H_∞ and Mixed H_2/H_∞ Control of Dual-Actuator Hard Disk Drive via LMIs

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***Abstract*—In this paper, H_∞ and mixed H_2/H_∞ controllers are designed for a hard disk drive (HDD) using linear matrix inequalities (LMIs). In the proposed algorithm, the problem of minimizing track misregistration (TMR) in the presence of unmodeled dynamics of voice-coil motor (VCM) is formulated as robust H_∞ and mixed H_2/H_∞ problems. Additional stack variables are introduced to characterize H_2 and H_∞ performances. The simulation and comparison results demonstrate the significant performance improvement of TMR under unmodeled dynamics of VCM.**

***Keywords*—Linear matrix inequalities(LMIs), hard disk drive, robust control**

I. Introduction

The dual-actuator structure for hard disk drives allows faster data access with relatively slower spindle speed. However, the mechanical interaction between the actuators tends to degrade the head positioning accuracy and thus decreases the track density performance. The hard disk drives industry continues to strive for larger areal densities, faster data transfer rate and lower cost. The dual-actuator HDD architecture was proposed for a cost-effective solution [1]. This contains two independent actuators mounted in the diagonal comers of the disks. The dual-actuator system requires one additional actuator instead of one whole HDD in the disk array system. Thus, the cost is undoubtedly reduced. Furthermore, the dual-actuator system allows faster data access than the conventional single-actuator HDD. In HDD industry, the access time is evaluated by the seek time and latency. The seek time measures the amount of time required for the actuator to position the read-write heads between tracks over the disk surface. One of the most important performance measures for hard disk drives is track misregistration, which is the variance of the deviation of the center of the read-write head from the center of a data track. TMR should be minimized via proper design of the servo system in order to achieve a higher storage capacity in HDDs. As a means of obtaining smaller TMR, dual- actuators, which combine a conventional voice coil motor and a secondary micro-actuator (MA) placed close to the head, have been studied intensively [2] and [3]. The design and optimization of track-following controllers for dual-stage actuators have been studied by many researchers over the past years.

The main objective of the HDD servo system is to make the position error signal (PES) as small as possible, in order to achieve high areal densities and low-readout error rates. Since the entire system is adequately modeled as a stochastic system, i.e., all external disturbances can be considered as random signals with Gaussian distribution, the tracking performance is normally characterized by the 3σ-value of the PES. The track-following control of the magnetic read/write head in hard disk drives is of great importance in meeting recent and future requirements of extremely high track density. For a given system consisting of several components such as a suspension, sensors, and actuators, the track-following servo system control should attain the smallest possible track-misregistration, which is generally measured by the variance of the PES, in the presence of measurement noise, track run out, wind age, and external shock. In this paper, we propose two types of controllers for dual-stage track-following in hard disk drives. These approaches formulate the control objectives as a robust stabilizer controller design problems. H_2 and H_∞ performance constraints are expressed as a set of linear matrix inequalities. The minimization of TMR is formulated as a H_2 performance problem.

The rest of this paper is organized as follows. In Section II, the LMI formulation of H_2 and H_∞ controllers design are presented. The application of the proposed approaches to a typical HDD servo loop is given in Section III. Finally, Section IV concludes the paper.

II. Problem Formulation via LMIs

In recent years, the LMI based approaches to controller design have attracted significant interests due to their numerical tractability. However, existing LMI based solutions for the mixed control present sufficient solvability conditions which are generally conservative with various degrees of conservatism [6]. Consider the following state-space representation for linear time invariant systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_u u(t) + B_w w(t) \\ z_1(t) = L_1 x(t) + H_{12} u(t) + H_{11} w(t) \\ z_2(t) = L_2 x(t) + H_{22} u(t) + H_{21} w(t) \\ y(t) = Cx(t) + D_w w(t) \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state, $y(t) \in \mathcal{R}^r$ is the measurement output, $z_i(t) \in \mathcal{R}^p (i = 1,2)$ are the controlled outputs, $w(t) \in \mathcal{R}^l$ is the disturbance input, $u(t) \in \mathcal{R}^m$ is the control input, and $A, B_1, B_2, C, D_1, L_1, L_2, H_{11}, H_{12}, H_{21}, H_{22}$ are matrices of appropriate dimensions.

Let a controller of the same dimension as the one of system (1) be of the form:

$$\begin{cases} \dot{x}_k(t) = A_k x_k(t) + B_k y(t) \\ u(t) = C_k x_k(t) + D_k y(t) \end{cases} \tag{2}$$

where the matrices (A_k, B_k, C_k, D_k) are to be determined. $\mathrm{T}_{z_1 w}$ denotes the transfer function matrix from w to z_1, and $T_{z_2 w}$ is the transfer function matrix from w to z_2 [7] and [11].

