Fault Detection for Networked Control System with Random Delays

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Abstract—This paper deals with the fault detection problem for a class of discrete-time networked systems with random delays and disturbance. With the Maximum allowable delay, a binary stochastic switching sequence is used to simplify the FDI for NCS. Attention is focused on the design of a fault detection filter, which can be robust for the unknown disturbance. A sufficient condition for the existence of the desired fault detection filter is established in terms of a linear matrix inequality (LMI). A numerical example is provided to illustrate the feasibility and applicability of the proposed techniques.

Keywords—fault detection; NCS; random switch; LMI

I. INTRODUCTION

With the development of networked technology, more and more networks (e.g. Internet) have been applied to distributed control system, whose feedback loop is closed through a real-time network, which is termed network control system (NCS). The usage of the network gives rise to new interesting and challenging issues including network-induced delays and packets dropout, which are mainly caused or deteriorated by the limited bandwidth of the network. Therefore, networked control systems (NCSs) have received increasing attentions in recent years [1-6].

One of the particular problems with analyzing and designing NCSs is the network-induced delays, which has become a crucial factor for the stability of the NCS. Furthermore, the continuous plant coupled with the digital controller can be named hybrid system, especially, when there is time delay happening, the system also should be modeled as a discrete time delay system. The time delays are usually random and time-varying by nature, which will make the research and design of NCSs complex and different from that in traditional systems. Network-induced delays can vary widely according to the transmission time of messages and the overhead time [5]. Recently, the network-induced delays have been modeled in various probabilistic ways [2,7,8]. In these and other networked control system applications, the delay can be modeled as a random process described by a Markov chain with a finite number of states. Each state in this Markov chain reflects the loading of the network or a processor at a given time [17]. It is worth mentioning that the binary random delay has gained much research interest because of its simplicity in describing network-induced delays [9], and the references therein. The binary switching sequence is viewed as a Bernoulli distributed white sequence taking on values of 0 and 1. The packet losses phenomenon can also be described as a binary switching sequence.

On the other hand, the fault detection and isolation (FDI) problem has attracted persistent attention because of the increasing complexity and safety demand of real time systems [10]. The basic idea of FDI is to construct a residual signal, based on which, a residual evaluation function can be determined. If the residual has a value larger than the predefined threshold, an alarm signal of fault is generated. Due to the advantages of Networked Control System, such as low cost, reduced weight and simple installation and maintenance, the networked control has been a trend. There is an urgent need to consider the FDI problem for networked systems. Recently, there have been some research interests on network based FDI problems, see the survey paper [11-15] and the references therein. It is complex to monitor the NCS. To reduce the calculation delay and complexity of FDI, it is feasible to take binary switching sequence to deal with the random networked delay.

In this paper, we aim to solve the fault detection problem for a class of Networked Control Systems with random sensor delays and disturbances, where the measured data is transmitted over communication network. Assume that the Maximum allowable delay [16] – τMad is gotten prior. With the delay, the system can still maintain stability. State 1 denotes delay happening, state 0 denotes no delay. The system is stochastic switching between the two states. A sufficient condition for the existence of the desired fault detection filter is established in terms of certain linear matrix inequality (LMI). When this LMI is feasible, the explicit expression of the desired fault detection filter can be determined. A numerical example is provided to illustrate the feasibility and applicability of the present methods.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following class of discrete-time linear system:

\[
\begin{align*}
\dot{x}(k+1) &= Ax(k) + Bu(k) + Ed(k) + Ff(k) \\
y(k) &= Cx(k) + Gd(k) + Hi(f(k) )
\end{align*}
\]  

---(2-1)

- \( x(k) \) is the state vector of the system, \( u(k) \) is the control input, \( d(k) \) is the disturbance, \( f(k) \) is the fault, \( y(k) \) is the output measurable. The matrices \( A, B, C, D, E, F, G, H \) are constant matrices.
where $x \in R^n$ denotes the state vector, $u \in R^p$ denotes the control input vector, $y \in R^m$ denotes the measurable output vector, $d \in R^l$ denotes the unknown disturbance vector and $f \in R^r$ denotes the vector of faults to be detected. $A, B, C, E, F, G, H$ are known matrices of compatible dimensions. Without considering the control of the system, $A$ is assumed to be stable. If we can get the full information of the system, we can achieve fault detection using model based methods. As to the NCS, especially the medium is Ethernet, from the Figure 1, we can see that the network delay denotes stochastic. To achieve the fault detection for the system, it’s obvious that the traditional model-based residual generator cannot be directly applied, because it is designed under the assumption of perfect communication. It should be either modified or completely redesigned to accommodate the unusual situation of random delays.

