Delay-Dependent Robust Reliable Guaranteed Cost Control for Nonlinear Systems with Time-Varying State Delays

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***Abstract*—This paper concerns the reliable guaranteed cost control problem of nonlinear systems with time-varying state delays and actuator failures for a given quadratic cost function. The problem is to design a delay-dependent reliable guaranteed cost state feedback control law which can tolerate actuator failures, such that the closed-loop cost function value is not more than a specified upper bound. Based on the linear matrix inequality (LMI) approach, a sufficient condition for the existence of reliable guaranteed cost controllers is derived. Furthermore, a convex optimization problem with LMI constraints is formulated to design the optimal reliable guaranteed cost controller which minimizes the upper bound of the closed-loop system cost. A numerical example is given to illustrate the proposed method.**

***Keywords*—Actuator failure, Guaranteed cost control, Nonlinear systems, Reliable control**

I. INTRODUCTION

The problem of designing robust controllers for systems with parameter uncertainties has drawn considerable attention in recent control system literatures. It is also desirable to design a control system which is not only stable but also guarantees an adequate level of performance. One approach to this problem is the so-called guaranteed cost control approach first introduced by Chang and Peng [1]. This approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by the uncertainties is guaranteed to be less than this bound. Based on this idea, some significant results have been proposed for the continuous-time case [2, 3] and for the discrete-time case [4].

In practical application, actuators are very important in transforming the controller output to the plant. Actuator failures may be encountered sometimes. Furthermore, how to preserve the closed-loop system performance in the case of actuator failures will be tougher and more meaningful. Recently, there have been some efforts to tackle the reliable guaranteed cost controller design problem, and some good results have also been obtained for the continuous-time case [5] and for the discrete-time case [6]. However, up to our knowledge, there have been few results in the literature of an investigation for the reliable guaranteed cost controller design of nonlinear uncertain systems with time-varying state delay and actuator failure.

In this paper, the problem of reliable guaranteed cost control for nonlinear systems with time-varying state delays is considered. In Section 2, the problem under consideration and some preliminaries are given. In section 3, several stability criteria for the existence of the reliable guaranteed cost controller are derived in terms of LMI, and their solutions provide a parameterized representation of the controller. A numerical example is given in Section 4. Finally, Section 5 concludes the paper.

II. PROBLEM STATEMENT

Consider the following nonlinear systems with time-varying state delays

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{k} A_i x(t-\tau_i(t)) + Bu(t) + f(t, x(t), x(t-\tau_1(t)), \cdots, x(t-\tau_k(t))), t \in R^+,$$
$$x(t) = \phi(t), \quad t \in [-h, 0], \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) = [u_1(t) \quad u_2(t) \quad \cdots \quad u_m(t)]^T \in R^m$ is the control vector, A, A_i and B are known real constant matrices of appropriate dimensions, $f(\cdot): R_+ \times R^n \to R^n$ is the nonlinear uncertainties, being denoted f in the following. $\tau_i(t)$ is the time-varying bounded delay satisfying

$$0 \le \tau_i(t) \le h_i < \infty, \ \dot{\tau}_i(t) \le d_i < 1, \ i = 1, \cdots, k$$
$$h = \max\{h_i : i = 1, \cdots, k\}, \ d = \max\{d_i : i = 1, \cdots, k\}. \tag{2}$$

ϕ is a given continuous vector-valued initial function on $[-h, 0]$.

Assumption 1. The nonlinear uncertainty f satisfies

$$f^T f \le \Xi^T H \Xi, \tag{3}$$

where $\Xi = col\{x(t) \quad x(t-\tau_1(t)) \quad \cdots \quad x(t-\tau_k(t))\}$, and H is a known constant matrix satisfying $H = \text{Block-diag}\{H_0^T H_1, H_1^T H_1, \cdots, H_k^T H_k\} > 0$.

Associated with this system is the cost function

$$J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt, \tag{4}$$

where Q and R are given positive-definite matrices.

For the control input $u_i(t)$, $i = 1,2,\cdots,m$, let $u_i^F(t)$ denote the signal from the actuator that has failed. The following failure model is adopted in this paper:

$$u_i^F(t) = \alpha_i u_i(t), \quad i = 1,2,\cdots,m, \tag{5}$$

where

$$0 \le \hat{\alpha}_i \le \alpha_i \le \breve{\alpha}_i, \quad i = 1,2,\cdots,m \tag{6}$$

with $\hat{\alpha}_i \le 1$ and $\breve{\alpha}_i \ge 1$.