A. Problem Formulation

Fig. 1 shows the standard representation of the closed-loop system.

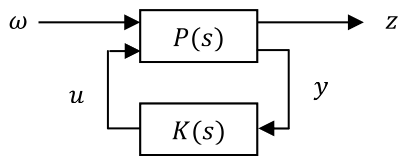

Figure 1. Standard representation of closed-loop system.

The notation is fairly standard. The compact notation $P(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ is used to denote the transfer function $P(s) = C(sI - A)^{-1}B + D$, where all of variables are defined in (1), and:

$$K(s) = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right] \tag{3}$$

This paper deals with multiobjective output-feedback synthesis MIMO linear time invariant (LTI) systems. These specifications are defined for particular channels or combinations of channels. Each specification or objective is formulated relative to some closed loop transfer function of the form:

$$T_j = L_j T R_j \tag{4}$$

where matrices L_j, R_j select the appropriate input/output (I/O) channels or channel combinations [11]. The specifications and objectives under consideration include H_∞ performance, H_2 performance, time-domain constraints and regulation. The H_∞ performance is convenient to enforce robustness to model uncertainty and to express frequency-domain specifications such as unmodeled high frequency dynamics of the VCM actuator. The H_2 performance is useful to handle stochastic aspects such as measurement noise and random disturbance.

With the plant P and controller K defined as above, the closed-loop system admits the realization:

$$T: \begin{cases} \dot{x}_{cl} = A_{cl} x_{cl} + B_{cl} w(t) \\ u(t) = C_{cl} x_{cl} + D_{cl} w(t) \end{cases} \tag{5}$$

where:

$$T: \begin{bmatrix} A + BD_k C & BC_k & B_w + BD_k D_w \\ B_k C & A_k & B_k D_w \\ C_z + D_z D_k C & D_z C_k & D_{zw} + D_z D_k D_w \end{bmatrix} \tag{6}$$

and:

$$C_z = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \quad \& \quad D_z = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}.$$

$T_j(s)$ is the transfer function from w_j to z_j if specifying the input and output signals in (5) as $w = R_j w_j$ and $z_j = L_j z$.

B. H_∞ Controller Design

The constraint $\|T_j(\mathrm{s})\|_\infty < \gamma$ can be interpreted as a disturbance rejection performance where:

$$\|T_j(\mathrm{s})\|_\infty = \mathrm{Sup}_\omega\, \overline{\sigma}\, [T_j(j\omega)] \tag{7}$$

This constraint is also useful to enforce robust stability. Specifically, it guarantees that the closed-loop system remains stable for all perturbations. According to the BR Lemma, A_{cl} is stable and $\|T_j(\mathrm{s})\|_\infty < \gamma$ if and only if there exists a symmetric matrix $\mathrm{X_{cl}} > 0$ such that:

$$\begin{bmatrix} A_{cl}{}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl}{}^T \\ B_{cl}{}^T X_{cl} & -I & D_{cl}{}^T \\ C_{cl} & D_{cl} & -I \end{bmatrix} < 0 \tag{8}$$

Since the requirement $X_{cl} > 0$ is common to all analysis results of Section III, the constraint:

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{9}$$

should always be included in the list of synthesis LMI's, either explicitly or as part of some other LMI constraint [11].

The analysis LMI (8) with finding nonsingular matrices M and N to satisfy $MN^T = I - XY$, are leads to the synthesis LMI (10), where the state-space matrices of controller are:

$$\begin{cases} D_k = \widehat{D} \\ C_k = (\widehat{D} - D_k C X) M^{-T} \\ B_k = N^{-1}(\hat{B} - YBD_k) \\ A_k = N^{-1}(\hat{A} - NB_k CX - YBC_k M^T) - Y(A + BD_k C) M^{-T} \end{cases}$$

and:

$$B_j = B_w R_j,\ C_j = L_j C_z,\ D_j = L_j D_{zw} R_j,\ E_j = L_j D_z,$$
$$F_j = D_w R_j$$

and ■ replaces blocks that are readily inferred by symmetry. Since γ enters linearly, it can be directly minimized by LMI optimization to find the smallest achievable H_∞ norm. As a single objective problem, no conservatism is involved. Imposing independent H_∞ constraints on several different channels just amounts to incorporating (10) for each individual channel and introduces conservatism.

