Extension of the Second Paden-Kahan Sub-problem and its' Application in the Inverse Kinematics of a Manipulator

Tan Yue-sheng
Industry school
Beijing Forest University
Beijing, China
tanyuesheng@163.com

Xiao Ai-ping
Institute of Automation
Chinese Academy of Sciences
Beijing, China

***Abstract*—Three basic sub-problems of screw theory are acceptable for some particular configuration manipulators' inverse kinematics, which can not solve the inverse kinematics of all configuration manipulators. A new sub-problem is extended based on sub-problem 2, that is, rotation about two disjoint axes, and the inverse kinematics thereof is solved in this paper. Based on the extended sub-problem, a manipulator, the inverse kinematics of which can not be solved by the three sub-problems without the participation of the new sub-problem, is constructed. The inverse kinematics of the manipulator is solved with the help of the extended sub-problem, therefore a close-form solution expressed by motion screw is gained.**

***Keywords*—motion screw, Paden-Kahan sub-problem, inverse kinematics**

I. INTRODUCTION

In recent years, screw theory is widely used in kinematics, dynamics and control of the robot and gets a fine effect. For a general configuration manipulator, Denavit-Hartenberg parameter method is employed to make mathematical model for kinematics and dynamics of the robot, and then a number of value iterative methods [1,2] or neural network methods [3,4,5] or genetic algorithm [6] are taken to solve the inverse kinematics or dynamics of the robot, which not only cost much calculating time for its' high computational complexity, but also the integrality and convergency of the solution can not be warranted. On the other hand, the model based on D-H parameter method requires that each joint and the end-effector has to be connected with a local coordinate in order to represent four parameters of each joint, and different configurations correspond to different solution methods, thus it is not a general method [9]. There are two main advantages to using screw theory. The first is that they allow a global description of rigid body motion which does not suffer from singularities due to the use of local coordinates. Such singularities are inevitable when one chooses to represent rotation via Euler angles, for example. The second advantage is that screw theory provides a very geometric description of rigid motion which greatly simplifies the analysis of mechanisms [7]. The method to solve the inverse kinematics problem based on screw theory generally reduces the full inverse kinematics problem into appropriate sub-problems whose solutions are known. For any configuration manipulator, the inverse kinematics of the manipulator is rather simple if the sub-problems employed are not out of the existing ones. Some scholars such as Chen and Gao [8] , Zhao Jie [9], etc, have solved the inverse kinematics for some manipulators having a special configuration basing on the following three sub-problems and exponential equation. For a manipulator only including rotary joints, the typical sub-problems includes

Sub-problem 1: Rotation about a single axis;
Sub-problem 2: Rotation about two subsequent axes;
Sub-problem 3: Rotation to a given distance.

The above-mentioned three sub-problems are basic ones, which have limitations in application, that is, they can not solve the inverse kinematics of some manipulator in the case all the axes of the adjacent joints are not crossed. In order to solve this problem, a new sub-problem based on the sub-problem 2 is developed, that is, rotation about two disjoint subsequent axes. The solution for the new sub-problem is solved in the paper, and then a special manipulator whose solution has to employ the new sub-problem is configured, finally the inverse kinematics is made to the manipulator based on the screw theory.

II. EXTENSION OF SUBPROBLEM 2: ROTATION ABOUT TWO DISJOINT SUBSEQUENT AXES

Where ξ_1 and ξ_2 are two zero-pitch, unit magnitude twists with non-intersecting axes and $p, q \in \Re^3$ two points, find θ_1 and θ_1 such that

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q$$

The problem is similar as sub-problem 2 described in reference [1] except that the two axes here are non-intersecting.