In the NCS, to make the system stable, we have to consider the delays of feedback and forward as the Figure 2. But with the strong computing ability of the smart nodes, we can avoid considering the delay of forward for locating the controller and fault detection module together.

In this paper, we make some assumption for convenience. It is assumed that the control and supervision stations for process (1) are located together;

- We assume that the delays are random but bounded;
- The Maximum Allowable Delay ($\tau_{MAD}$) is gotten;

**Remark**: If the parameters of fault detection algorithm are re-computed according to the delay gotten on-line, it is difficult for the smart nodes to achieve it for the complexity of the algorithm every time. Thus the bounded delay - $\tau_{MAD}$ is used to simplify the design. Though some conservatism is brought, it will be a feasible practical application scheme for Networked Control Systems.

- Sensor delay doesn’t affect the disturbance.

Under these assumptions, the process (2-1) will be changed to the following equation:

$$
\begin{align*}
\dot{x}(k+1) &= Ax(k) + Bu(k) + Ed(k) + Ff(k) \\
y(k) &= Cx(k)
\end{align*}
$$

---(2-2)

### III. ROBUST FAULT DETECTION OBSERVER DESIGN

#### A. The modeling of time delay in the feedback channel

Because the time delay happens randomly, one of the difficulties of fault detection for NCS is how to model the time delay. According to the assumption above, the fault detection observer is designed at the controller site, which is based on two different modes. One is ‘0’ denotes there is no delay; the other is ‘1’ denotes there happens delay, and the delay is Maximum Allowable Delay ($\tau_{MAD}$). From the Figure 3,

$$
(1-\delta)y_k + \hat{y}_{k-\tau_{MAD}} \approx \tau_{MAD}
$$

---(3-1)

where $\delta$ is modeled as a discrete-time Markov chain with two state space $\varnothing = \{0,1\}$.

#### B. Design of Robust Fault Detection Observer

Based on the output (3-1) of the plant, we constructed the following equation as the injection output of the observer.

$$
\dot{v}_k = (1-\delta)v_k + \hat{y}_{k-\tau_{MAD}} \approx \tau_{MAD}
$$

---(3-2)
Thus the observer is designed as following,
\[
\dot{x}(k+1) = Ax(k) + Bu(k) + H(w_k - \nu_k) = A\hat{x}(k) + Bu(k) + H[(1 - \delta)(y(k) - \hat{y}(k)) - (3-3)] + \delta(y(k - \tau_{\text{MAD}}) - \hat{y}(k - \tau_{\text{MAD}}))
\]
Let \( m_k = \delta \), we can let \( m_k = 1 \) denotes that the delay occurs at the \( kth \) time step, and \( m_k = 0 \) denotes no delay happens. Where \( m_k \) is modeled as a Markov chain with two states taking values as \( \varphi = \{0, 1\} \), and the transition probability matrix at time \( k \) is given by:
\[
\Pi_k := \begin{bmatrix} p_{11}(k) & p_{12}(k) \\ p_{21}(k) & p_{22}(k) \end{bmatrix} \quad (k = 0, 1, 2, \ldots) \quad (3-4)
\]
\[
P(m_k = j \mid m_{k-1} = i) = p_{ij}
\]
Where \( p_{ij}(k)(i, j = 1, 2; k = 0, 1, 2, \ldots) \) is the conditional probability. Hence the \( p_{ij}(k) \) satisfies the relation:
\[
0 \leq p_{ij}(k) \leq 1, \quad p_{1j}(k) + p_{2j}(k) = 1,
\]
\[
i, j = 1, 2; k = 0, 1, 2, \ldots
\]
From the plant (2-2) and model (3-3), we can get the state error equation:
\[
e(k + 1) = (A - HC(1 - m_k))e(k) - HCM_k e(k - \tau_{\text{MAD}}) + Ed(k) + Ff(k)
\]
\[
+ Ed(k) + Ff(k)
\]
\[
(3-6)
\]
and the residual is
\[
r(k) = W(w_k - \nu_k) = WC((1 - m_k)e(k) + m_k e(k - \tau_{\text{MAD}})) \quad (3-7)
\]
where \( W \) is the weighting gain. The coefficient matrices are functions of the random parameter- \( m_k \).