In the above model of actuator failure, if $\hat{\alpha}_i = \breve{\alpha}_i$, then it corresponds to the normal case $u_i^F(t) = u_i(t)$. When $\breve{\alpha}_i = 0$, it covers the outage case. If $\hat{\alpha}_i > 0$, it corresponds to the partial failure case, namely, partial degradation of the actuator.

Denote

$$\begin{aligned} u^F(t) &= \begin{bmatrix} u_1^F(t) & u_2^F(t) & \cdots & u_m^F(t) \end{bmatrix}^T, \\ \breve{\alpha} &= \text{diag}\{\breve{\alpha}_1, \breve{\alpha}_2, \cdots, \breve{\alpha}_m\}, \\ \hat{\alpha} &= \text{diag}\{\hat{\alpha}_1, \hat{\alpha}_2, \cdots, \hat{\alpha}_m\}, \\ \alpha &= \text{diag}\{\alpha_1, \alpha_2, \cdots, \alpha_m\}, \end{aligned} \tag{7}$$

α is said to be admissible if α satisfies $\hat{\alpha} \le \alpha \le \breve{\alpha}$.

The objective of this paper is to develop a procedure to design a memoryless state feedback control law

$$u(t) = Kx(t), \tag{8}$$

such that for any admissible uncertain α, the resulting closed-loop system

$$\begin{aligned} &\dot{x}(t) = (A + B\alpha K)x(t) + \sum_{i=1}^{k} A_i x(t-\tau_i(t)) + f, \quad t \in R^+, \\ &x(t) = \phi(t), \quad t \in [-h, 0] \end{aligned} \tag{9}$$

is asymptotically stable and the cost function (4) satisfies $J \le J^*$, where J^* is some specified constant.

Definition 1. If there exists a control $u(t) = Kx(t)$ and a positive scalar J^* such that for all admissible α, the closed-loop system (9) is asymptotically stable and $J \le J^*$, then J^* is said to be a guaranteed cost and $u(t) = Kx(t)$ is said to be a reliable guaranteed cost control law for system (1) and cost function (4).

Define

$$\begin{aligned} \beta &= \text{diag}\{\beta_1, \beta_2, \cdots, \beta_m\}, \\ \beta_0 &= \text{diag}\{\beta_{10}, \beta_{20}, \cdots, \beta_{m0}\}, \end{aligned} \tag{10}$$

where

$$\beta_i = \frac{\hat{\alpha}_i + \breve{\alpha}_i}{2}, \quad \beta_{i0} = \frac{\breve{\alpha}_i - \hat{\alpha}_i}{\breve{\alpha}_i + \hat{\alpha}_i}, \quad i = 1,2,\cdots,m. \tag{11}$$

From (7) and (10), we define

$$\alpha = (I + \alpha_0)\beta \tag{12}$$

and

$$|\alpha_0| \le \beta_0 \le I, \tag{13}$$

where $\alpha_0 = \text{diag}\{\alpha_{01}, \alpha_{02}, \cdots, \alpha_{0m}\}$, and $|\alpha_0| = \text{diag}\{|\alpha_{01}|, |\alpha_{02}|, \cdots, |\alpha_{0m}|\}$.

Lemma 1.(Barmish [7]) *Given matrices Y, H, E of appropriate dimensions and with Y symmetric, then for all F satisfying $F^T F \le I$, $Y + HFE + E^T F^T H^T < 0$ holds, if and only if there exists $\varepsilon > 0$ such that $Y + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$.*

Lemma 2.(Moon et al. [8]) *Assume that $a \in R^p$, $b \in R^q$, and $N \in R^{p\times q}$, then for any matrices $Z \in R^{p\times p}$, $Y \in R^{p\times q}$, $\bar{R} \in R^{q\times q}$, the following holds:*

$$-2a^T Nb \le \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} Z & Y - N \\ Y^T - N^T & \bar{R} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix},$$

if

$$\begin{bmatrix} Z & Y \\ Y^T & \bar{R} \end{bmatrix} \ge 0.$$

III. MAIN RESULTS

Since it holds that

$$x(t-\tau_i(t)) = x(t) - \int_{t-\tau_i(t)}^{t} \dot{x}(s)ds$$

Then, rewrite system (9) in an equivalent form

$$\begin{aligned} \dot{x}(t) &= (A + B\alpha K + \sum_{i=1}^{k} A_i)x(t) \\ &\quad - \sum_{i=1}^{k} A_i \int_{t-\tau_i(t)}^{t} \dot{x}(s)ds + f, \quad t \in R^+, \\ x(t) &= \phi(t), \quad t \in [-h, 0], \end{aligned} \tag{14}$$