In Fig.1, c is a point such that

$$e^{\hat{\xi}_2\theta_2} p = c = e^{-\hat{\xi}_1\theta_1} q \tag{1}$$

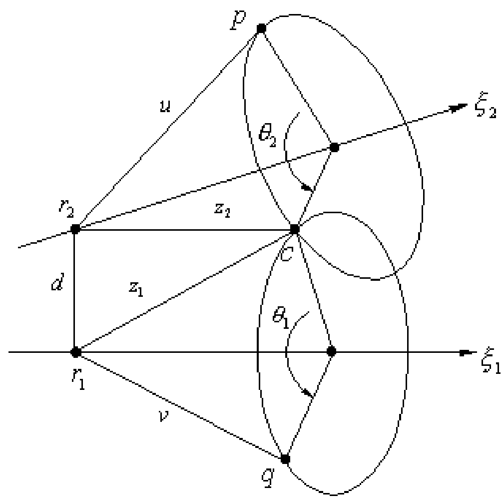

Fig.1 an extended sub-problem of the 2th Paden-Kahan sub-problem

and as shown in Fig.1, d is a common normal between axes ξ_1 and ξ_2, and $d = \|r_2 - r_1\|(\omega_1 \times \omega_2)$, wherein r_1 and r_2 is the point of intersection between the common normal and axes ξ_1 and ξ_2 respectively, hence we can get

$$e^{\hat{\xi}_2\theta_2}(p - r_2) = c - r_2 = z_2 = e^{-\hat{\xi}_1\theta_1} q - r_2$$
(2)

And

$$c - r_1 = z_1 = e^{-\hat{\xi}_1\theta_1} q - r_1, \; d = r_2 - r_1 = z_1 - z_2$$

so that equation (2) becomes

$$e^{\hat{\xi}_2\theta_2}(p - r_2) = e^{-\hat{\xi}_1\theta_1} q - d - r_1 = e^{-\hat{\xi}_1\theta_1}(q - r_1) - d$$
(3)

Let $u = p - r_2$, $v = q - r_1$, $z_1 = c - r_1$, $z_2 = c - r_2$。Substituting these expressions into equation (3) give

$$e^{\hat{\xi}_2\theta_2} u = z_2 = z_1 - d = e^{-\hat{\xi}_1\theta_1} v - d$$
(4)

From equation (4) we can get

$\omega_1^T v = \omega_1^T z_1$ 及 $\omega_2^T u = \omega_2^T z_2 = \omega_2^T(z_1 - d)$, and $\|u\|^2 = \|z_2\|^2 = \|z_1 - d\|^2$, $\|v\|^2 = \|z_1\|^2$
(5)

Since ω_1, ω_2 and $\omega_1 \times \omega_2$ are linely independent, we can write

$$z_2 = \alpha\omega_1 + \beta\omega_2 + \gamma(\omega_1 \times \omega_2)$$
(6)

$$z_1 = \alpha\omega_1 + \beta\omega_2 + \gamma(\omega_1 \times \omega_2) + d = \alpha\omega_1 + \beta\omega_2 + (\gamma + \|d\|)(\omega_1 \times \omega_2)$$
(7)

$$\|z_2\|^2 = \alpha^2 + \beta^2 + 2\alpha\beta\omega_1^T\omega_2 + \gamma^2\|\omega_1 \times \omega_2\|^2$$
(8)

Substituting equation (6) and (7) into equation (5) gives

$$\omega_1^T v = \alpha + \beta\omega_1^T\omega_2 + \omega_1^T d$$
(9)

$$\omega_2^T u = \beta + \alpha\omega_2^T\omega_1$$
(10)

From equation (9) and (10) we can get

$$\alpha = \frac{(\omega_1^T\omega_2)\omega_2^T u - \omega_1^T v}{(\omega_1^T\omega_2)^2 - 1} \tag{11}$$

$$\beta = \frac{(\omega_1^T\omega_2)\omega_1^T v - \omega_2^T u}{(\omega_1^T\omega_2)^2 - 1} \tag{12}$$

Substituting equation (11) and (12) into equation (8) we can get

$$\gamma^2 = \frac{\|u\|^2 - \alpha^2 - \beta^2 - 2\alpha\beta\omega_1^T\omega_2}{\|\omega_1 \times \omega_2\|^2} \tag{13}$$

Substituting α, β, and γ into equation(6) and (7) we can find z_1, z_2, and hence c in the case that a solution exists. Obviously the equation (1) has a same form as that of the sub-problem 1, so we can solve the equation (1) to find θ_1 and θ_2

$$\begin{cases} \theta_2 = a\tan 2(\omega_2^T(u' \times z_2'), u'^T z_2') \\ \theta_1 = a\tan 2(\omega_1^T(z_1' \times v'), z_1'^T v') \end{cases} \tag{14}$$

Where u' and z_2' are the projections of u and z_2 on ω_2, and z_1', v' are the projections of z_1, v on ω_1. If there are multiple solutions for c, each of these solutions gives a value for θ_1 an θ_2.Two solutions exist in the case where the circles in Figure 1 intersect at two points, one solution when the circles are tangential, and none when the circles fail to intersect.