The error equation (3-6) can be reduced to a discrete-time Markov jump system with time delay. From the equation, we can see that if the equation is robust stability without the fault term, the observer can be used to detect fault during the system running and the residual (3-7) can be signal of the occurrence of the fault. Without fault term, the equation (3-6) is changed as equation (3-8):
\[
e(k + 1) = (A - HC(1 - m_k))e(k) - HCM_k e(k - \tau_{\text{MAD}}) + Ed(k)
\]
\[
(3-8)
\]
The equation is consistent with the discrete time jump linear state-delay systems of Cao [18][19]. When the system operates on the \( ith \) mode ( \( d_k = i \) ), the matrices can be denoted as following:
\[
A_{ti} = A - HC(1 - m_k), \quad A_{2i} = -HCM_k \quad (3-9)
\]
We can use a stochastic Lyapunov function \( V(\cdot) \) (Kushner [20]),
\[
V_k(x_k, m_k) = x_k^T P(m_k) x_k + \sum_{i=k-d}^{k-1} x_i^T Q x_i. \quad (3-10)
\]
With the proof in Cao [18], we can get the

**Theorem 1.** [18]. The free-jump system is stochastically stable if there exist \( Q > 0 \) and \( P_i > 0, i = 1, \ldots, s \), satisfying the coupled LMIs
\[
M_i = \begin{bmatrix} A_{ii}^T P_i A_{ii} - P_i + Q & A_{ii}^T P_i A_{2i} \\ A_{2i}^T P_i A_{ii} & -Q + A_{2i}^T P_i A_{2i} \end{bmatrix} < 0.
\]
\[
i = 1, \ldots, s \quad (3-11)
\]
Where
\[
\hat{P}_i = \sum_{j=1}^{i} p_{ij} P_j, i = 1, \ldots, s \quad (3-12)
\]
To get the robust \( H_\infty \) disturbance attenuation for the discrete-time residual above, we can get the following theorem.

**Theorem 2.** The residual system possesses the \( \gamma \) -disturbance attenuation property, that is
\[
\|r\|_\infty < \gamma \|d\|_1 \quad (3-13)
\]
for all \( d \in l_1[0, N-1], d \neq 0 \), if there exist \( Q > 0 \) and \( P_i > 0, i = 1, \ldots, s \), satisfying the following coupled LMIs,
\[
\Theta_i = \begin{bmatrix} \psi_i & A_{ii}^T \tilde{P}_i A_{ii} + C_{ii}^T C_{ii} & A_{ii}^T \tilde{P}_i E \\ * & -Q + A_{2i}^T \tilde{P}_i A_{2i} + C_{2i}^T C_{2i} & A_{2i}^T \tilde{P}_i E \\ * & * & -\gamma^2 I + E^T \tilde{P}_i E \end{bmatrix} \quad (3-14)
\]
where * represents blocks that are readily inferred by symmetry and
\[
\psi_i = A_{ii}^T \tilde{P}_i A_{ii} - P_i + Q + C_{ii}^T C_{ii} \quad (3-15)
\]
Proof: As to the residual system (3-7) and (3-8), in the following, we assume zero initial condition that is
\[
e_k = 0, k \in \{-\tau_{\text{MAD}}, \ldots, 0\},
\]
And define
\[ J_N \Delta = \| \tilde{d} \|_2^2 - \gamma^2 \| d \|_2^2 = E \{ \sum_{j=0}^{N-1} (r_j^T r_j - \gamma^2 d_j^T d_j) \} \]

From the Theorem 1, we can see that
\[
E\{V_N(e_N, m_k)\} = E\{ \sum_{k=0}^{N-1} [V_{k+1}(e_{k+1}, m_{k+1}) - V_k(e_k, m_k)] \}
\]