C. H_2 Controller Design

The H_2 norm of a stable and strictly proper transfer function $T_j(s)$ is defined as [4] and [5]:

$$\left\|T_j(j\omega)\right\|_2^2 \triangleq \frac{1}{2\pi}\int_{-\infty}^{+\infty} Trace\left[T_j(j\omega)^H T_j(j\omega)\right]d\omega. \tag{11}$$

It is well known that this norm can be computed as $\left\|T_j(j\omega)\right\|_2^2 = \mathrm{Tr}\left(C_j P_0 C_j^T\right)$, where P_0 is the solution of the Lyapunov equation:

$$A_{cl}P_0 + P_0 {A_{cl}}^T + B_j {B_j}^T = 0 \tag{12}$$

It is readily verified that $\left\|T_j(j\omega)\right\|_2^2 < \delta$ if and only if there exists $P_0 > 0$ satisfying above equations. Upshot we obtain the following analysis result that A_{cl} is stable and $\left\|T_j(j\omega)\right\|_2^2 < \delta$ if there exist symmetric $X_{cl} := {P_0}^{-1} > 0$ and Q such that:

$$\begin{cases} \begin{bmatrix} A_{cl}^T X_{cl} + A_{cl} X_{cl} & X_{cl} B_j \\ B_j^T X_{cl} & -I \end{bmatrix} < 0 \\ \begin{bmatrix} X_{cl} & C_j^T \\ C_j & Q \end{bmatrix} > 0 \\ Tr(Q) < \delta \end{cases} \tag{13}$$

The synthesis LMI's for generalized H_2 control have been given above. With the same congruence we obtain the synthesis LMI's (14) from the analysis LMI's (13) for the standard problem.

$$\begin{bmatrix} AX + XA^T + B\hat{C} + (B\hat{C})^T & \hat{A}^T + (A + B\hat{D}C) & B_j + B\hat{D}F_j \\ \hat{A} + (A + B\hat{D}C)^T & A^T Y + YA + \hat{B}C + (\hat{B}C)^T & YB_j + \hat{B}F_j \\ (B_j + B\hat{D}F_j)^T & (YB_j + \hat{B}F_j)^T & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} X & I & (C_j X + E_j \hat{C})^T \\ I & Y & (C_j + E_j \hat{D} C)^T \\ C_j X + E_j \hat{C} & C_j + E_j \hat{D} C & Q \end{bmatrix} > 0$$

$$Tr(Q) < \delta \tag{14}$$

In this problem, these involve the auxiliary variables Q and δ which enter linearly. Hence, δ can be directly minimized by LMI optimization [11].

III. Application to HDD Servo Loop

A. Problem Formulation

A block diagram of a typical HDD servo loop is shown in Fig. 2. $P(z)$ and $C(z)$ represent transfer functions of the plant and controller, respectively. v represents all torque disturbances. d represents disturbances that are due to no repeatable disk and slider motions. n denotes the PES demodulation and measurement noise. z_1 is the true position error, and y is the measured position error. V,D and N are the disturbance and noise models, and w_1, w_2 and w_3 are white noises of zero mean and unit variance.

Through experiments, the frequency responses of the actual VCM are obtained. A fifth-order model is used to approximate the actual frequency responses of the VCM actuator and is given by (19), shown at the bottom of next page.

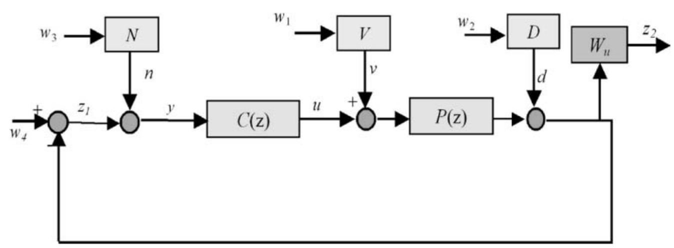

Figure 2. HDD servo loop block diagram.

To capture the unmodeled dynamics in high frequencies dozens of frequency response measurement are carried out and multiplicative uncertainty of the VCM actuator defined by $W_u(s)$. An approximate bounding function $W_u(s)$, is obtained as:

$$W_u(s) = \frac{3s^2 + 2.903 \times 10^4 s + 1.433 \times 10^8}{s^2 + 3.016 \times 10^4 s + 1.421 \times 10^9} \tag{15}$$

By discretization using the zero-order hold, the corresponding z-domain models of the VCM and the bounding function, i.e., $P(z)$ and $W_u(z)$ can be obtained.

