Research of Time-frequency Analysis Method of Nonstationary Periodic Signal

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***Abstract*—Nonstationary periodic signal(NPS) is a sort of stochastic signal whose period follow normal distribution, and the statistic average value of period (SAVP) of NPS is very important but difficult to yield. Several time-frequency analysis method including short time power spectrogram(STPS), Wigner-Ville distribution(WVD) and wavelet transform(WT) are employed to obtain the SAVP of NPS, then Gauss white noise and color noise with large amplitude are used to verify the anti-interference capability of these methods. The analytic results show that STPS with inertia filtering can separate NPS from noise efficiently and has good time-frequency resolution of NPS, which means high measurement precision of SAVP. Smooth pseudo Wigner-Ville Distribution (SPWVD) and redistribution SPWVD have excellent time-frequency aggregation and resolution, but their anti-interference capability is a flaw. WT has powerful ability of anti-disturbance. In sum, all the three methods can compute SAVP of NPS, among them, STPS and WVD have higher measurement precision while WT has more perfect dynamic characteristic.**

***Keywords*—nonstationary periodic signal, short time power spectrogram, Wigner-Ville distribution, wavelet transform, time-frequency analysis**

I. INTRODUCTION

Nonstationary periodic signal (NPS) is a kind of important stochastic signal which can be expressed as a periodic function with normal distribution of period and magnitude. Doppler signal, cockpit voice signal and many fault diagnosis signals in industrial process all can be regarded as the NPS[1][2].

The period and frequency of NPS are stochastic variable, while the statistic average value of period (SAVP) or frequency of NPS are very important to practice, for example, the statistic average value of frequency (SAVF) of Doppler signal is capable of reflecting the relative velocity between signal source and observation point[3], the SAVP of fault diagnosis signal can be employed to predict the fault occurrence[1], The SAVP of fluctuation component of paper-making machine velocity signal is commonly taken as accident diagnostic criterion to judge whether paper breaking happens or not. But it is very difficulty to calculate SAVP of NPS in time domain. In view of the time-variation of the SAVP of NPS, time-frequency analysis methods are employed to obtain them.

Currently the methods of time-frequency analysis of nonstationary signal can be classified as linear and nonlinear, the linear methods mainly include Short Time Fourier Transform(STFT), Wavelet Transform(WT) and Gabor expansion, while the nonlinear methods is mainly Wigner-Ville Distribution based on Cohen type[4].

In the paper, we aimed at the ultrasonic echo signal which is yielded by ultrasonic sensor through measuring a two-phase mixture with solid and gas moving in pipeline. STPS has time-frequency statistic characteristic, WVD is a global analysis of energy distribution of a signal at certain moment, can also provides the capability of time-frequency statistics. Therefore STPS and WVD can transform SAVF into a peak frequency where STPS or WVD reachs its maximum, that is, the peak frequency is just the exact estimation of SAVF of NPS, so all above two methods can get the SAVP of NPS. Because WT has favorable local character, the WT of NPS appears clear periodicity in different scales, so the SAVP of NPS can be calculated directly according to the period of WT. In the paper, preceding time-frequency analyses methods are discussed from the aspects of time-frequency resolution, time-frequency aggregation, anti-disturbance ability and dynamic performance.

II. NONSTATIONARY PERIODIC SIGNAL AND ITS STATISTIC AVERAGE VALUE OF PERIOD

Some stochastic signal can be expressed as following form:

$$s(t) = As(t + nT) + \nu(t) \tag{1}$$

In (1), the period T and amplitude A are stochastic variable which follow normal distribution around SAVP $\overline{T}$ and value EA (=1) respectively, $\nu(t)$ is a noise with zero mean. We employ mathematic expectation to the (1), since A and T have no correlation each other, so we yield

$$Es(t) = E[As(t + nT) + \nu(t)] = Es(t + n\overline{T}) \tag{2}$$

It is obvious that $s(t)$ is a nonstationary signal whose statistic average value has periodicity, it is called nonstationary periodic signal(NPS). NPS exist extensively, its SAVP and SAVF has great significance in practice. The calculation of SAVP $\overline{T}$ is of no ease because NPS is a nonstationary signal, i.e. $\overline{T}$ can be different at different time.

