Robust Control Based on LS-SVM for Uncertain Nonlinear System

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***Abstract*—In this paper, a new stable robust adaptive control approach is presented for SISO uncertain nonlinear system. The key assumption is that LS-SVM approximation errors and external disturbances satisfy certain bounding conditions. The LS-SVM can find a global minimum and avoid local minimum. Its weights in the observer can be tuned after training phase and find optimistic value automatically. By combining LS-SVM, the system state vector is estimated by an observer efficiently. A simulation example demonstrates the feasibility of the proposed approach.**

***Keywords*—nonlinear system, control, observer, LS-SVM, uncertainty**

I. Introduction

The design of nonlinear system observer is one of the essential problems in control theory. With the development of the Kalman Filter [1] and Luenberger observer [2], there have been many works on nonlinear observer for nonlinear system. Most of the early attempts were based on extending the linear methodology through various kinds of linearization techniques [3-5]. The research results during that period were required to satisfy some assumptions, such as matching conditions, or extended matching conditions. R. Marino and P. Tomei presented a global adaptive observer for a class of single-output nonlinear systems which are linear with respect to an unknown constant parameter vector [6]. However, it is difficulty for many physical control systems to satisfy the assumption of linearity because of their more and more complexity.

Whereas neural network possesses a lot of advantage, observers based on neural network have greatly been developed recently. The design methods are divided into two catalogues: non-adaptation and adaptation. The main drawback of the non-adaptive neural networks is that the weight updating laws utilize information on the local data structures (local optima) and the function approximation is sensitive to the training data [7]. Dynamic neural network was first introduced by Hopfield [8] and may successfully overcome these disadvantages because of feedback structure. Dynamic neural network can change its weight matrix using the estimate error between observer and observed object and then has ability to adapt the circumstances. On the other hand, the neural networks have some problems such as converge to local minimum, the over-fitting and the structure of NN is always decided by experience because it doesn't have a good guiding theory.

To overcome these problems of NN, Support Vector Machine (SVM) was proposed by Vapnik. SVM is a newly developed technique which based on statistical learning theory. It adopts structure Risk Minimization principle, avoids local minimum, effectively solves the over learning, assures good generalization ability, and better predict accuracy. The special predominance of SVM in resolving limited samples, non-linear function and multidimensional pattern recognition make it become a kind of excellent machine learning method. Suykens first put forward Least Squares Support Vector Machine (LS-SVM). As a result the solution follows directly from solving a set of linear equation, instead of quadratic programming. While in classical SVM's many support values are zero, in least squares SVM's, the support values are proportional to the errors.

This paper utilizes LS-SVM to design a nonlinear observer to overcome the problems of NN. A SVM is inserted in the feedback path to capture the nonlinear characteristic of the observer system. We will show that the state estimation errors are suitable small and bounded.

II. Observer Based Controller Design

Before the description of the observer system and problem formulation, some notation and definition are stated as follow:

$$\|x\| = \sqrt{x^T x} \qquad x \in R^n$$

$\lambda_{\min}[\cdot]$ represents the minimum eigenvalue of a matrix.

A. Dynamic Model of Nonlinear Uncertain System

Consider the nonlinear system

$$\begin{aligned} \dot{x} &= Ax + B[g(x)u + f(x) + m(t)] \\ y &= C^T x \end{aligned} \tag{1}$$

With $x \in R^n$, $y \in R$ and $u \in R$, $m(t)$ is the unknown disturbances with a known upper bound m_d , and $f, g : R^n \to R$ unknown smooth functions. We assume $f(x)$ and $g(x)$ contain parameter uncertainties which are not necessarily linear. Note that nonlinearities of functions $f(x)$ and $g(x)$ depend on the system state x and y .

$$A=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 0 & 1\\ 0 & 0 & 0 & \cdots & 0\end{bmatrix}, B=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}, C=\begin{bmatrix}1\\0\\ \vdots\\0\\0\end{bmatrix}$$

A nonlinear observer for the states in (1) is

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x}+B[\hat{g}(\hat{x})u+\hat{f}(\hat{x})]+K(y-C^T\hat{x})\\ \hat{y} &= C^T\hat{x}\end{aligned} \tag{2}$$

Where $\hat{x}$ denotes the estimates of the state x and $K=[k_1,k_2,\cdots,k_n]^T$ is the observer gain matrix chosen so that the characteristic polynomial of $A+KC^T$ is strictly Hurwitz. The functions $\hat{f}(\hat{x})$ and $\hat{g}(\hat{x})$ are estimated for $f(x)$ and $g(x)$ respectively.

Lemma1: If a strictly proper rational function $H(s)=C^T(sI-A)^{-1}B$ with A a Hurwitz matrix SPR, then there exists a positive, definite symmetric matrix P such that

$$A^TP+PA+Q=0,\ PB=C$$

Here Q is a positive definite symmetric matrix.

