Consensus-based Strategy for Parallel Motion of Multiple Mobile Robots

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***Abstract*—A decentralized coordination strategy is developed to generate parallel motion of multiple mobile robots. All robots communicate with each other to form a robotic network the topology of which is modelled by a weighted digraph. The linear consensus algorithm is extended to design the coordinated control strategy, in which bounded velocity specifications are considered to avoid actuator satuation. A design method of information flow topology for two modes of parallel motion, leader-follower and leaderless, is investigated. The effectiveness of control laws and effects of different information flow topologies and sampling rates on the group behavior are shown in nontrivial computer simulations.**

***Keywords*—multirobot system, parallel motion, consensus, decentralized coordination, information flow**

I. Introduction

In the field of distributed mobile robotics, *group of wheeled mobile robots* is widely used as a testbed of multi-vehicle system and mobile wireless sensor network in which an individual has capabilities of moving, sensing, computing and communicating. For the case that robots can be networked with each other by communication links or sensing, we name the whole system *mobile robotic network* (or *MRN*) to emphasize the role of information flow in the group.

Motion coordination of MRN is a critical problem in its potential applications in surveillance, environment monitoring, border security, search and rescue, where the objective is to make individuals rendezvous at a common point or move in a synchronized manner [1]-[5]. In this paper we focus on the group behavior of *parallel motion* meaning that all the robots move in the same direction. And decentralized strategies are adopted to generate this objective behavior for its superiority to centralized ones in terms of robustness to failure of a robot and independence on the system scale.

Consensus algorithm, an inherently decentralized strategy, has a long history in the field of distributed computing [6], [7] and recently is applied to coordination of multi-agent systems. Its basic idea is that each agent changes the value of a shared variable based on those of its local neighbors in such a way that the final value of each agent converges to a common constant or trajectory. In the literature, A. Jadbabaie et al [8] rigorously prove the convergence of alignment algorithm that updates agent's orientation in the context of undirected network. Olfati-Saber et al [9] discuss continuous consensus algorithms for network of single integrators with fixed and switching directed topologies and present sufficient/necessary conditions for reaching consensus and average consensus. Ren et al [10] extend the work of [8] to the case of directed topology. Gao et al [11] investigate linear consensus algorithm over acyclic directed graph. R. Olfati-Saber et al [12] and Ren et al [13] survey the results available for consensus algorithms of single integrator network and its applications. Most of the research activities in consensus problem have focused on theoretical aspects, where the shared variables are defined in a general form that might not be related to the agent's motion.

The main purpose of this paper is to build a description framework of differential-driven mobile robotic networks and develop a design methodology of decentralized control strategy for parallel motion behavior based on consensus algorithm. The model of MRN is composed of three parts: individual kinematics, information flow structure and objective group behavior. The kinematics is described by the unicycle model and information flow structure by weighted digraph. With the coordinated variable denoting the orientation of each robot, the emergence of objective group behavior can be formulated by consensus problem. Then the linear consensus algorithm over weighted digraph is extended to design the decentralized control strategy. Considering the practical velocity constraints, we present a method that corrects the control laws to prevent actuator saturation. The orientation of a robot is regulated by its own local controller using states of its neighbors specified in the information flow structure. Thus various structures can produce different final modes of parallel motion, two kinds of which, *leaderless* and *leader-follower*, are reported in this paper.

An outline of this paper is as follows. Section II presents the model of mobile robotic network and the objective system behavior. Section III investigates the design of decentralized control laws and discusses the effect of information flow topology on the system behavior. Section IV describes the simulated experiments and conclusion is presented in Section V.

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II. System and Problem Description

Here a MRN $\mathcal{S}$ is a group of N differential driven mobile robots indicated by index set $\mathcal{I}=\{1,2,\cdots,N\}$. The robots can control their own actions to move in 2D space, rotate on a spot and actively communicate with local neighbors to attain their information, e.g. the value of current orientation.

A. Individual and Group Kinematics

We assume that a differential driven mobile robot moves without longitudinal slip and lateral slide and is equipped with efficient low-level PID controller to regulate the speed of wheels. Hence the kinematics is considered to describe robot's motion instead of its dynamics.

The configuration and size of robot i are shown in Fig. 1. The geometric center Q_i locates at the centre of wheel axis, L_w is the distance between the two wheels and R_w the radius of a wheel. Let $\boldsymbol{q}_i=[x_i,y_i]^T\in\mathbb{R}^2$ be the position of Q_i in the workspace $\mathcal{C}\subset\mathbb{R}^2$ and $\theta_i\in(-\pi,\pi]$ is the robot's orientation with respect to the global coordinate frame, as shown in Fig. 1(a). Then the configuration of robot i is denoted by

$$\boldsymbol{p}_i=[\boldsymbol{q}_i^T,\theta_i]^T\in\mathbb{R}^2\times(-\pi,\pi]$$

We use a nonholonomic unicycle to describe the motion of robot i, $\forall i\in\mathcal{I}$. The *Individual kinematics* f_i is written as follows:

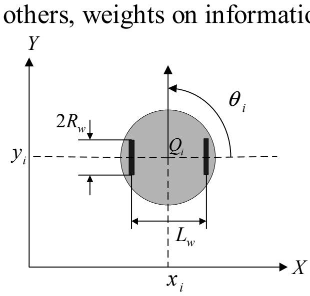
$$f_i:\begin{cases}\dot{x}_i=v_i\cos\theta_i\\ \dot{y}_i=v_i\sin\theta_i\\ \dot{\theta}_i=\omega_i\end{cases}\qquad i\in\mathcal{I}\qquad(1)$$

where v_i, ω_i is translational and rotational velocity of robot i respectively. Let $\boldsymbol{u}_i=[v_i,\omega_i]^T$ be robot's local control input, the value of which is actually the desired reference given to low-level speed controllers. Then the individual kinematics can be simply written as $\dot{\boldsymbol{p}}_i=f_i(\boldsymbol{p}_i,\boldsymbol{u}_i)$.