III. TIME-FREQUENCY ANALYSIS OF NONSTATIONARY PERIODIC SIGNAL

In this paper, Doppler signal $s_1(t)$ is the research target as a representative NPS, the Fig. 1 is pictured according to partial signals of $s_1(t)$. It is evident that it is extremely difficult to compute SAVP or SAVF in time domain.

The 2-D spectrum $S_1(t,f)$ is the time-frequency analysis of $s_1(t)$, and peak frequency $f_{\max}$ is a frequency which make $S_1(t,f)$ reach maximum. It is found that $f_{\max}$ is the exact estimation of SAVF at time t. Therefore the time-frequency analysis of NPS can be the valid method to obtain SAVF and SAVP.

A. Short time power spectrum

Short time power spectrum (STPS) combines the characteristics of short time Fourier transform and power spectrum estimation. Its calculating step is that, Firstly, signal $s_1(t)$ is stationarized through adding a time window function $h(t)$, then power spectrum estimation is curried out, finally with the shift of $h(t)$, yield STPS $P_x(t,f)$ at every time.

$$P_x(t,f)=\left|\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} s_1(u)h^*(u-t)e^{-j2\pi fu}du\right|^2 \tag{3}$$

In (3), $h(t)$ is a analysis window with the center time t=0, can constrain efficiently the signals which exist outside the adjacent region where u is equal to t, so STPS $P_x(t,f)$ is the local spectrum in the region of time t. STPS neglects the phase message and has strong statistic characteristic. The following figures show the STPS of Doppler signal $s_1(t)$.

In Fig. 2(a), It shows that the spectral line of peak frequency (pointed by white arrow) is clear, which reflects excellent time-frequency resolution, by this way, we can achieve the exact SAVF $\overline{f}$ at every time t. At the same time, we can note that the spectral line appears fracture at time interval from 9 to 11 millisecond, and at time 13.5 millisecond the nearby spectral line appears bifurcation and shift. The Fig. 2(b) is the STPS after inertia filtering, we can see the spectrum with fracture and bifurcation in Fig. 2(a) has been restored perfectly in spite of a little broadening of spectral line, it is clear that the resolution is slightly reduced and inertia filtering takes inevitably some effects on the dynamic performance of the calculation process. From Fig. 2(b), we can obtain the SAVF which is approximate from 57.8 to 58.6 Kilohertz, the relative error to the real frequency 58.4 Kilohertz is just only 1%.

In a word, STPS with inertia filtering can be applied to the SAVF calculation of NPS with high precision but slow analysis process.

B. Wigner-Ville distribution

It is Wigner who presented Wigner-Ville distribution (WVD) from the research of quantum mechanics in 1932, and in 1984 Ville applied WVD to signal analysis field. The definition of WVD for signal $s(t)$ is expressed as follows

$$W_x(t,f)=\int_{-\infty}^{\infty} s(t+\frac{q}{2})s^*(t-\frac{q}{2})e^{-j2\pi fq}dq \tag{4}$$

Here q is a time variable, because of the periodicity of triangle function $e^{-j2\pi fq}$ in (4), WVD has the capacity of extracting SAVT of a signal. However the WVD definition of (4) may produce cross-term among the initial frequencies so as to bring false frequency. Therefore (4) can not used directly and we can resort to the improved WVD such as SPWVD and redistribution SPWVD to process time-frequency analysis.