The following Lyapunov-Krasovskii functional is applied

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{15}$$

where

$$V_1(t) = x^T(t)Px(t)\,, \tag{16}$$

$$V_2(t) = \sum_{i=1}^{k}\int_{-\tau_i(t)}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\bar{R}_i\dot{x}(s)dsd\theta\,, \tag{17}$$

$$V_3 = \sum_{i=1}^{k}\int_{t-\tau_i(t)}^{t}x^T(\tau)S_i x(\tau)d\tau\,. \tag{18}$$

Then, the following theorem gives the delay-dependent reliable guaranteed cost control for the systems (1) and (4).

Theorem 1. $u(t) = Kx(t)$ *is a reliable guaranteed cost control law if there exist positive-definite matrices* P, S_i, $\bar{R}_i$, *matrices* Y_i, Z_i, *and a scalar* $\varepsilon_1 > 0$, *such that for any admissible* α, *the following matrix inequalities hold:*

$$\Gamma = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \cdots & \Psi_{1,k+2} \\ * & \Psi_{22} & \cdots & \Psi_{2,k+2} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & * & \Psi_{k+2,k+2} \end{bmatrix} < 0\,, \tag{19}$$

and

$$\begin{bmatrix} Z_i & Y_i \\ * & \bar{R}_i \end{bmatrix} \geq 0\,, \quad i = 1,2,\cdots,k\,, \tag{20}$$

where $(*)$ *denotes the symmetric element of a matrix, and*

$$\begin{aligned}
\Psi_{11} &= \sum_{i=1}^{k}\left[A + B\alpha K\right]^T h_i\bar{R}_i\left[A + B\alpha K\right] \\
&\quad + \left[A + B\alpha K\right]^T P + P\left[A + B\alpha K\right] \\
&\quad + \sum_{i=1}^{k}(h_i Z_i + Y_i + Y_i^T + S_i) + \varepsilon_1^{-1}H_0^T H_0 \\
&\quad + Q + K^T\alpha R\alpha K, \\
\Psi_{1j} &= \sum_{i=1}^{k}\left[A + B\alpha K\right]^T h_i\bar{R}_i A_{j-1} + PA_{j-1} - Y_{j-1}, \\
\Psi_{1,k+2} &= \sum_{i=1}^{k}\left[A + B\alpha K\right]^T h_i\bar{R}_i + P, \\
\Psi_{lj} &= \sum_{i=1}^{k}A_{l-1}^T h_i\bar{R}_i A_{j-1}, \Psi_{l,k+2} = \sum_{i=1}^{k}A_{l-1}^T h_i\bar{R}_i, \\
\Psi_{jj} &= -S_{j-1}(1 - d_{j-1}) + \varepsilon_1^{-1}H_{j-1}^T H_{j-1} + \sum_{i=1}^{k}A_{j-1}^T h_i\bar{R}_i A_{j-1}, \\
\Psi_{k+2,k+2} &= \sum_{i=1}^{k}h_i\bar{R}_i - \varepsilon_1^{-1}I, \\
l &= 2,3,\cdots k+1, \quad j = 2,3,\cdots k+1, \quad l \neq j.
\end{aligned} \tag{21}$$

Moreover, the cost function (4) *satisfies the following bound:*

$$\begin{aligned}
J &\leq x^T(0)Px(0) + \sum_{i=1}^{k}\int_{-\tau_i(t)}^{0}\int_{\theta}^{0}\dot{x}^T(s)\bar{R}_i\dot{x}(s)dsd\theta \\
&\quad + \sum_{i=1}^{k}\int_{-\tau_i(t)}^{0}x^T(\tau)S_i x(\tau)d\tau = J^*.
\end{aligned} \tag{22}$$

Proof. Taking $u(t) = Kx(t)$ in the system (1), the resulting closed-loop system is given by (9).

