller recognize the speed error and get the control input  $u$  ( the fuel injected every cycle  $q$  (g) ) at first. Then, inquires the MAP together with feedback of the diesel speed to obtain the mean indicated pressure(M.I.P.). M.I.P. subtracts mean friction pressure(M.F.P.) is M.E.P.. Multiplies M.E.P. by coefficient  $k''$  is the power torque. Finally, according to the error that the power torque  $M$  subtracts the load torque  $M_l$ , employing the power mechanical equation principle, we can get the actual speed.

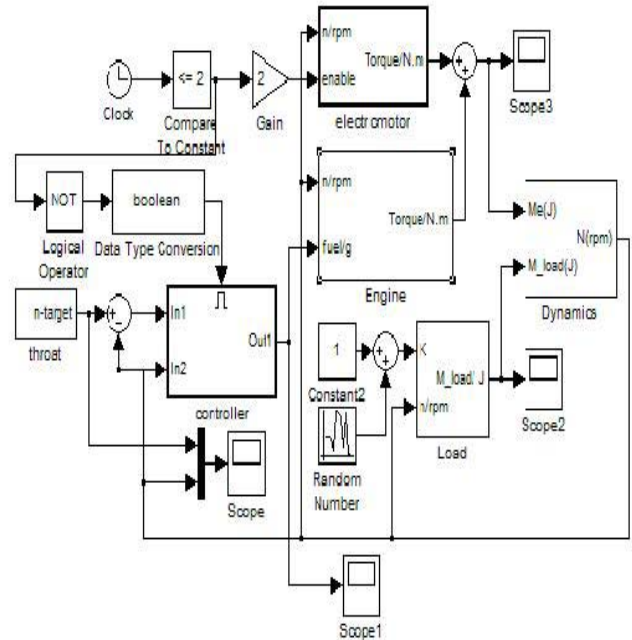


Figure 5. VSC structure of simulation in-ring

The simulation results are shown in Figs.6-9:

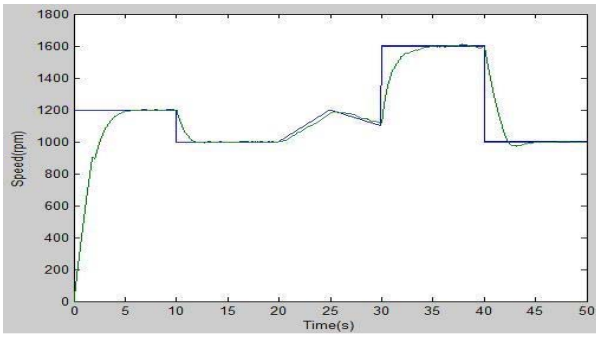


Figure 6. VSC simulation result

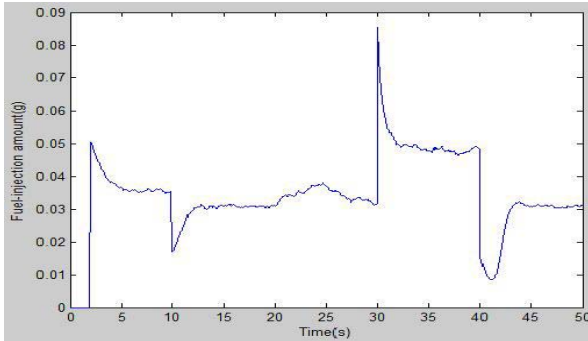


Figure 7. The output of controller (the fuel injected)

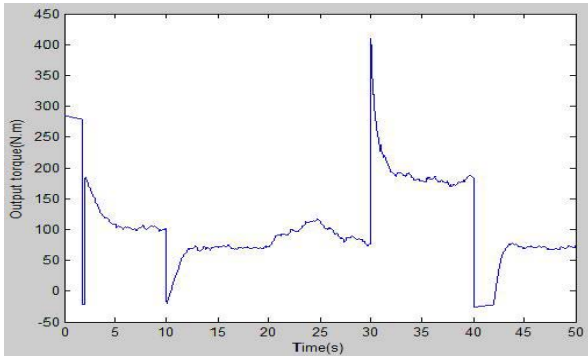


Figure 8. The output of the power torque

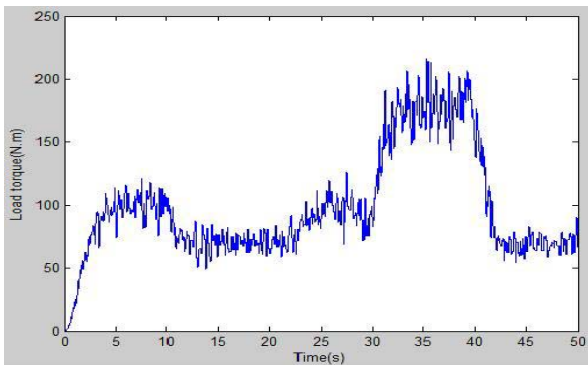


Figure 9. The load torque (disturbances)