Based on the power spectrum of the measured PES and the disturbance modeling method in [8], the identified $V(z)$, $D(z)$ and $N(z)$ are given by:

$$V(z) = \frac{8.199 \times 10^{-5} z + 8.008 \times 10^{-5}}{z^2 - 1.931z + 0.9319} \tag{16}$$

$$\begin{bmatrix} AX + XA^T + B\hat{C} + (B\hat{C})^T & \hat{A}^T + (A + B\hat{D}C) & ■ & ■ \\ \hat{A} + (A + B\hat{D}C)^T & A^T Y + YA + \hat{B}C + (\hat{B}C)^T & ■ & ■ \\ (B_j + B\hat{D}F_j)^T & (YB_j + \hat{B}F_j)^T & -\gamma I & ■ \\ C_j X + E_j \hat{C} & C_j + E_j \hat{D} C & D_j + E_j \hat{D} F_j & -\gamma I \end{bmatrix} < 0 \tag{10}$$

$$D(z) = \frac{0.07333z - 0.007806}{z - 0.8058} \quad (17)$$

$$N(z) = \frac{8.58 \times 10^{-5}z^2 - 1.802 \times 10^{-5}z - 3.937 \times 10^{-5}}{z^3 - 1.958z^2 + 1.265z - 0.2702} \quad (18)$$

As mentioned, one of the most important performance measures for HDDs is TMR, the total amount of random fluctuation about the desired track location. TMR is used to judge the required accuracy of positioning and thus, to scale the disk capacity. To achieve a high-capacity disk drive, one way in servo control is to minimize TMR, which is expressed as the standard deviation of the true PES [7].

Now, we introduce the state-space representation (1) for the system in Fig. 2.

$$x = \begin{bmatrix} x_p^T & x_v^T & x_d^T & x_n^T & x_u^T \end{bmatrix}^T$$
$$w = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}^T$$
$$A = \begin{bmatrix} A_p & B_pC_v & 0 & 0 & 0 \\ 0 & A_v & 0 & 0 & 0 \\ 0 & 0 & A_d & 0 & 0 \\ 0 & 0 & 0 & A_n & 0 \\ B_uC_p & 0 & B_uC_d & 0 & A_u \end{bmatrix}$$
$$B_1 = \begin{bmatrix} B_pD_v & 0 & 0 & 0 \\ B_v & 0 & 0 & 0 \\ 0 & B_d & 0 & 0 \\ 0 & 0 & B_n & 0 \\ 0 & B_uD_d & 0 & 0 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} B_p \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} -C_p & 0 & -C_d & C_n & 0 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} 0 & -D_d & -D_n & 1 \end{bmatrix}$$
$$L_1 = \begin{bmatrix} -C_p & 0 & -C_d & 0 & 0 \end{bmatrix}$$
$$H_{11} = \begin{bmatrix} 0 & -D_d & 0 & 0 \end{bmatrix}$$
$$H_{12} = 0$$
$$L_2 = \begin{bmatrix} D_uC_p & 0 & B_uD_d & 0 & C_u \end{bmatrix}$$
$$H_{21} = \begin{bmatrix} 0 & D_uD_d & 0 & 0 \end{bmatrix}$$

B. Simulation Results

In this section, we apply the proposed approach to solve the control problem of HDD. We need to ensure the system stability against the unmodeled high frequency dynamics of the VCM actuator. For the purpose of comparisons, we also design robust H_∞ and mixed $H_2\backslash H_\infty$ controllers for the disk drive. In order to minimize TMR, the control design problem can be treated as an H_2 optimal control problem. Suppose that $\widetilde{w} = [w_1 \quad w_2 \quad w_3]^T$ and $T_{z_2\widetilde{w}}$ denotes the transfer function matrix from $\widetilde{w}$ to z_2. By minimizing the H_2 norm of this transfer function, we want to reject the disturbance effect and minimize TMR. For stability against the unmodeled high-frequency dynamics of the VCM, the constraint $\|TW_u\|_\infty < 1$ is to be met, where T is the closed loop transfer function and W_u is the bounding function of the unmodeled dynamics.

Fig. 3 shows the sensitivity function of the closed loop system using H_∞ controller that achieves $\gamma = 0.9727$. Also, the modulus margin of this system is better than the one of [7].