1) Smooth pseudo Wigner-Ville Distribution:

Smooth pseudo Wigner-Ville Distribution(SPWVD) sets windows to the variables both in time domain and frequency domain at the same time so that cross-term can be reduced. It is defined as follows

$$\begin{aligned} SPW_x(t,f) = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-u+\frac{q}{2}) \\ &\cdot x(t-u-\frac{q}{2})h(q)g(u)e^{-j2\pi qf}\,dqdf \end{aligned} \tag{5}$$

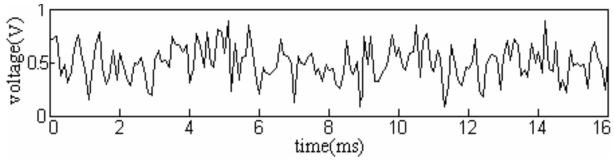
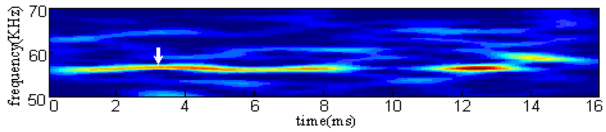
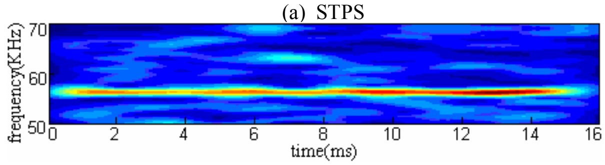


Figure 1. The curve of Doppler signal



(a) STPS



(b) STPS after inertia filtering

Figure 2. STPS to Doppler signal

in (5) $h(q)$ and $g(u)$ are both real symmetrical windows, and submit to $h(0)=g(0)$ [5]. In fact this spectrum is a compromise between the time-frequency aggregation and attenuation of cross-term. It attenuates the cross-term of WVD signal at the expense of time-frequency resolution descent.

2) Redistribution smooth pseudo Wigner-Ville Distribution:

As a bilinear energy distribution, spectrum of WVD can be written as the form of 2D-convolution with the signal WVD and smooth kernel function.

$$C_x(t,v,M)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W_x(s,p) \cdot M_h(t-s,v-p)dsdp \tag{6}$$

Notice that the geometrical center (t, v) of smooth kernel function $M(t-s,v-p)$ has no theoretical dependence, so the new geometrical center ($\hat{t}$, $\hat{v}$) needs to be recalculated as follows.

$$\hat{t}=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}sM(t-s,v-p)W_x(s,p)dsdp}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}M(t-s,v-p)W_x(s,p)dsdp} \tag{7}$$

$$\hat{v}=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}pM_h(t-s,v-p)W_x(s,p)dsdp}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Pi(t-s,v-p)W_x(s,p)dsdp} \tag{8}$$

So we can yield the rearranged spectrum

$$C_x'(t',v';M)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C(t,v;M) \cdot D(t'-\hat{t})D(v'-\hat{v})dtdv \tag{9}$$

Here $D(\cdot)$ is a impact function, selecting appropriate smooth kernel, the rearranged SPWVD can restrain efficiently the cross-term as well as enhance the capability of time-frequency aggregation [6].

3) Time-frequency analysis of NPS based on Wigner-Ville distribution:

Fig. 3 shows several WVD spectrums of Doppler signal $s_1(t)$, Fig. 3(a) is WVD of $s_1(t)$, Fig. 3(b) is SPWVD of $s_1(t)$, Fig. 3(c) is redistribution SPWVD, and Fig. 3(d) is redistribution SPWVD of $s_1(t)$ after inertia filtering.

In Fig. 3(a) the cross-terms of different frequency signal bring an amount of pseudo frequencies, it will produce that WVD can't be directly employed. In Fig. 3(b) SPWVD has better time-frequency aggregation, and the cross-terms can almost be eliminated, but the spectral line of SPWVD is somewhat obscure especially at time interval 2.8 to 3.6 millisecond, which shows the peak value of SPWVD spectrum is not enough prominent, so its time-frequency analysis capability to NPS is relatively low. In Fig. 3(c) the spectral line of redistribution SPWVD is quite clear, which reflects that the method has the excellent character of time-frequency aggregation, but the spectral line is not enough smooth and it shows that redistribution SPWVD has lower time-frequency resolution compared with SPWVD. The frequency measured according Fig. 3(d) at time interval 0 to 16 millisecond is between 63.1 Kilohertz and 64.4 Kilohertz, which shows that the relative error to the real frequency 63.6 Kilohertz is only 1.25%.