Differentiating $V_1(t)$ with respect to t gives

$$\begin{aligned}
&\dot{V}_1(t) \\
&= x^T(t)\left\{[A + B\alpha K + \sum_{i=1}^{k}A_i]^T P \right. \\
&\quad \left. + P[A + B\alpha K + \sum_{i=1}^{k}A_i]\right\}x(t) \\
&\quad - 2\sum_{i=1}^{k}x^T(t)PA_i\int_{t-\tau_i(t)}^{t}\dot{x}(s)ds + x^T(t)Pf + f^T Px(t)
\end{aligned} \tag{23}$$

Using lemma 2, taking

$$\begin{aligned}
&N = N_i = PA_i\,, Z = Z_i\,, Y = Y_i\,, \\
&\bar{R} = \bar{R}_i\,, a = x(t)\,, b = \dot{x}(s)\,,
\end{aligned} \tag{24}$$

we obtain

$$\begin{aligned}
&-2x^T(t)PA_i\int_{t-\tau_i(t)}^{t}\dot{x}(s)ds \\
&\leq \int_{t-\tau_i(t)}^{t}\begin{bmatrix} x^T(t) & \dot{x}^T(s)\end{bmatrix}\begin{bmatrix} Z_i & Y_i - PA_i \\ Y_i^T - A_i^T P & \bar{R}_i \end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(s)\end{bmatrix}ds \\
&\leq x^T(t)\left(Y_i^T - A_i^T P + Y_i - PA_i\right)x(t) \\
&\quad + \int_{t-\tau_i(t)}^{t}\dot{x}^T(s)\bar{R}_i\dot{x}(s)ds + h_i x^T(t)Z_i x(t) \\
&\quad - x^T(t-\tau_i(t))\left(Y_i^T - A_i^T P\right)x(t) \\
&\quad - x^T(t)\left(Y_i - PA_i\right)x(t-\tau_i(t))
\end{aligned} \tag{25}$$

Substituting (25) into (23) and using (3), we have

$$\begin{aligned}
&\dot{V}_1(t) \\
&\leq x^T(t)\Big[(A + B\alpha K)^T P + P(A + B\alpha K) \\
&\quad + \sum_{i=1}^{k}(h_i Z_i + Y_i^T + Y_i)\Big]x(t) + x^T(t)Pf \\
&\quad + f^T Px(t) - \sum_{i=1}^{k}x^T(t)\left(Y_i - PA_i\right)x(t-\tau_i(t)) \\
&\quad - \sum_{i=1}^{k}x^T(t-\tau_i(t))\left(Y_i^T - A_i^T P\right)x(t)
\end{aligned}$$

$$+\sum_{i=1}^{k}\int_{t-\tau_i(t)}^{t}\dot{x}^T(s)\bar{R}_i\dot{x}(s)ds-\varepsilon_1^{-1}f^Tf+\varepsilon_1^{-1}\Xi^T H\Xi$$

Differentiating $V_2(t)$ and $V_3(t)$ with respect to t gives

$$\begin{aligned}&\dot{V}_2(t)\\ &\le\sum_{i=1}^{k}h_i[(A+B\alpha K)x(t)+\sum_{i=1}^{k}A_ix(t-\tau_i(t))+f]^T\\ &\quad\bullet R_i[(A+B\alpha K)x(t)+\sum_{i=1}^{k}A_ix(t-\tau_i(t))+f]\\ &\quad-\sum_{i=1}^{k}\int_{t-\tau_i(t)}^{t}\dot{x}^T(s)\bar{R}_i\dot{x}(s)ds\end{aligned}$$

$$\begin{aligned}&\dot{V}_3(t)\\ &=\sum_{i=1}^{k}x^T(t)S_ix(t)-\sum_{i=1}^{k}(1-\dot{\tau}_i(t))x^T(t-\tau_i(t))S_ix(t-\tau_i(t))\\ &\le\sum_{i=1}^{k}x^T(t)S_ix(t)-\sum_{i=1}^{k}(1-d_i)x^T(t-\tau_i(t))S_ix(t-\tau_i(t))\end{aligned}$$

From the above inequalities, we can obtain

$$\begin{aligned}\dot{V}(t)&\le\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)\\ &\le\zeta^T(t)\Psi\zeta(t)-x^T(t)[Q+K^T\alpha R\alpha K]x(t),\end{aligned}\qquad(26)$$

where

$$\zeta(t)=col\{x(t)\quad x(t-\tau_1(t))\quad\cdots\quad x(t-\tau_k(t))\quad f\}.\qquad(27)$$

