Resilient Robust H_∞ Fuzzy Controller Design for a Class of Nonlinear Systems with Time-Varying Delays in States

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***Abstract*—This paper deals with resilient robust H_∞ fuzzy control problem for a class of nonlinear time-delayed systems with norm-bounded and time-varying uncertainties in the matrices of state, delayed state and control gain via state feedback controllers. Firstly, the nonlinear system is represented by Takagi-Sugeno (T-S) fuzzy model. Sufficient conditions are derived for the existence of resilient robust H_∞ fuzzy controllers in terms of linear matrix inequalities (LMI), which can be solved by convex optimization method. Finally, numerical example is presented to demonstrate the effectiveness of the proposed controller design.**

I. Introduction

During the past years, one of the most challenging problem for many engineers is to design resilient controller, which is also called non-fragile controller [1]–[4]. Fragility refers to the performance debasement of the closed-loop system due to small perturbations in the coefficients of the controller design. Some examples in [1] had been presented to show that small perturbations in the coefficients of the controller designed by using robust H_2, H_∞, l_1 and μ approaches can destabilize the closed-loop control system. The authors therein had suggested taking into account both uncertainties in the controller structure and in the system structure, so as to make a good trade-off between fragility and optimality.

On the other hand, dynamical systems with time delays are common and constitute basic mathematical models of real phenomena, for instance in chemical processes, communication network, and mechanics. Since time delays frequently cause serious deterioration of the performance and even stability of the system, there are many approaches to solve this problem over the last decades [5]–[8]. Particularly, resilient state feedback controller had been discussed in consideration of implementation errors for linear system with time delays in [8]. How ever, the efforts in [1]–[3] , [8] were mainly focused on linear systems. Non-fragility of the controller for nonlinear system was discussed in [4]. However, the method therein needs positive-definite solution to a pair of coupled Hamilton-Jacobi inequalities, which are much complicated and only have solutions for a special kind of systems.

It has been shown in [9]–[11] that Takagi-Sugeno (T-S) fuzzy model can act as a universal approximator of any smooth nonlinear systems having a first order that is differentiable. T-S fuzzy logic controller design and parallel-distributed compensation (PDC) scheme had been proposed and developed in [12]. Fuzzy model-based controller can combine the merits of both fuzzy controller and conventional linear theory, and furthermore guarantee stability in the sense of Lyapunov and control performance theoretically. Moreover, linear matrix inequality (LMI) techniques [13] also make model-based fuzzy controller design more convenient. Therefore, it is meaningful to consider applying the fuzzy model to approximate the nonlinear system with time delays. To stabilize the nonlinear time-delayed system, some researchers considered T-S fuzzy system with time delays [14], [15], which had studied the designs of delay-independent and delay-dependent controller, respectively.

The main contribution of this paper is to propose a resilient robust H_∞ fuzzy controller design for a class of nonlinear systems with time delays and norm-bounded time-varying uncertainties. First, the nonlinear system with time delays is described by T-S fuzzy model. Then, the sufficient conditions for resilient robust H_∞ fuzzy controller are presented through PDC scheme. And the conditions are reduced to a set of LMIs, which can nowadays resort to some popular commercial software. Finally, numerical example is given to illustrate the efficiency of the controller design.

The rest of this paper is organized as follows. T-S fuzzy system with time delays is constructed in Section II. Resilient robust H_∞ fuzzy controller is proposed in Section III. In Section IV, the proposed scheme is applied to a numerical example. Some conclusions are collected in Section V.

II. Fuzzy System with Time Delays

A general T-S model employs an affine model with a constant term in the consequent part of each rule, based on a fuzzy partition of input space. In each fuzzy subspace a linear input-output relation is formulated. The output of fuzzy reasoning is given by the aggregation of the values inferred by some implications that were applied to an input. This is often referred to as an affine T-S model. However, what we are mostly interested in is another type of T-S fuzzy model, in which the consequent part for each rule is represented by a linear model (without a constant term). This type of T-S fuzzy model is called a linear T-S model.