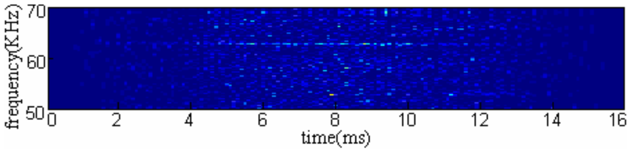
Anyway, among all WVD methods, redistribution SPWVD and redistribution SPWVD with inertia filtering can be applied to calculate the SAVP of NPS with high precision and good dynamic performance.

C. Continuous wavelet transform

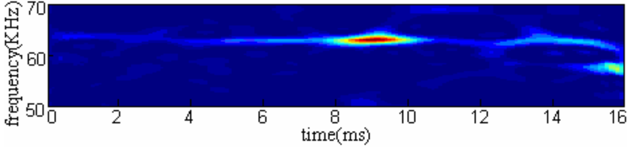
The definition of Continuous wavelet transform(CWT) is to employ a zero mean wavelet $N_{a,b}(t)$ in a quadratic integrable space $L^2(R)$ to analyze signals by cross-correlation, and the mother wavelet $N_{a,b}(t)$ comes from the scale and shift changes of mother wavelet $N(t)$.

$$N_{a,b}(t)=\frac{1}{\sqrt{a}}N(\frac{t-b}{a}) \qquad (a>0) \tag{10}$$

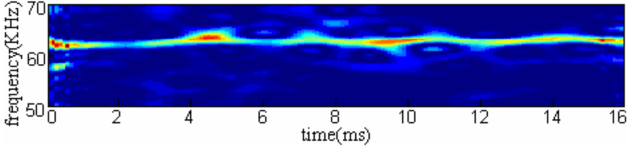
Analysis wavelet $N_{a,b}(t)$ relates with mother wavelet



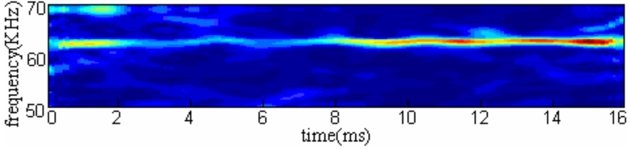
(a) WVD



(b) SPWVD



(c) Redistribution SPWVD



(d) Redistribution SPWVD after inertia filtering

Figure 3. ExamplSeveral WVD to Doppler signal

$N(t)$ according to scale factor a and shift factor b, a and b reflect the frequency and time characteristics of $N_{a,b}(t)$ respectively. Utilizing wavelet analysis $s_1(t)$, we yield

$$W_s(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}s_1(t)N^*(\frac{t-b}{a})dt \tag{11}$$

Wavelet transform provides the ability of autofocus which can regulate time resolution and frequency resolution automatically according to the frequency change of nonstationary signal so that the contradiction between time resolution and frequency resolution can be solved. Fig. 4 is CWT of Doppler signal $s_1(t)$ with Morlet mother wavelet.

We can see that $W_s(a,b)$ changes periodically in X-axis direction at the scale where b is equal to 31, and its period T is lined out in Fig. 4. The SAVP $\overline{T}$ of $s_1(t)$ can be computed as the follow.

$$\overline{T}=T_0\frac{T}{a} \tag{12}$$

In (12), T_0 is the equivalent period of mother wavelet. In Fig. 4, the range of T is from 182 to 191 microsecond, compared to the real period 186 microsecond, the relative error is 2.7%.

The method of measuring SAVP of NPS based on CWT is simple, direct and quick with good time-frequency resolution but ordinary time-frequency aggregation, which means ordinary measurement precision of SAVP.

IV. THE EFFECT OF NOISE ON TIME-FREQUENCY ANALYSIS OF NONSTATIONARY PERIODIC SIGNAL

NPS is disturbed inevitably by noise, and in (1) $v(t)$ may be white noise or color noise, we discuss the anti-interference capability of above methods.