Then, the matrix inequality (19) implies that

$$\dot{V}(t)<-x^T(t)[Q+K^T\alpha R\alpha K]x(t)<0.\qquad(28)$$

Noting that $Q>0$ and $R>0$, this implies that the system (9) is asymptotically stable by Lyapunov stability theory. Moreover, from (28) we have

$$x^T(t)[Q+K^T\alpha R\alpha K]x(t)<-\dot{V}(t),\qquad(29)$$

by integrating both sides of (29) from 0 to T_f, we obtain

$$\int_0^{T_f}x^T(t)[Q+K^T\alpha R\alpha K]x(t)dt<V(0)-V(T_f).\qquad(30)$$

As the closed-loop systems (9) is asymptotically stable when $T_f\to\infty$, we get $V(T_f)\to\infty$, that is

$$\int_0^{\infty}(x^T(t)Qx(t)+u^T(t)Ru(t))dt\le V(0)=J^*.\qquad(31)$$

This completes the proof.

The criterion for the existence of guaranteed cost controller is equivalent to the feasibility of a LMI.

Theorem 2. For system (1) with cost function (4), if there exist scalars $\varepsilon_1>0$, matrices $X>0$, W, $M_i>0$, $R_0>0$, such that the following LMI is feasible:

$$\left[\begin{array}{cccccccccc}
\Omega_1 & A_1M_1 & \cdots & A_kM_k & \varepsilon_1 I & \Omega_2 & \cdots & \Omega_2 & \beta W^T & BR_0\\
* & (d_1-1)M_1 & \cdots & 0 & 0 & (A_1M_1)^T & \cdots & (A_1M_1)^T & 0 & 0\\
* & * & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots\\
* & * & * & (d_k-1)M_k & 0 & (A_kM_k)^T & \cdots & (A_kM_k)^T & \vdots & 0\\
* & * & * & * & -\varepsilon_1 I & -\varepsilon_1 I & \cdots & -\varepsilon_1 I & \vdots & 0\\
* & * & * & * & * & -h_1^{-1}\bar{R}_1^{-1} & \cdots & 0 & \vdots & BR_0\\
* & * & * & * & * & * & \ddots & \vdots & \vdots & \vdots\\
* & * & * & * & * & * & * & -h_k^{-1}\bar{R}_k^{-1} & 0 & BR_0\\
* & * & * & * & * & * & * & * & -R^{-1} & R_0\\
* & * & * & * & * & * & * & * & * & -R_0\\
* & * & * & * & * & * & * & * & * & *\\
* & * & * & * & * & * & * & * & * & *\\
* & * & * & * & * & * & * & * & * & *\\
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* & * & * & * & * & * & * & * & * & *\\
* & * & * & * & * & * & * & * & * & *
\end{array}\right.$$

$$\left.\begin{matrix} W^T\beta\beta_0 & (H_0X)^T & 0 & \cdots & 0 & \Omega_3 \\ 0 & 0 & (H_1M_1)^T & 0 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & 0 & (H_kM_k)^T & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ -R_0 & 0 & \vdots & \vdots & \vdots & \vdots \\ * & -\varepsilon_1 I & 0 & \vdots & \vdots & \vdots \\ * & * & -\varepsilon_1 I & \vdots & \vdots & \vdots \\ * & * & * & \ddots & \vdots & \vdots \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & \Omega_4 \end{matrix}\right] < 0 \tag{32}$$

where

$$X = P^{-1}, \quad W = KP^{-1}, \quad M_i = S_i^{-1},$$

$$\Omega_1 = (AX + BW)^T + AX + BW,$$

$$\Omega_2 = (AX + B\beta W)^T, \quad \Omega_3 = [\underbrace{X^T, \cdots, X^T}_{2k+1}],$$

$$\Omega_4 = \left[-h_1^{-1}Z_1^{-1}, \cdots, -h_k^{-1}Z_k^{-1}, -M_1, \cdots - M_k, -Q^{-1}\right],$$

then

$$u(k) = WX^{-1}x(k) \tag{33}$$

is a reliable guaranteed cost controller of system (1), and the corresponding closed-loop value of the cost function satisfies

$$J \le x^T(0)Px(0) + \sum_{i=1}^{k}\int_{-\tau_i(t)}^{0}\int_{\theta}^{0}\dot{x}^T(s)\bar{R}_i\dot{x}(s)dsd\theta$$
$$+ \sum_{i=1}^{k}\int_{-\tau_i(t)}^{0}x^T(\tau)M_i^{-1}x(\tau)d\tau = J^*.$$

Proof. Letting $Y \equiv 0$, $\bar{R}_i > 0$ and $Z_i > 0$ in (20), in light of Lemma 1 and using the Schur complement, we can obtain the results. The proof is omitted.