When applying traditional PID algorithm, the simulation result is as Fig.10:

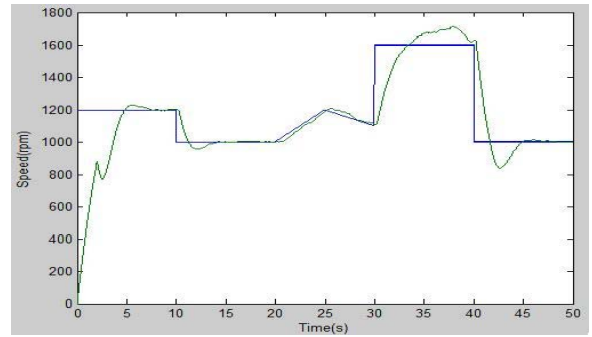


Figure 10. The simulation result of the PID algorithm

When strengthening disturbances (increasing or decreasing 50% load at random), as shown in Fig. 11,

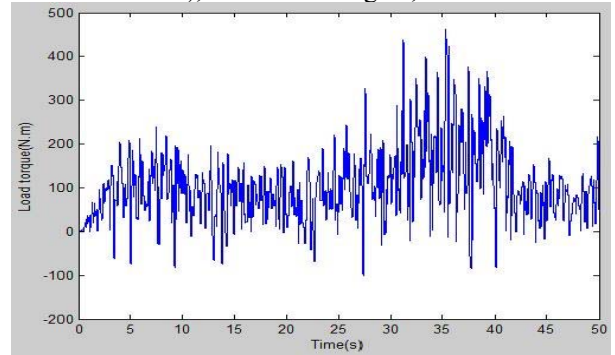


Figure 11. 50% disturbances variation

simulation results of VSC and PID are respectively shown in Fig.12 and Fig. 13.

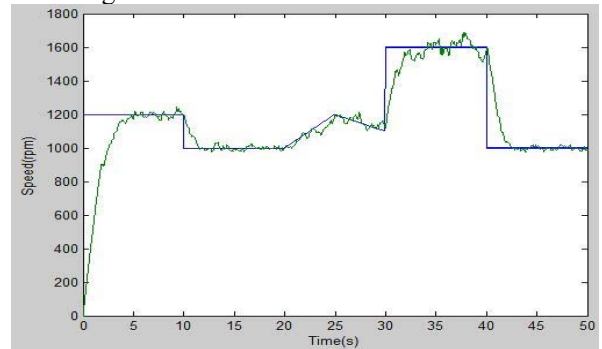


Figure 12. VSC simulation result of increasing load

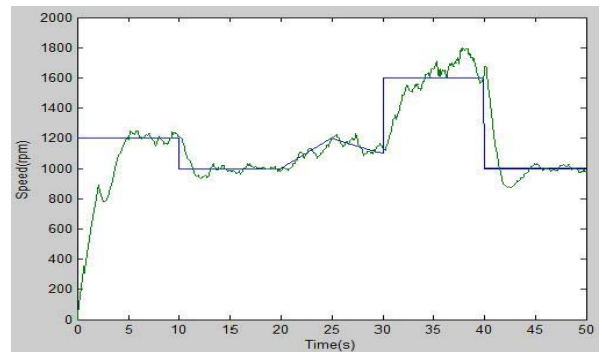


Figure 13. PID simulation result of increasing load

Comparing two controllers of the nonlinear engine, it shows that for VSC, overshoot is very tiny, regulating time is short.



Thus, VSC system responds fast and has good dynamic and static quality.