T-S fuzzy system can be used to approximate a class of nonlinear time-delayed systems with norm-bounded parametric uncertainties [11], which is constructed as follows

$$\begin{aligned}&\text{Plant Rule } i: \text{ IF } \theta_1(t) \text{ is } N_{i1}, \ \cdots, \text{ and } \theta_q(t) \text{ is } N_{iq},\\ &\quad \text{THEN } \dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})\\ &\qquad\qquad \times x(t-d(t)) + (B_i + \Delta B_i)u(t)\\ &\qquad\qquad + B_{2i}\omega(t),\\ &\qquad z(t) = Cx(t),\\ &\qquad x(t) = \phi(t), \ t \in [-\sigma_0, \ 0], \ i = 1, \ 2, \ \cdots, \ r.\end{aligned} \tag{1}$$

where $\theta(t) = \{\theta_1(t), \ \theta_2(t), \ \ldots, \ \theta_q(t)\}$ denote the variables of premise part, $A_i, \ A_{di} \in \Re^{n\times n}$, $B_i \in \Re^{m\times n}$, $x(t) \in \Re^n$ denotes state vector, $u(t) \in \Re^m$ denotes control input vector, and N_{il} denotes fuzzy sets, the real-valued functional $d(t)$ is the time-varying delay in the state and satisfies $d(t) \leq \sigma_0$, a real positive constant representing the upper bound of the time-varying delay. It is further assumed that $\dot{d}(t) \leq \beta < 1$ and β is a known constant. $\phi(t)$ are continuous vector-valued initial functions, and r denotes the number of IF–THEN rules. $\Delta A_i, \ \Delta A_{di} \in \Re^{n\times n}$, $\Delta B_i(t) \in \Re^{m\times n}$are the system's uncertainty matrices and satisfy Assumption 1.

Assumption 1: Uncertainty matrices ΔA_i and ΔB_i in system (1) take the following structures

$$\begin{bmatrix} \Delta A_i & \Delta B_i & \Delta A_{di} \end{bmatrix} = M_i F_i(t) \begin{bmatrix} N_{1i} & N_{2i} & N_{di} \end{bmatrix} \tag{2}$$

where M_i, N_{i1} and N_{i2} are constant real matrices of appropriate dimensions, and $F_i(t) \in \Re^{i\times j}$ is unknown matrix-valued functions with Lebesgue-measurable elements and satisfies

$$F_i^T(t)F_i(t) \leq I \tag{3}$$

where I is the identity matrix of appropriate dimensions.

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifiers, the final output of the T-S fuzzy model is inferred as follows

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})\\ \times x(t-d(t)) + (B_i + \Delta B_i)u(t) + B_{2i}\omega(t)]\end{aligned} \tag{4}$$

where

$$h_i(\theta(t)) = \frac{w_i(\theta(t))}{\sum_{i=1}^{r} w_i(\theta(t))}, \ w_i(\theta(t)) = \prod_{j=1}^{q} N_{ij}(\theta(t)),$$

and $N_{ij}(\theta(t))$ denotes the degree of membership of $z(t)$ on N_{ij}. It is assumed that the degree of membership satisfies

$$\sum_{i=1}^{r} w_i(\theta(t)) > 0, \quad w_i(t) \geq 0, \quad i = 1, \ 2, \ \cdots, \ r.$$

Note that for all t, there exists

$$\sum_{i=1}^{r} h_i(\theta(t)) = 1, \quad h_i(\theta(t)) \geq 0,$$

where $h_i(\theta(t))$ can be taken as the weights of normalized IF-THEN rules.

For PDC scheme, resilient robust H_∞ fuzzy controller and the fuzzy model (1) possess the same premises. Then, supposing that all the states are observable, the i-th controller rule can be expressed by

$$\begin{aligned}&\text{Controller Rule } i:\\ &\text{IF } \theta_1(t) \text{ is } N_{i1}, \ \cdots, \text{ and } \theta_q(t) \text{ is } N_{iq},\\ &\text{THEN } u(t) = (K_i + \Delta K_i)x(t), \ i = 1, \ 2, \ \cdots, \ r.\end{aligned} \tag{5}$$

where $u(t)$ is the actually implemented local controller, K_i is the local nominal gain, ΔK_i represents drifting from the nominal solution.