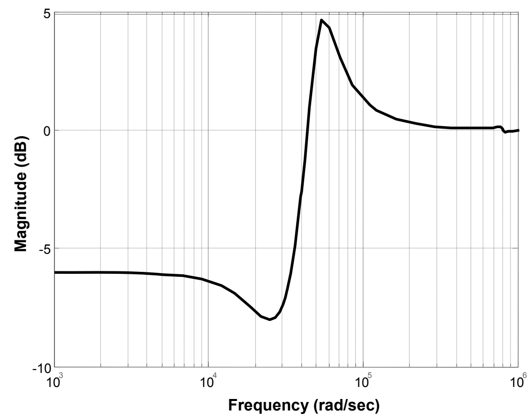

Figure 3. Sensitivity function using H_∞ controller.

The step response of the closed-loop system using robust H_∞ controller han been shown in Fig. 4. The overshoot is %50 and the settling time is $3.9 \times 10^{-4} sec$.

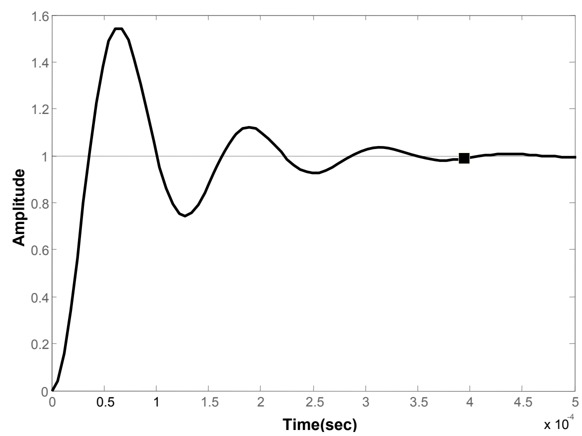

Figure 4. Step response of the closed-loop system using H_∞ controller.

Now, we use the both of the H_2 and H_∞ controllers. Fig. 5 shows the sensitivity function using the mixed $H_2\backslash H_\infty$ controller. The modulus margin is bigger than of the H_∞ controller and one of in [7]. Also, the mixed $H_2\backslash H_\infty$ controller achieves $\gamma = 0.9705$. Also, by supposing the w_4 as a disturbance input, the disturbance response of the closed-loop system using the mixed $H_2\backslash H_\infty$ controller has been shown in Fig. 6. The settling time is $t_s = 3 \times 10^{-4} sec$ that is very good.

$$P(s) = \frac{5.172 \times 10^{12}s^2 + 1.82 \times 10^{17}s + 3{,}267 \times 10^{21}}{s^5 + 2.117 \times 10^4 s^4 + 1.032 \times 10^9 s^3 + 1.906 \times 10^{13}s^2 + 8.587 \times 10^{15}s + 7.345 \times 10^{18}} \quad (19)$$

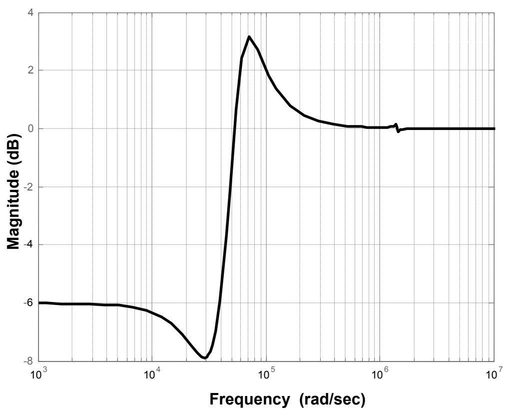

Figure 5. Sensitivity function using mixed $H_2\backslash H_\infty$ controller.

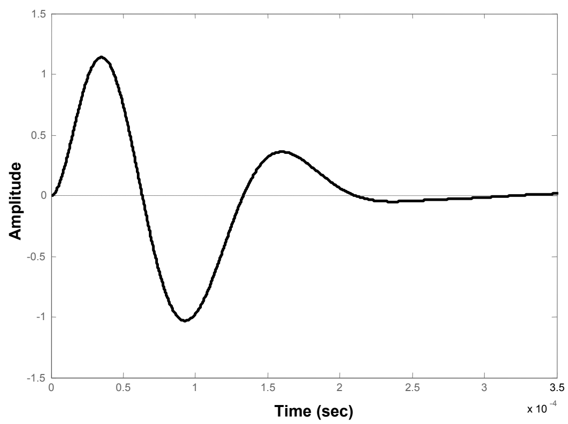

Figure 6. Disturbance response of the closed- loop system using mixed $H_2\backslash H_\infty$ controller.

IV. Conclusion

In this paper, two types of controllers have been designed for a dual-actuator HDD via LMIs. By comparing the simulation results, it find that the mixed $H_2\backslash H_\infty$ controller has a better stability and performance than H_∞ controller. In future works, one can be include the pole-placement and performance limitations constraints in the control problem to improve the results.

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