III. EXPERIMENTS

To verify the correctness of the solution the extended sub-problem, we configure a 5-DOF manipulator for which the extended sub-problem must be employed, as shown in Figure2.

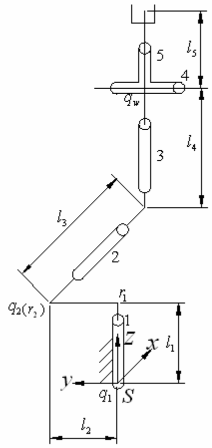

Fig. 2 a 5-DOF manipulator for experiment in its reference configuration

In figure 2, The axes of joint 1 and joint 2 are perpendicular to each other and not intersecting, and the axes of joint 4,5,6 intersect at a point q_w。Where，$l_1 = 355$，$l_2 = 245$，$l_3 = 90$，$l_4 = 300$，$l_5 = 180$。

If $\theta = 0$, the configuration of the fixed coordinate relative to the end-effector coordinator is given by

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & l_1 + l_4 + l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and we can choose axis points

$$q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} q_2 = \begin{bmatrix} l_3 \\ l_2 \\ l_1 \end{bmatrix} q_w = \begin{bmatrix} l_3 \\ l_2 \\ l_1 + l_4 \end{bmatrix}$$

$$\omega_1 = \omega_3 = \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \omega_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

With the provided q_i and ω_i $(i = 1,2,...5)$, we can get the twists of the manipulator. Based on the specific configuration, we determine the requisite joint angles in two steps.

(1) Solve for θ_1 and θ_2

$$e^{\xi_1\theta_1} e^{\xi_2\theta_2} e^{\xi_3\theta_3} e^{\xi_4\theta_4} e^{\xi_5\theta_5} = g_d g_{st}^{-1}(0) = g_1 \qquad (15)$$

Apply both sides of equation (15) to a point p_w which is the common point of intersection for the wrist axes 4, 5 and 6, this yields

$$e^{\xi_1\theta_1} e^{\xi_2\theta_2} e^{\xi_3\theta_3} e^{\xi_4\theta_4} e^{\xi_5\theta_5} q_w = g_1 q_w \Rightarrow e^{\xi_1\theta_1} e^{\xi_2\theta_2} q_w = g_1 q_w \qquad (16)$$

Obviously the equation (16) is in the form of the extended subproblem. Hence applying the extended subproblem, we solve for θ_1 and θ_2.

(2) Solve for θ_3 , θ_4 and θ_5

Since θ_1 and θ_2 are known, so

$$e^{\xi_3\theta_3} e^{\xi_4\theta_4} e^{\xi_5\theta_5} = e^{-\xi_2\theta_2} e^{-\xi_1\theta_1} g_1$$

Let $e^{-\xi_2\theta_2} e^{-\xi_1\theta_1} g_1 = g_2$, we get

$$e^{\xi_3\theta_3} e^{\xi_4\theta_4} e^{\xi_5\theta_5} = g_2 \qquad (17)$$

in this case, we can apply subproblem 2 to solve for θ_3 and θ_4 , and get θ_5 finally with subproblem 1, the concrete steps can refer to reference [8].

To verify the correctness of the algorithm, we give arbitrary groups of joint angles, and get the positions and orientations of the end-effector by applying the forward kinematics. Taking the computed positions and orientations of the end-effector as a known condition, we solve for the joint angles of the manipulator by applying the inverse kinematics. Comparing the given joint angles with the computed joint angle, we found that they are identical, with which the correctness of the algorithm is verified.

IV. CONCLUSION

Based on sub-problem 2 of Paden-Kahan, a new sub-problem is developed and its solving method is provided herein, and its solution is given, which can be applied directly in the inverse kinematics of a manipulator, providing a new approach for the inverse kinematics of a general configuration manipulator. The inverse kinematics process based on the screw theory is more explicit and direct than that of D-H parameters method.

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