The following theorem presents a method of selecting a controller minimizing the upper bound of the guaranteed cost (34).

Theorem 3. *Consider the systems* (1) *with performance index* (4), *if the following optimization problem*

$$\min \quad \xi + \sum_{i=1}^{k}(tr(\Gamma_{1i}) + tr(\Gamma_{2i}))$$

(i) (32),

(ii) $\begin{bmatrix} -\xi & x^T(0) \\ x(0) & -X \end{bmatrix} < 0$,

(iii) $\begin{bmatrix} -\Gamma_{1i} & C^T \\ C & -\bar{R}_i^{-1} \end{bmatrix} < 0$,

(iv) $\begin{bmatrix} -\Gamma_{2i} & D^T \\ D & -M_i \end{bmatrix} < 0$, (37)

has a solution set $(X, \xi, \Gamma_{1i}, \Gamma_{2i})$, *then the controller* (33) *is an optimal reliable guaranteed cost control law which ensures the minimization of the guaranteed cost* (34) *for system* (1), *where*

$$\int_{-\tau_i(t)}^{0}\int_{\theta}^{0}\dot{x}(s)\dot{x}^T(s)dsd\theta = C^TC,$$
$$\int_{-\tau_i(t)}^{0}x(\tau)x^T(\tau)d\tau = D^TD. \tag{38}$$

Proof. By Theorem 2, (i) in (37) is clear. Using the Schur complement, (ii)-(iv) in (37) are equivalent to

$$x^T(0)X^{-1}x(0) < \xi, \quad C^T\bar{R}_iC < \Gamma_{1i}, D^TM_i^{-1}D < \Gamma_{2i}, \tag{39}$$

respectively. Furthermore,

$$\int_{-\tau_i(t)}^{0}\int_{\theta}^{0}\dot{x}^T(s)\bar{R}_i\dot{x}(s)dsd\theta = tr(C^T\bar{R}_iC) < tr(\Gamma_{1i}), \tag{40}$$

and

$$\int_{-\tau_i(t)}^{0}x^T(\tau)M_i^{-1}x(\tau)d\tau = tr(D^TM_i^{-1}D) < tr(\Gamma_{2i}). \tag{41}$$

Hence, it follows from (34) that

$$J^* = x^T(0)X^{-1}x(0) + \int_{-\tau_i(t)}^{0}\int_{\theta}^{0}\dot{x}^T(s)\bar{R}_i\dot{x}(s)dsd\theta$$
$$+ \int_{-\tau_i(t)}^{0}x^T(\tau)M_i^{-1}x(\tau)d\tau$$
$$\le \xi + tr(\Gamma_{1i}) + tr(\Gamma_{2i}) \tag{42}$$

Thus, the minimization of (42) implies the minimization of the guaranteed cost for the system (1). This completes the proof.

IV. Simulations

Consider the nonlinear dynamic systems with multiple time-varying delays in (1)-(4) with

$$A = \begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.15 \\ 0.8 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0.23 \\ 0.95 & 0 \end{bmatrix},$$

$$B=\begin{bmatrix}3\\1\end{bmatrix},\ H_0=H_1=H_2=\begin{bmatrix}0.1 & 0\\0 & 0.1\end{bmatrix},$$

$$Q=\mathrm{diag}\{1,\ 1\},\ \ R=\bar{R}_1=\bar{R}_2=0.2,$$

$$h=1,\ \ d=0.5,\ \ Z_1=Z_2=\begin{bmatrix}0.1 & 0\\0 & 0.1\end{bmatrix}.$$

It is assumed that the single input to the system has partial failure as follows:

$\hat{\alpha}=0.8$, $\breve{\alpha}=1.2$.

By applying Theorem 3 and solving the corresponding optimization problem (37), the optimal reliable guaranteed cost controller is given by

$$u(k)=\begin{bmatrix}-7.4532 & -2.8194\end{bmatrix}x(k),$$

and the upper bound of the corresponding closed-loop cost function is $J^*=70.4173$.

V. Conclusion

In this paper, based on the Lyapunov method, we have presented a design method to the reliable guaranteed cost controller via memoryless state feedback control for nonlinear systems with time-varying state delays in an LMI framework. The parameterized representation of a set of the controller, which guaranteed not only the robust stability of the closed-loop system but also the cost function bound constraint, has been provided in terms of the feasible solutions to the LMIs. Furthermore, a convex optimization problem has been introduced to select the optimal reliable guaranteed cost controller.

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