Defining the state error as $e\equiv x-\hat{x}$, then gain the estimation error dynamics

$$\begin{aligned}\dot{e} &= (A+KC^T)e\\ &+B[\underline{f}(x,\hat{x})u+\underline{g}(x,\hat{x})+m(t)]\end{aligned} \tag{3}$$

Where the functional estimation error $\underline{f}(x,\hat{x})$ and $\underline{g}(x,\hat{x})$ are defined by

$$\begin{aligned}\underline{f}(x,\hat{x}) &= f(x)-\hat{f}(\hat{x})\\ \underline{g}(x,\hat{x}) &= g(x)-\hat{g}(\hat{x})\end{aligned} \tag{4}$$

This paper utilizes SVM to form the functions $\hat{f}(\hat{x})$ and $\hat{g}(\hat{x})$, and assures this system has excellent performance.

B. LS-Support Vector Machin[9]

Given a training samples set of $\{x_k,y_k\}_{k=1}^N$, $x_k\in R^n$, $y_k\in R$, N is the samples number, n is the number of input dimension. In LS-SVM the optimization problem is formulated

$$\min_{\omega,b,\xi_t}\left\{J(\omega,\xi)=\tfrac{1}{2}\omega^T\omega+\tfrac{\gamma}{2}\sum_{t=1}^{l}\xi_t^2\right\} \tag{5}$$

Subject to the equality constrains

$$y_k=\omega^T\varphi(x_k)+b+\xi_k\quad k=1,2,\cdots,n \tag{6}$$

The solution is obtained after constructing Lagrangian

$$L(\omega,b,\xi,\alpha)=J(\omega,\xi)-\sum_{k=1}^{N}\alpha_k\{\omega^T\Phi(x_K)+b-y_k+\xi_t\} \tag{7}$$

With Lagrange multipliers $\alpha_k\in R$. Then, due to the Karush-Kuhn-Tucher (KKT) conditions, we can gain the following equations and constrain conditions

$$\begin{aligned}&\tfrac{\partial L}{\partial\omega}=0\rightarrow\omega=\sum_{k=1}^{N}\alpha_k\varphi(x_k)\\ &\tfrac{\partial L}{\partial b}=0\rightarrow\sum_{k=1}^{N}\alpha_k=0\\ &\tfrac{\partial L}{\partial\xi_k}=0\rightarrow\alpha_k=\gamma\xi_k\\ &\tfrac{\partial L}{\partial\alpha_k}=0\rightarrow\omega\varphi(x_k)+b+\xi_k-y_k=0\end{aligned} \tag{8}$$

Meantime we can gain the following set of linear Equations from (6).

$$\begin{bmatrix}0 & 1^T\\ 1 & \varphi(x_k)^T\varphi(x_k)+\gamma^{-1}I\end{bmatrix}\begin{bmatrix}b\\ \alpha\end{bmatrix}=\begin{bmatrix}0\\ y\end{bmatrix} \tag{9}$$

So, the non-linear regression function is defined as

$$y(x)=\sum_{i=1}^{N}\alpha_i^*k(x_i,x) \tag{10}$$

Here we use Radial Basis Function Kernel (RBF)

$$K(x,x_i)=\exp\left\{-\tfrac{\|x-x_i\|^2}{2\sigma^2}\right\}$$

Here the Kernel parameter is the width σ of the radial basis function.

C. LS-SVM Observer Design

Thus, $\hat{f}(\hat{x})$ and $\hat{g}(\hat{x})$ in (2) can been replaced by the observer of the nonlinear system

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x}+B[\sum_{i=1}^{N}\alpha_{gi}^*k_g(\hat{x}_i,\hat{x})u\\ &+\sum_{i=1}^{N}\alpha_{fi}^*k_f(\hat{x}_i,\hat{x})]+K(y-C^T\hat{x})\\ \hat{y} &= C^T\hat{x}\end{aligned} \tag{11}$$

Here we have

$$\hat{g}(\hat{x})=\sum_{i=1}^{N}\alpha_{gi}^*k_g(\hat{x}_i,\hat{x})$$

$$\hat{f}(\hat{x}) = \sum_{i=1}^{N} \alpha_{fi}^{*} k_f(\hat{x}_i, \hat{x})$$

Here α_{ki}^{*} $(i = 1,\cdots,N, k = f, g)$ are the constant according to the result of the training set. In addition, the continuous nonlinear functions $f(x)$, $g(x)$ can be represented by LS-SVM with "ideal" parameters α_{ki} $(i = 1,\cdots,N, k = f, g)$ and $K(x, x_i)$

$$\begin{aligned} f(x) &= \sum_{i=1}^{N} \alpha_{fi} k_f(x_i, x) + b_f \\ g(x) &= \sum_{i=1}^{N} \alpha_{gi} k_g(x_i, x) + b_g \end{aligned} \tag{12}$$