Hence, for any nonzero \( w \in l_2[0, N-1] \),
\[
J_N = E\{ \sum_{k=0}^{N-1} (r_j^T r_j + V_{k+1}(e_{k+1}, m_{k+1}) - V_k(e_k, m_k) - \gamma^2 d_j^T d_j) \}
\]

So
\[
J_N = E\{ \sum_{k=0}^{N-1} (r_j^T r_j + V_{k+1}(e_{k+1}, m_{k+1}) - V_k(e_k, m_k) - \gamma^2 d_j^T d_j) \}
\]

\[
= \sum_{i=0}^{N-1} (\sigma_i^T \Theta, \sigma_i) < 0
\]

where \( \sigma_k = [e_k^T, e_{k-MAD}^T, q_k^T]^T \). Therefore, the dissipativity inequality (3-13) holds for \( N > 0 \). In other words, we have \( r \in l_2[0, N-1] \) for any nonzero \( d \in l_2[0, N-1] \), and
\[
\| \tilde{d} \|_2 < \gamma \| d \|_2
\]

To design the foregoing observer, we will make some transition for the equation (3-14) to be easy to calculate the observer gain \( H \). Define \( X_i = P_i^{-1} \). Pre and post-multiplying (3-14) by \( \text{diag}(X_i, I) \), it is easy to find that matrix inequalities (3-14) are feasible if and only if
\[
\begin{bmatrix}
-X_i + X_i Q X_i & 0 & 0 & U_{ti}^T (C_i X_i)^T \\
0 & -Q & 0 & U_{2i}^T C_{2i}^T \\
0 & 0 & -\gamma^2 I & U_{3i}^T \\
U_{ti} & U_{2i} & U_{3i} & -Z \\
C_{1i} & X_i & C_{2i} & 0 & -I
\end{bmatrix} < 0
\]

Where
\[
U_{ti}^\Delta = [\sqrt{p_{ti}} (A_{ti} X_i)^T \cdots \sqrt{p_{ti}} (A_{ti} X_i)^T] \quad (3-17)
\]
\[
U_{2i}^\Delta = [\sqrt{p_{ti}} A_{ti}^T \cdots \sqrt{p_{ti}} A_{ti}^T] \quad (3-18)
\]
\[
U_{3i}^\Delta = [\sqrt{p_{ti}} E^T \cdots \sqrt{p_{ti}} E^T] \quad (3-19)
\]
\[ Z = \text{diag}(X_i, \cdots X_s) \]

Let \( R = Q^{-1} \) and continue to simplify the matrix inequalities (3-16) as following inequality equation (3-21) with Schur Lemma. We can see that (3-21) is not a typical LMI because there is an inverse matrix \( Q^{-1} \). However it can be solved using a cone complementarity linearization algorithm (Ghaoui, L. El 1997 [21]) as following:

\[
\min_{X_{i}, X_{s}, H, Q, R} tr(QR)
\]

And let \( R = Q^{-1} \), subject to
\[
\begin{bmatrix}
-X_i & 0 & 0 & U_{ti}^T (C_i X_i)^T & X_i \\
0 & -Q & 0 & U_{2i}^T C_{2i} & 0 \\
0 & 0 & -\gamma^2 I & U_{3i}^T & 0 \\
U_{ti} & U_{2i} & U_{3i} & -Z & 0 \\
C_{1i} & X_i & C_{2i} & 0 & -I
\end{bmatrix} < 0
\]

\[
i = 1, \cdots, s
\]

The feasible solution of above minimization problem can be found by the following iterative algorithm.

**Algorithm 1:**

1. Find a feasible solution \([X_{10}, X_{20}, H_0, Q_0, R_0]\) satisfying (3-21) and (3-22).
2. Solve the following LMI problem for \([X_i, X_s, H, Q, R]\), \( \min_{X_{i}, X_{s}, H, Q, R} tr(QR) \) subject to (3-21) and (3-22).
3. Set \( Q_{i+1} = Q \) and \( R_{i+1} = R \)
4. If the condition (3-21) is satisfied, there exits the observer gain \( H \), which is gotten by the equation (3-9). If the condition (3-21) is not satisfied, \( i = i + 1 \) and go to step (2). If \( i \) is bigger than the specified maximum of iterations, then the program exits. Where matrix \( H \) is the required observer gain. The algorithm can be readily implemented with the aid of Matlab LMI control toolbox.