A. White noise

We add Gauss white noise to Doppler signal $s_1(t)$ with the same magnitude, so the synthetic white noise Doppler signal $s_1'(t)$ can be employed to analyze the anti-disturbance capability of above time-frequency analysis methods. The following figures show the effect of white noise on Doppler signal.

From Fig. 5(a) and Fig. 5(c) we can see, STFT and SPWVD lose the ability to analyze Doppler signal because of the effect of white noise. Fig. 5(b) is STFT after inertia filtering, which has good time-frequency resolution and clear spectrum line, and can separate absolutely noise from signal as well as can slightly restrain the noise. In Fig. 5(d) SPWVD after inertia filtering has good character of time-frequency aggregation, and noise can be successively separated and restrained, the time-frequency resolution is destroyed but satisfy tolerably the requirement. In Fig. 5(e), we can find the evident trail caused by noise, but wavelet can analyze signals from several frequency bands, so it still can get the period of wavelet transform from different scales even though some wavelet is disturbed at a certain scale. For this reason, CWT has strong ability of anti-disturbance.

In a word white noise has different extent effects on STFT and SPWVD, and STPS with reinforced inertia filtering can overcome the effect of noise and SPWVD also can satisfy basically the requirement. At the same time CWT is little affected by noise.

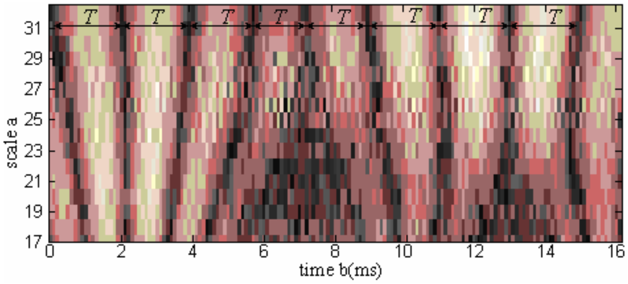
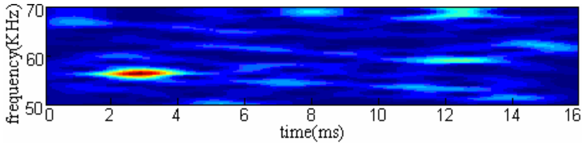
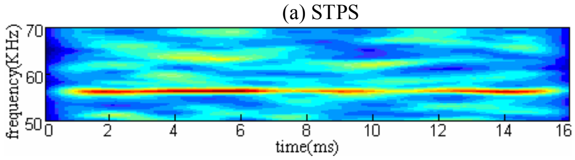


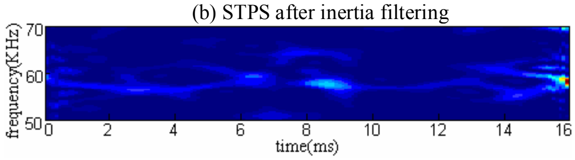
Figure 4. CWT to Doppler signal



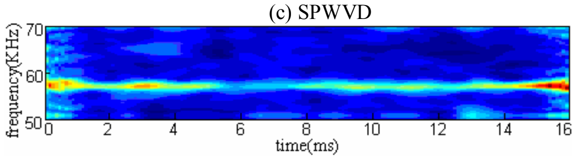
(a) STPS



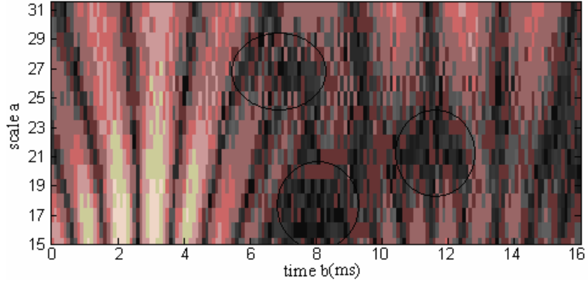
(b) STPS after inertia filtering



(c) SPWVD



(d) SPWVD after inertia filtering



(e) CWT

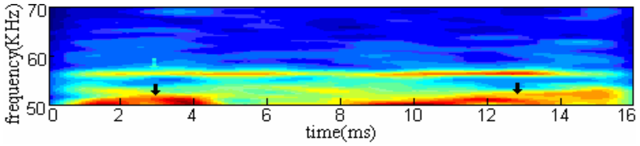
Figure 5. Several time-frequency analysis method to white Doppler signal