So we can gain the $\dot{e}$

$$\begin{aligned} \dot{e} &= (A + KC^T)e + B[(\sum_{i=1}^{N} (\alpha_{fi} - \alpha_{fi}^{*}) k_f(\hat{x}_i, \hat{x}) + b_g))u \\ &+ (\sum_{i=1}^{N} (\alpha_{gi} - \alpha_{gi}^{*}) k_g(\hat{x}_i, \hat{x}) + b_g) + m(t)] \\ \tilde{y} &= C^T e \end{aligned} \tag{13}$$

Lemma 2: Let $V : [0, \infty) \to R$, $\dot{V} \le -2\lambda V + \beta, \forall t > 0$, We can get

$$V(t) \le V(t_0)\exp[-2\lambda(t - t_0)] + \tfrac{\beta}{2\lambda},\ \forall t > t_0$$

Where λ and β are constants, $\lambda > 0$, $\beta > 0$ [10].

Assumption1: To α_{ki}, α_{ki}^{*}, $k = f, g$, $i = 1,\cdots,N$, we can assume that $\|\alpha_{fi} - \alpha_{fi}^{*}\| \le m_f$, $\|\alpha_{gi} - \alpha_{gi}^{*}\| \le m_g$.

Assumption2: The mismatch values of b_f and b_g are bounders according to $|b_f| \le c_f$ and $|b_g| \le c_g$. Then, we can design controller

$$u = -\frac{m_g \left|\sum_{i=1}^{N} k_g(x_i, x)\right| + c_g + d_m}{m_f \left|\sum_{i=1}^{N} k_f(x_i, x)\right| + c_f + \varepsilon_m} \tag{14}$$

Theorem: For the nonlinear system (1), under the condition of assumption 1 and 2, we can choose to controller u (13), and make the nonlinear system stable.

Proof: consider the follow Lyapunov function

$$V = \tfrac{1}{2} e^T P e$$

With $P = P^T > 0$

$$\begin{aligned} \dot{V} &= -\tfrac{1}{2} e^T Q e + \tilde{y}[(\sum_{i=1}^{N} (\alpha_{fi} - \alpha_{fi}^{*}) k_f(\hat{x}_i, \hat{x}) + b_g))u \\ &+ (\sum_{i=1}^{N} (\alpha_{gi} - \alpha_{gi}^{*}) k_g(\hat{x}_i, \hat{x}) + b_g) + m(t)] \\ &\le -\tfrac{1}{2}\lambda_{\min}(Q)\|e\|^2 + |\tilde{y}|[(\sum_{i=1}^{N} (\alpha_{fi} - \alpha_{fi}^{*}) k_f(\hat{x}_i, \hat{x}) - b_f))u \\ &+ (\sum_{i=1}^{N} (\alpha_{gi} - \alpha_{gi}^{*}) k_g(\hat{x}_i, \hat{x}) - b_g) + m(t)] \\ &\le -\tfrac{1}{2}\lambda_{\min}(Q)\|e\|^2 + |\tilde{y}|[\|\alpha_{fi} - \alpha_{fi}^{*}\| \left|\sum_{i=1}^{N} k_f(\hat{x}_i, \hat{x})\right| u + c_f u \\ &+ \|\alpha_{gi} - \alpha_{gi}^{*}\| \left|\sum_{i=1}^{N} k_g(\hat{x}_i, \hat{x})\right| + c_g + d_m] \\ &\le -\tfrac{1}{2}\lambda_{\min}(Q)\|e\|^2 + |\tilde{y}|[m_f \left|\sum_{i=1}^{N} k_f(\hat{x}_i, \hat{x})\right| u + c_f u \\ &+ m_g \left|\sum_{i=1}^{N} k_g(\hat{x}_i, \hat{x})\right| + c_g + d_m] \end{aligned}$$

And then, u is replaced by (13)