Remark 1: Generally speaking, there are two types of structured gain uncertainties, i.e. additive and multiplicative norm-bounded uncertainties. Haddad and Corrado [2] extended the robust fixed-structure guaranteed cost controller synthesis framework to synthesize resilient controllers for additive controller gain variations and system parametric uncertainty. Multiplicative controller gain variations were addressed in [4]. In this paper, only additive gain uncertainties are taken into consideration. It is assumed that $\Delta K_i = H_i F_{Ki}(t) E_{Ki}$, where H_i, E_{Ki} are constant real matrices of appropriate dimensions.

At the consequent part, fuzzy control rules have linear state feedback gain. It has been proved that the controller using the PDC scheme (5) is an approximator for any nonlinear state feedback controller [11]. The overall fuzzy controller can be represented as follows

$$u(t) = \sum_{i=1}^{r} h_i(\theta(t))(K_i + \Delta K_i)x(t) \tag{6}$$

Applying the controller (6) to the system (4) will result in the following closed-loop system

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))\{[(A_i + \Delta A_i) + (B_i + \Delta B_i)\\ \times (K_i + \Delta K_i)]x(t) + (A_{di} + \Delta A_{di})\\ \times x(t-d(t)) + B_{2i}\omega(t)\}\\ x(t) = \phi(t), \ t \in [-\sigma_0, \ 0].\end{aligned} \tag{7}$$

Next, we will introduce a definition for the closed-loop system (7).

Definition 1: The closed-loop system (7) is asymptotically stable with disturbance attenuation level γ and stable, if the following is fulfilled for all time delay and the uncertainties therein satisfy (2) and (3):

1. The closed-loop system (7) is asymptotically stable;

2. The closed-loop system guarantees, under zero initial conditions, $\|z(t)\|_2 \leq \gamma^2 \|\omega(t)\|_2$, for all non-zero $\omega(t) \in L_2[0, \infty)$.

Then, the objective of this paper is to design a resilient robust H_∞ controller in the presence of time-varying delays, parameter uncertainties of system and additive uncertainty of controller. Also the controller guarantees disturbance attenuation of the closed-loop system from $\omega(t)$ to $z(t)$.

III. Resilient Robust H_∞ Fuzzy Controller Design

In this section, we will present stability conditions for the closed-loop fuzzy system (7). Some useful matrix inequalities are introduced first, which will be used in the proof of our main results.

Lemma 1: [7]

1. For any real vectors x, y and matrix $P > 0$ of compatible dimensions,

$$2x^T y \le x^T P^{-1} x + y^T P y \tag{8}$$

2. Let A, D, E and $F(t)$ be real matrices of appropriate dimensions. Then we have

(a) For any scalar $\mu > 0$,

$$DFE + (DFE)^T \le \mu^{-1} DD^T + \mu E^T E \tag{9}$$

(b) For any real matrix $P = P^T > 0$, scalar $\mu > 0$, F satisfying $FF^T \le I$. For any scalar $\mu > 0$ such that $\mu I - EPE^T > 0$,

$$\begin{aligned}(A+DFE)P(A+DFE)^T &\le \mu DD^T + APA^T \\ &+ APE^T(\mu I - EPE^T)^{-1} EPA^T\end{aligned} \tag{10}$$

(c) For any real matrix $P = P^T > 0$, and scalar $\mu > 0$ such that $P - \mu DD^T > 0$,

$$\begin{aligned}(A+DFE)^T P^{-1}(A+DFE) &\le \mu^{-1} E^T E \\ &+ A^T (P - \mu DD^T)^{-1} A\end{aligned} \tag{11}$$

Lemma 2: [13] Suppose that $\Lambda = \Lambda^T \in \Re^{(l+k)\times(l+k)}$ is partitioned as

$$\Lambda = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

where $C \in \Re^{k+k}$ is nonsingular, then $\Lambda > 0$ if and only if $C > 0$ and $A - BC^{-1}B^T > 0$.

Now we are in a position to present the main result in this paper. Firstly, stability conditions are presented for the systems (7) without external disturbances.