B. Color noise

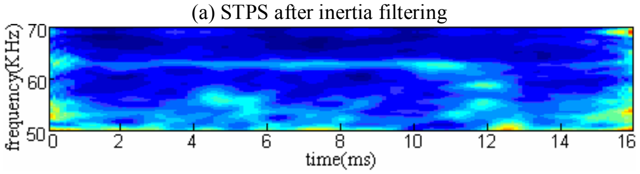
Doppler signal $s_1(t)$ is added to color noise with magnitude of $0.8|s_1(t)|_{\max}$, and become a color noise Doppler signal $s_1''(t)$. $s_1''(t)$ can be used to analyze the ability of anti-disturbance of the foregoing time-frequency analysis methods.

From Fig. 6(a) we can see, STPS after inertia filtering has good time-frequency resolution and clear spectral line, and can absolutely separate noise from signal but can not restrain the noise, even the peak of noise spectrum is larger than that of signal spectrum. Some further process needs to be done for noise spectrum. In Fig. 6(b) SPWVD after inertia filtering suffers the effect of color noise and the spectral line is in chaos with no worth. In Fig. 6(c), we can find that color noise can not affect CWT evidently.

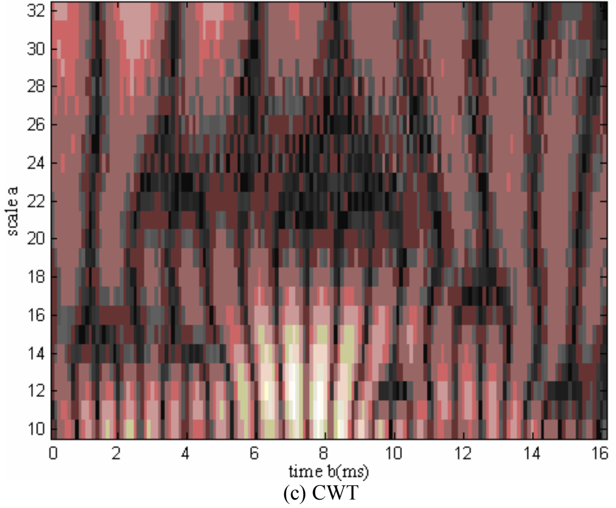
In a word color noise has relative strong effects on STPS and SPWVD with inertia filtering. STPS can attenuate the effect of noise according to strengthening inertia filtering but at the cost of dynamic performance of analysis process. CWT has strong ability of anti-disturbance to color noise. At the same time color noise has narrower frequency band than that of white noise, so the effect of color noise to WT is smaller than that of white noise.



(a) STPS after inertia filtering



(b) SPWVD after filtering



(c) CWT

Figure 6. Several time-frequency analysis method to $s_1''(t)$ color noise Doppler signal

V. CONCLUSION

The SAVP or SAVF of NPS have considerable physical significance. Among all forgoing methods of computing SAVP and SAVF, STPS is not good at time-frequency analysis and anti-disturbance, but if we add moderate inertia filtering to STPS according to the strength of noise, relatively good time-frequency resolution and anti-disturbance can be obtained at the expense of certain dynamic performance. WVD has dispersive spectrums because of the effect of disturbance terms and is of no use, but the improved methods of WVD such as SPWVD and redistribution SPWVD bring better character of time-frequency aggregation and resolution. At the same time we must notice that all the methods of WVD present the question of bad anti-disturbance and inertia filtering can only improve anti-disturbance in some extent. In spite of ordinary performance of time-frequency aggregation, WT has powerful anti-disturbance ability to any noise. Anyway, STPS with inertia filtering can achieve high precision SAVF of NPS but with long calculation time, WT has common precision of SAVP with a short time and strong anti-disturbance ability, and WVD can only be employed to the occasions where disturbance is feeblish.

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