$$\begin{aligned} \dot{V} &\le -\tfrac{1}{2}\lambda_{\min}(Q)\|e\|^2 + |\tilde{y}|\{[m_f \left|\sum_{i=1}^{N} k_f(\hat{x}_i, \hat{x})\right| + c_f] \\ &* [-\frac{m_g \left|\sum_{i=1}^{N} k_g(x_i, x)\right| + c_g + d_m}{m_f \left|\sum_{i=1}^{N} k_f(x_i, x)\right| + c_f + \varepsilon_m}] + m_g \left|\sum_{i=1}^{N} k_g(\hat{x}_i, \hat{x})\right| + c_g + d_m\} \\ &= -\tfrac{1}{2}\lambda_{\min}(Q)\|e\|^2 \\ &+ |\tilde{y}|\{\frac{-[m_f \left|\sum_{i=1}^{N} k_f(\hat{x}_i, \hat{x})\right| + c_f] * [m_g \left|\sum_{i=1}^{N} k_g(x_i, x)\right| + c_g + d_m]}{m_f \left|\sum_{i=1}^{N} k_f(x_i, x)\right| + c_f + \varepsilon_m} \\ &+ \frac{[m_g \left|\sum_{i=1}^{N} k_g(\hat{x}_i, \hat{x})\right| + c_g + d_m] * [m_f \left|\sum_{i=1}^{N} k_f(x_i, x)\right| + c_f + \varepsilon_m]}{m_f \left|\sum_{i=1}^{N} k_f(x_i, x)\right| + c_f + \varepsilon_m}\} \\ &= -\tfrac{1}{2}\lambda_{\min}(Q)\|e\|^2 + |\tilde{y}| \frac{m_g \left|\sum_{i=1}^{N} k_g(\hat{x}_i, \hat{x})\right| + c_g + d_m]\varepsilon_m}{m_f \left|\sum_{i=1}^{N} k_f(x_i, x)\right| + c_f + \varepsilon_m} \\ &= -\tfrac{1}{2}\lambda_{\min}(Q)\|e\|^2 + \varepsilon \\ &= -2\mu V + \varepsilon \end{aligned}$$

Here $\varepsilon = |\tilde{y}| \frac{m_g \left|\sum_{i=1}^{N} k_g(\hat{x}_i, \hat{x})\right| + c_g + d_m]\varepsilon_m}{m_f \left|\sum_{i=1}^{N} k_f(x_i, x)\right| + c_f + \varepsilon_m} > 0$ and $\mu = \frac{1}{2}\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$.

According to the lemma 2, we gain $V(t) \le V(0)e^{-2\mu t} + \frac{\varepsilon}{2\mu}$, $\forall t > 0$. This completes the proof.

III. Simulation Example

In this section, to illustrate the application of the proposed approach, we considered a nonlinear system

$$\begin{aligned} &\ddot{x} + (x^2 - 1)\dot{x} + x = (1 + x^2 + \dot{x}^2)u \\ &y = x \end{aligned} \tag{15}$$

Let $x_1 = x$ and $x_2 = \dot{x}$, then the state-space description of the system is

$$\begin{aligned} &\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\{(1 - x_1^2)x_2 - x_1 + (1 + x_1^2 + x_2^2)u\} \\ &y = x_1 \end{aligned} \tag{16}$$

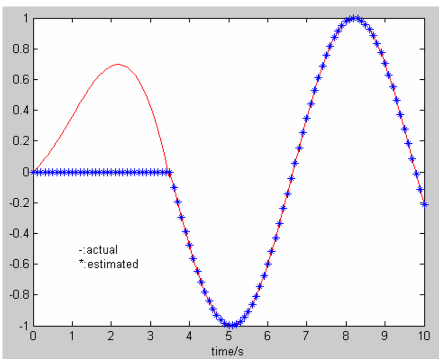

Figure 1. State estimation and actual(Trajectory of state $x_1, \hat{x}_1$)

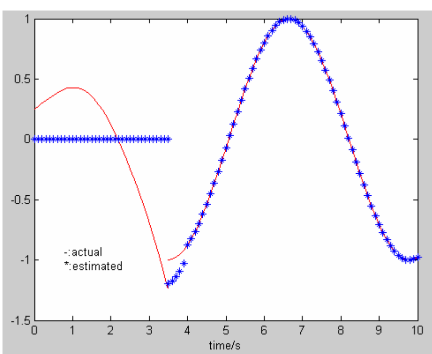

Figure 2. State estimation and actual(Trajectory of state $x_2, \hat{x}_2$)

The observer for the above system is given by (11). In this example, the data of x_1 and x_2 can be divided into two sections. The first section is used to train α_{ki} $(k = f, g, i = 1, \cdots, N)$ of LS-SVM observer. These data are sampled in the period of 0 to 3.5s. In this meantime, u equals zero. Then the second section of data is used to observe the $\hat{x}$. The process lasts from 3.5 to 10s and u is used in this process. In this example, $x(0) = [0, 0.25]^T$ .Figure 1 and 2 show bound of the estimation errors and indicate $\hat{x}$ can track x accurately. From Figure 1, 2, we can see the approach is effect.

IV. Conclusion

We have presented a nonlinear system control approach based on LS-SVM. The approach has some advantages for uncertain nonlinear systems. The reason is that LS-SVM can adopt structure Risk Minimization principle, avoids local minimum. The simulation experiment shows that the approach is effective.

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