Theorem 1: Consider the uncertain nonlinear systems with time-varying delays (7) and suppose that the disturbance input is zero for all the time. The closed-loop system (7) is asymptotically stable if there exist positive definite matrix P, and controller gains K_i such that

$$\begin{bmatrix} \tilde{\Pi}_1 & * \\ A_{di}^T P & \Gamma_1 \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \tilde{\Pi}_2 & * \\ A_{di}^T P + A_{dj}^T P & \Gamma_2 \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned}
\tilde{\Pi}_1 &= PA_i + PB_iK_i + A_i^T P + K_i^T B_i^T P + (1-\beta)^{-1} R_1, \\
&+ (\varepsilon_{1i} + \varepsilon_{3i}\cdot\varepsilon_{2i}) B_i^T P + \varepsilon_{1i}^{-1}(E_{1i} + E_{2i}K_i)^T, \\
&\times (E_{1i} + E_{2i}K_i) + \varepsilon_{2i}^{-1} E_{Ki}^T E_{Ki}, \\
\tilde{\Pi}_2 &= PA_i + PB_iK_j + A_i^T P + K_j^T B_i^T P + PA_j + PB_jK_i. \\
&+ A_j^T P + K_i^T B_j^T P + \frac{2}{1-\beta} R_1 + (\varepsilon_{1ij} + \varepsilon_{2ij} \\
&\times \varepsilon_{3ij} + \varepsilon_{2ij}) PB_i (I - \varepsilon_{3ij}^{-1}(E_{2i}H_j)^T (E_{2i}H_j))^{-1} \\
&\times B_i^T P + (\varepsilon_{5ij}\cdot\varepsilon_{6ij} + \varepsilon_{4ij} + \varepsilon_{4i}) PD_iD_i^T P \\
&+ \varepsilon_{4j} PD_jD_j^T P + \varepsilon_{1ij}^{-1}(E_{1i} + E_{2i}K_j)^T (E_{1i} \\
&+ E_{2i}K_j) + \varepsilon_{2ij}^{-1} E_{Kj}^T E_{Kj} + \varepsilon_{5ij} PB_j (I - \varepsilon_{6ij}^{-1} \\
&\times (E_{2j}H_i)^T (E_{2j}H_i))^{-1} B_j^T P + \varepsilon_{5ij}^{-1} E_{Ki}^T E_{Ki} \\
&+ \varepsilon_{4ij}^{-1}(E_{1j} + E_{2j}K_i)^T (E_{1j} + E_{2j}K_i), \\
\Lambda_1 &= \varepsilon_{4i}^{-1} E_{di}^T E_{di} - \frac{1}{1-\beta} R_1, \\
\Lambda_2 &= \varepsilon_{4i}^{-1} E_{di}^T E_{di} + \varepsilon_{4j}^{-1} E_{dj}^T E_{dj} - \frac{1}{1-\beta} R_1,
\end{aligned}$$

where $1 \le i < j \le r$, ε_{ci} $(1 \le c \le 4)$, ε_{dij} $(1 \le d \le 6)$ are arbitrary positive scalars, * denotes the transposed element in the symmetric position, and I is identity matrix with appropriate dimension.

Define the following Lyapunov functional candidate for the system (7) as follows

$$V(x(t)) = x^T(t) P x(t) + \frac{1}{1-\beta} \int_{t-d(t)}^{t} x^T(s) R_1 x(s) ds \tag{14}$$

where P is a time-invariant, symmetric positive definite matrix. It is straightforward that $V(x(t))$ is positive definite and radially unbounded.