### V. SHELF TESTS

To prove the sliding mode controller to be correct, simulations for 2135 diesel engine in ring are done, as shown in Fig. 14 and Fig. 15. Finally, shelf tests are carried out to test the responding ability in real-time. In order to control the oil-injection amount, a step-by-step electrical device which is the executive organization of the electronic governor to decide rack position of the oil pump. The whole course includes choosing 0.1s as sample time and gathering 4000 data. And the upper and lower fluctuation of 15rpm range in Fig. 15 is because that speed of the diesel engine is instantaneous speed. Test results of VSC and PID are respectively shown in Fig. 16 and Fig. 17.

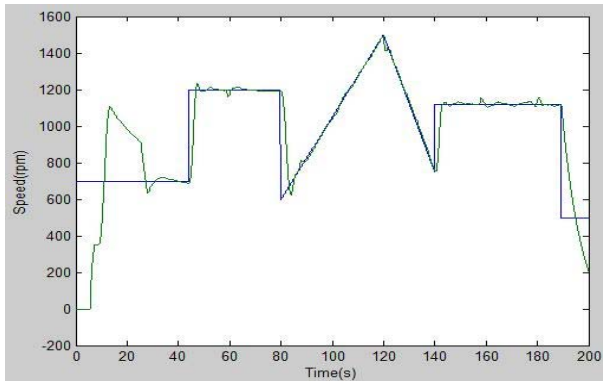


Figure 14. VSC simulation result of 2135 diesel engine

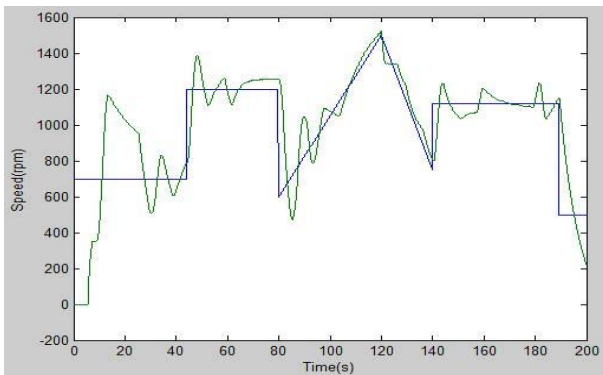


Figure 15. PID simulation result of 2135 diesel engine

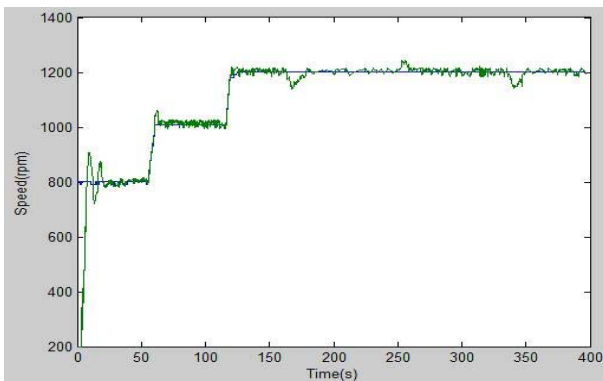


Figure 16. Shelf tests of 2135 diesel engine with VSC algorithm

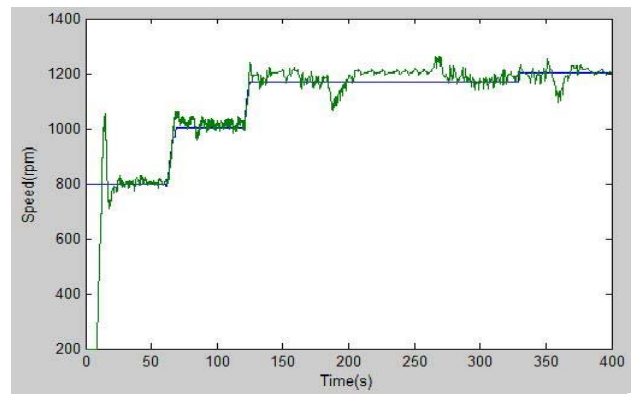


Figure 17. Shelf tests of 2135 diesel engine with PID algorithm

### SUMMARY

The paper proposes a robust control system for the diesel engine by employing VSC. The practical limitations of the conventional PID approach for the speed control system are investigated and VSC method is proposed as an effective way of overcoming the problems in the conventional PID approach. The distinctive results of this research are summarized.

Unlike the conventional methods, VSC is suitable for nonlinear systems[6], and is able to respond quickly against disturbances without overshoot. These applications are being considered as our further research topics.

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