With the prior specified Maximum Allowable Delay (\( \tau_{\text{MAD}} \)), the fault detection module can be designed according to the Algorithm 1. The design can make fully use of the smart node, and promote the condition monitoring for the NCS, on the other hand, it can avoid calculating the parameters of the monitoring system every time.

**IV. NUMERICAL EXAMPLE**

To illustrate the proposed fault detection scheme, a numerical example is considered in this section. The parameters of the discrete-time linear system with two modes are given as the follows:

\[
A = \begin{bmatrix} 0.006738 & 0.506 \\ 0 & 0.9983 \end{bmatrix}, 
B = \begin{bmatrix} 0.002261 \\ 0.005507 \end{bmatrix}, 
C = \begin{bmatrix} 2.21 & 0 \end{bmatrix}
\]

Assume that the transition probability matrix is given by
\[ p = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}, \]  \hspace{1cm} \text{(4-1)}

The Maximum Allowable Delay - \( \tau_{MAD} = 10 \) steps.

\[ w_k = (1-m_k)y_k + m_k y_{k-\tau_{MAD}} \]  \hspace{1cm} \text{(4-2)}

where \( m_k \in \{0,1\} \), we can get the state error equation.

With it we can get the observer gain \( H \) under fault-free condition.

\[ e_{k+1} = [A-HC(1-m_k)]e_k - HCm_k e_{k-\tau_{MAD}} + Ed(k) \]  \hspace{1cm} \text{(4-3)}

Thus, as to the discrete-time Markov jump system with time delay,

For mode \( m_k = 0 \), the system matrices are given by

\[ A_{11} = A, A_{21} = HC \]  \hspace{1cm} \text{(4-4)}

For mode \( m_k = 1 \), the system matrices are given by

\[ A_{12} = A + HC, A_{22} = 0 \], and

\[ \bar{P}_1 = p_{11}P_1 + p_{12}P_2 \]  \hspace{1cm} \text{(4-5)}

\[ \bar{P}_2 = p_{21}P_1 + p_{22}P_2 \]

Using the LMI tool we can achieve the observer gain from the matrix equality (3-17) as following:

\[ H_1 = [0.4282; 0.4260], \text{ for the mode } m_k = 0; \]

\[ H_2 = [0.0038; 0.0016], \text{ for the mode } m_k = 1. \]

Update to now, the designed fault detection module can be used to monitor the distributed system by running on the smart node.

In addition, the unknown disturbance \( d(k) \) is supposed to be random noise which is uniform distribution over \([-0.3,0.3]\), and the sample period is 5\( ms \), and the fault signal \( f(k) \) is given as:

\[ f(k) = \begin{cases} 1, & \text{for } k = 400, 401, \ldots, 1000 \\ 0, & \text{others} \end{cases} \]

It is shown in Figure 4.

A two states stochastic variable \( \varphi \) is used to describe the measurement mode with random delays, as is shown in Figure 5. Figure 6 shows the generated residual signal \( r_k \). We selected a threshold as \( J_{th} = E\left\{\sum_{k=0}^{2000} r^T(k)r(k)\right\}^{1/2} \). After the simulations, we get an average value \( J_{th} = 8.4800 \). Through comparison, we can see that \( 9.411 = r(405) > J_{th} \), thus the fault can be detected in the Maximum Allowable Delay - \( \tau_{MAD} \) after its occurrence.

V. CONCLUSION

In this paper, we have studied the fault detection problem for a class of NCS with stochastic time delays, which is modeled as a Markov jump process. The Maximum Allowable Delay is introduced to simplify the practical application on the smart node. We then formulate the fault detection observer design with LMI, and the stability of the error system is tested. Assumptions have brought conservatism which can reduce the performance of the fault detection system. One of our future research topics would be how to reduce the conservatism.

Figure 4: Fault \( f(k) \)

Figure 5: Measurement mode over network
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