Then, the time derivative of the Lyapunov candidate $V(x(t))$ along the trajectory of (7) is given by

$$\begin{aligned}
\frac{dV(x(t))}{dt} &= \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) + \frac{1}{1-\beta} x^T(t) \\
&\times R_1 x(t) - \frac{1-\sigma(t)}{1-\beta} x^T(t-d(t)) R_1 x(t-d(t)) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t)) h_j(\theta(t)) (x^T(t)(P((A_i + \Delta A_i), \\
&+ (B_i + \Delta B_i)(K_j + \Delta K_j)) + ((A_i^T + \Delta A_i^T) \\
&+ (K_j^T + \Delta K_j^T)(B_i^T + \Delta B_i^T))P) x(t) + x^T(t) \\
&\times P(A_{di} + \Delta A_{di}) x(t-d(t)) + x^T(t-d(t)) \\
&\times (A_{di}^T + \Delta A_{di}^T) P x(t)) + \frac{1}{1-\beta} x^T(t) R_1 x(t) \\
&- \frac{1-\sigma(t)}{1-\beta} x^T(t-d(t)) R_1 x(t-d(t))
\end{aligned}$$

After some manipulations, the above formulae can be rewritten as follows

$$\frac{dV(x(t))}{dt} = \sum_{i=1}^{r} h_i^2(\theta(t))x^T(t)(P((A_i+\Delta A_i)+(B_i + \Delta B_i)(K_i+\Delta K_i)+((A_i^T+\Delta A_i^T)+(K_i^T+\Delta K_i^T) \times (B_i^T+\Delta B_i^T))P)x(t) + \sum_{i=1}^{r} h_i(\theta(t))h_j(\theta(t)) \times (P(A_i+\Delta A_i)+(B_i+\Delta B_i)(K_j+\Delta K_j)) + ((A_i^T+\Delta A_i^T)+(K_j^T+\Delta K_j^T)(B_i^T+\Delta B_i^T)) \times P + P(A_j+\Delta A_j)+(B_j+\Delta B_j)(K_i+\Delta K_i)) + ((A_j^T+\Delta A_j^T)+(K_i^T+\Delta K_i^T)(B_j^T+\Delta B_j^T)) \times P)x(t) + x^T(t)P(A_{di}+\Delta A_{di})x(t-d(t)) + x^T(t-d(t))(A_{di}^T+\Delta A_{di}^T)Px(t)) + \frac{1}{1-\beta}x^T(t) \times R_1x(t) - \frac{1-\sigma(t)}{1-\beta}x^T(t-d(t))R_1x(t-d(t))$$

Applying Lemma 1 to the above formulae results in

$$\frac{dV(x(t))}{dt} \leq \Xi_1 + \Xi_2, \tag{15}$$

where

$$\Xi_1 = \sum_{i=1}^{r} h_i^2(\theta(t))\{x^T(t)[PA_i+PB_iK_i+A_i^TP + K_i^TB_i^TP+\varepsilon_{1i}PD_iD_i^T+\varepsilon_{1i}^{-1}(E_{1i}+E_{2i}K_i)^T \times (E_{1i}+E_{2i}K_i)+\varepsilon_{2i}PB_i(I-\varepsilon_{3i}^{-1}(E_{2i}H_i)^T \times (E_{2i}H_i))^{-1}B_i^TP+\varepsilon_{3i}\cdot\varepsilon_{2i}PD_iD_i^TP + \varepsilon_{2i}^{-1}E_{Ki}^TE_{Ki}]x(t)+x^T\varepsilon_{4i}PD_iD_i^TPx(t) + \varepsilon_{4i}^{-1}x^T(t-d(t))E_{di}^TE_{di}x(t-d(t))+x^T(t)P \times A_{di}x(t-d(t))+x^T(t-d(t))A_{di}^TPx(t) + \frac{1}{1-\beta}x^T(t)R_1x(t)-\frac{1}{1-\beta}x^T(t-d(t)) \times R_1x(t-d(t))\},$$

$$\Xi_2 = \sum_{i<j} h_i(\theta(t))h_j(\theta(t))\{x^T(t)[PA_i+PB_iK_j + A_i^TP+K_j^TBP+\varepsilon_{2ij}PB_i(I-\varepsilon_{3ij}^{-1}(E_{2i}H_j)^T \times (E_{2i}H_j))^{-1}B_i^TP+\varepsilon_{3ij}\cdot\varepsilon_{2ij}PD_iD_i^TP + \varepsilon_{2ij}^{-1}E_{Kj}^TE_{Kj}+PA_j+PB_jK_i+A_j^TP+K_i^TB_j^TP + \varepsilon_{4ij}PD_iD_i^TP+\varepsilon_{4ij}^{-1}(E_{1j}+E_{2j}K_i)^T(E_{1j} + E_{2j}K_i)+\varepsilon_{5ij}PB_j(I-\varepsilon_{6ij}^{-1}(E_{2j}H_i)^T(E_{2j}H_i))^{-1} \times B_j^TP+\varepsilon_{5ij}\cdot\varepsilon_{6ij}PD_jD_j^TP+\varepsilon_{5ij}^{-1}E_{Ki}^TE_{Ki})x(t) + \varepsilon_{4i}x^T(t)PD_iD_i^TPx(t)+\varepsilon_{4j}PD_jD_j^TPx(t) + \varepsilon_{4i}^{-1}x^T(t-d(t))E_{di}^TE_{di}x(t-d(t))+\varepsilon_{4j}^{-1}x^T(t - d(t))E_{dj}^TE_{dj}x(t-d(t))+x^T(t)PA_{di}x(t-d(t)) + x(t-d(t))E_{dj}^TE_{dj}x^T(t-d(t))+x^T(t-d(t))A_{di}^T \times Px(t)+x^T(t-d(t))A_{dj}^TPx(t)+\frac{2}{1-\beta}x^T(t)R_1x(t) - \frac{2}{1-\beta}x^T(t-d(t))R_1x(t-d(t)).$$

From the properties of quadratic form, the above formulae will lead to

$$\frac{dV(x(t))}{dt} = \sum_{i=1}^{r} h_i^2(\theta(t))\begin{bmatrix} x(t) \\ x(t-d(t))\end{bmatrix}^T \times \begin{bmatrix} \tilde{\Pi}_1 & PA_{di} \\ A_{di}^TP & \Gamma_1 \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d(t))\end{bmatrix} + \sum_{i<j} h_i(\theta(t))h_j(\theta(t))\begin{bmatrix} x^T(t) & x^T(t-d(t))\end{bmatrix} \times \begin{bmatrix} \tilde{\Pi}_2 & PA_{di}+PA_{dj} \\ A_{di}^TP+A_{dj}^TP & \Gamma_2 \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-d(t))\end{bmatrix}.$$

So far, if inequalities (12) and (13) hold, there exists $\frac{dV(x(t))}{dt} < 0$. Therefore, the closed-loop system (7) will asymptotically stable.

Next, resilient robust H_∞ fuzzy controller will be presented for the T-S fuzzy system (7) with external disturbances based on Theorem 1.

Theorem 2: Consider uncertain nonlinear system with time delays (7). (5) is resilient robust H_∞ fuzzy controller for the system (7), if there exist matrices M_i, symmetric positive definite matrix N, U such that LMIs (16) and (17) holds, where

$$\Omega_{ii} = A_iN+B_iM_i+NA_i^T+M_i^TB_i^T+\frac{U}{1-\beta}+(\varepsilon_{1i} + \varepsilon_{3i}\cdot\varepsilon_{2i}+\varepsilon_{4i})D_iD_i^T+\varepsilon_{2i}B_i(I-\varepsilon_{3i}(E_{2i}H_i)^T \times (E_{2i}H_i))^TB_i^T,$$

$$\Omega_{ij} = A_iN+B_iM_j+NA_i^T+M_j^TB_i^T+A_jN+B_jM_i + NA_j^T+M_i^TB_i^T+\frac{2U}{1-\beta}+(\varepsilon_{1ij}+\varepsilon_{2ij}\cdot\varepsilon_{3ij} + \varepsilon_{5ij}\cdot\varepsilon_{6ij}+\varepsilon_{4ij}+\varepsilon_{4i})D_iD_i^T+\varepsilon_{4j}D_jD_j^T + \varepsilon_{1ij}^{-1}(E_{1i}N+E_{2i}M_j)^T(E_{1i}N+E_{2i}M_j) + \varepsilon_{4ij}^{-1}(E_{1j}N+E_{2j}M_i)^T(E_{1j}N+E_{2j}M_i),$$

$1 \leq i < j \leq r$, ε_{ci} $(1 \leq c \leq 4)$, ε_{dij} $(1 \leq d \leq 6)$ are arbitrary positive scalars. Feedback gain K_is are obtained by

$$K_i = M_iN^{-1}, \quad P = N^{-1}. \tag{18}$$

Proof: For our convenience, we introduce

$$\Lambda = (A_i+\Delta A_i)x(t)+(A_{di}+\Delta A_{di})x(t-d(t)) + (B_i+\Delta B_i)(K_i+\Delta K_i)x(t),$$

then we have

$$\begin{bmatrix} \Omega_{ii} & * & * & * & * & * & * \\ NA_{di}^T & -\frac{U}{1-\beta} & * & * & * & * & * \\ B_{2i}^T & 0 & -\gamma^2 I & * & * & * & * \\ E_{1i}N + E_{2i}M_i & 0 & 0 & -\varepsilon_{1i}I & * & * & * \\ E_{Ki}N & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * \\ CN & 0 & 0 & 0 & 0 & -I & * \\ 0 & E_{di}N & 0 & 0 & 0 & 0 & -\varepsilon_{4i}I \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} \Omega_{ij} & * & * & * & * & * & * & * & * & * \\ N(A_{di}^T + A_{dj}^T) & -\frac{2U}{1-\beta} & * & * & * & * & * & * & * & * \\ B_{2i}^T + B_{2j}^T & 0 & -2\gamma^2 I & * & * & * & * & * & * & * \\ E_{1i}N + E_{2i}M_j & 0 & 0 & -\varepsilon_{1ij}I & * & * & * & * & * & * \\ E_{1j}N + E_{2j}M_i & 0 & 0 & 0 & -\varepsilon_{4ij}I & * & * & * & * & * \\ E_{Kj}N & 0 & 0 & 0 & 0 & -\varepsilon_{2ij}I & * & * & * & * \\ E_{Ki}N & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{5ij}I & * & * & * \\ EN & 0 & 0 & 0 & 0 & 0 & 0 & -0.5I & * & * \\ 0 & E_{di}N & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{4i}I & * \\ 0 & E_{dj}N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{4j}I \end{bmatrix} < 0 \tag{17}$$

$$\begin{aligned} J &= \int_0^\infty [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)]dt \\ &\le \int_0^\infty [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \frac{dV(x(t))}{dt}]dt \\ &= \int_0^\infty \{z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\ &\quad + \sum_{i=1}^r\sum_{j=1}^r h_i(\theta(t))h_j(\theta(t))[\Lambda^T Px(t) + x^T(t)P\Lambda \\ &\quad + \omega^T(t)B_{2i}^T Px(t) + x^T(t)PB_{2i}\omega(t)]\}dt \\ &= \int_0^\infty [\sum_{i<j}^r h_i(\theta(t))h_j(\theta(t))\xi^T(t)\Phi_2\xi(t) \\ &\quad + \sum_{i=1}^r h_i^2(\theta(t))\xi^T(t)\Phi_1\xi(t)]dt, \end{aligned}$$

where

$$\begin{aligned} \xi(t) &= \begin{bmatrix} x^T(t) & x^T(t-d(t)) & \omega^T(t) \end{bmatrix}^T \\ \Psi_1 &= \begin{bmatrix} \Pi_1 & * & * \\ A_{di}^T P & \Lambda_1 & * \\ B_{2i}^T P & 0 & -\gamma^2 I \end{bmatrix}, \\ \Psi_2 &= \begin{bmatrix} \Pi_2 & * & * \\ A_{di}^T P + A_{dj}^T P & \Lambda_2 & * \\ B_{2i}^T P + B_{2j}^T P & 0 & -2\gamma^2 I \end{bmatrix}, \\ \Pi_1 &= \tilde{\Pi}_1 + C^T C, \\ \Pi_2 &= \tilde{\Pi}_2 + 2C^T C. \end{aligned}$$

If there exist $\Psi_1 < 0$ and $\Psi_2 < 0$, then $J \le 0$, which implies that $\|z(t)\|_2 \le \gamma\|\omega(t)\|_2$, for any $\omega(t) \in L_2[0, \infty)$. Therefore, when $\Psi_1 \le 0$ and $\Psi_2 < 0$, the closed-loop system is asymptotically stable with disturbance attenuation level γ according to definition 1 in section 2. Then, multiply the resulting inequalities $\Psi_1 < 0$ and $\Psi_2 < 0$ by Θ = diag(P^{-1}, P^{-1}, I) both left and right side, respectively.

However, the conditions are not jointly convex in K_is and P in Theorem 1. Therefore, Schur complement is applied to the obtained matrix inequalities. Introduce new variables $N = P^{-1}$, $M_i = K_i P^{-1}$ and $U = NR_1N$. Then, the LMIs (16) and (17) can be obtained. The search for the common matrix P and K_is nowadays can resort to some efficient numerical methods [13] in terms of LMIs. So far, LMIs are tractable and N, M_i and U can be determined.

■

IV. Numerical Example

To demonstrate the use of our method, we consider a nonlinear system with time delays approximated by using the following IF-THEN fuzzy rules:

IF $x_1(t)$ is P, THEN

$$\begin{aligned} \dot{x}(t) &= (A_1 + \Delta A_1)x(t) + (A_{d1} + \Delta A_{d1})x(t - d(t)) \\ &\quad + (B_1 + \Delta B_1)u(t) + B_{21}\omega(t); \end{aligned}$$

IF $x_1(t)$ is N, THEN

$$\begin{aligned} \dot{x}(t) &= (A_2 + \Delta A_2)x(t) + (A_{d2} + \Delta A_{d2})x(t - d(t)) \\ &\quad + (B_2 + \Delta B_2)u(t) + B_{22}\omega(t); \end{aligned}$$

where the membership functions of 'P', 'N' are given as follows

$$M_1(x_1(t)) = 1 - \frac{1}{1 + \exp(-2x_1)} \tag{19}$$

$$M_2(x_1(t)) = 1 - M_1(x_1(t)) \tag{20}$$

The uncertainties ΔA_i, ΔA_{di} and ΔB_i are assumed to have the form of (2). Then, the relevant matrices in the T-S fuzzy model are given as follows

$$A_1 = \begin{bmatrix} -1 & 0.4 \\ 0 & -0.5 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.3 & -0.4 \\ 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.5 & 0 \\ 0.5 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.4 & 0 \\ 0.4 & 0.3 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix},$$
$$E_{11}^T = E_{12}^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad E_{d1} = E_{d2} = \begin{bmatrix} 0.1 & 0 \end{bmatrix},$$
$$E_{21} = 0.3, \quad E_{22} = 0.2,$$
$$F_1(t) = F_2(t) = \sin(t), \quad H_1 = H_2 = 0.5,$$
$$E_{K1} = E_{K2} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix},$$
$$\phi(t) = \begin{bmatrix} e^{t+1} & 0 \end{bmatrix}^T,$$

and $d(t) = h \sin t$. In Theorem 1, we choose the scalar coefficients $\varepsilon_{ci} = \varepsilon_{dij} = 1$, $1 \le c \le 4$, $1 \le d \le 6$, $\gamma = 1.5$. By using Matlab LMI Control Toolbox [16], positive definite matrices P, R_1 and feedback gain K_is can be obtained as follows

$$P = \begin{bmatrix} 5.8361 & 2.6938 \\ 2.3181 & 2.6022 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 1.9767 & 1.1741 \\ 1.6807 & 2.4234 \end{bmatrix},$$
$$K_1 = [\text{ - } 3.3698 \text{ - } 4.5295],$$
$$K_2 = [\text{ - } 2.4277 \text{ - } 0.2184].$$

V. Conclusions

In this paper, resilient robust H_∞ fuzzy controller design has been addressed for a class of nonlinear systems with time delays via fuzzy interpolation of a series of linear systems. The fuzzy controller is reduced to the solution of a set of LMIs, which make the design much more convenient. Furthermore, an example has demonstrated the use of the proposed fuzzy model-based controller.

Acknowledgment

This work was supported in part by National Natural Science Foundation of China under Grant No. 60474014 and National 973 Projects of China under Grant No. 2002CB31200.

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