Let $\boldsymbol{p}=[\boldsymbol{p}_1^T,\boldsymbol{p}_2^T,\cdots,\boldsymbol{p}_N^T]^T$ and $\boldsymbol{u}=[\boldsymbol{u}_1^T,\boldsymbol{u}_2^T,\cdots,\boldsymbol{u}_N^T]^T$ be the configuration and control input of MRN respectively. The configuration evolves according to *group kinematics* $\dot{\boldsymbol{p}}=F(\boldsymbol{p},\boldsymbol{u})$, where F is column-wise concatenation of individual kinematics f_i, $i\in\mathcal{I}_N$.

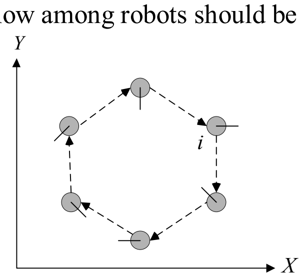
B. Information Flow Structure

All the robots in $\mathcal{S}$ exchange information over a wireless local network. An example of information flow topology is shown in Fig. 2. In order to show the impacts of a robot on others, weights on information flow among robots should be defined. The topology and weights constitute an information flow structure.



(a)

Fig. 1 Configuration of robot i



(b)

Fig. 2 Information flow topology

Let $\mathcal{G}=(\mathcal{I},\mathcal{E},\boldsymbol{A})$, a *weighted directed graph*, represent the information flow structure, where $\mathcal{I}=\{1,2,\cdots,N\}$ represents a *vertex set* with elements corresponding to N robots and $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ is a non-empty *edge set* with each element representing a communication link from one robot to another, e.g., $(i,j)\in\mathcal{E}$ mean that robot i requests information from j and data flow backwards. Note that $(i,j)\in\mathcal{E}$ doesn't mean $(j,i)\in\mathcal{E}$, and the digraph becomes undirected if $(j,i)\in\mathcal{E}$ anytime $(i,j)\in\mathcal{E}$. $\boldsymbol{A}$ is an $N\times N$ *weighted adjacency matrix* defined by

$$\begin{cases}a_{ij}>0 & (i,j)\in\mathcal{E}\\ a_{ij}=0 & (i,j)\notin\mathcal{E}\end{cases}\qquad\forall i,j\in\mathcal{I}$$

where a_{ij} denotes the weight on the communication link from i to j. If $a_{ij}=1$ for all $(i,j)\in\mathcal{E}$, $\mathcal{G}$ turns to a digraph and if $a_{ij}=a_{ji}$ for all $(i,j)\in\mathcal{E}$, $\mathcal{G}$ is undirected. So both digraph and (weighted) undirected graph are special cases of weighted digraph.

The graph $\mathcal{G}$ describes the relation between any two robots. All robots receiving requests from i are called out-neighbors of robot i and those sending requests to i in-neighbors. Then let $\mathcal{N}_i^+=\{j\,|\,(i,j)\in\mathcal{E}\}$ and $\mathcal{N}_i^-=\{j\,|\,(j,i)\in\mathcal{E}\}$ denote out-neighborhood and in-neighborhood of robot i respectively.

C. Objective Group Behavior

We define $\boldsymbol{c}\in\mathbb{R}^r, 1\le r\le 3$ to be the coordinated variable in $\mathcal{S}$ shared by all the robots to quantify the objective behavior and $\boldsymbol{c}_i\in\mathbb{R}^r$ the corresponding variable of robot i with initial value $\boldsymbol{c}_i(0)$. In motion coordination problem, this variable is related to the configuration of a robot. It mean that there exist a map $T:\mathbb{R}^2\times(-\pi,\pi]\to\mathbb{R}^r$ and $\boldsymbol{c}_i=T(\boldsymbol{p}_i),\forall i\in\mathcal{I}$. To encode the desired group behavior, we define an objective function

$$\Phi(c)=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(c_i-c_j)^2\qquad(2)$$

where a_{ij} is the element of adjacency matrix $\boldsymbol{A}$ of graph $\mathcal{G}$. When $c_i=c_j,\forall i,j\in\mathcal{I}$, i.e.

$$\lim_{t\to\infty}\|\boldsymbol{c}_i-\bar{\boldsymbol{c}}\|=0,\quad\forall i\in\mathcal{I}$$

the objective function reaches its unique minimum, where $\bar{\boldsymbol{c}}$ denotes the critical point. This situation is called reaching *consensus*. Specially, if

$$\bar{\boldsymbol{c}}=\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{c}_i(0)$$

it is called reaching *average consensus*.

In this paper, where parallel motion is the objective group behavior, θ_i is an instance of $\boldsymbol{c}_i$ and $\theta_i=[0\quad 0\quad 1]\boldsymbol{p}_i$. Let $\boldsymbol{p}_{\mathcal{N}_i^+}$ be the state of i's neighbors defined by the information flow structure. The design objective is to derive a proper decentralized control laws in the form of $\boldsymbol{u}_i=\varphi_i(\boldsymbol{p}_{\mathcal{N}_i^+},\boldsymbol{p}_i)$ that drive the robots moving in the same direction θ and find out the effect of information flow structure on system behavior.

III. DESIGN AND ANALYSIS OF COORDINATION STRATEGY

A. Preliminary Developments

Let $\mathcal{G}=(\mathcal{I},\mathcal{E},\boldsymbol{A})$ be a weighted digraph with the same definition given in Section II. The out-degree and in-degree of vertex i are defined by:

$$d_i^+=\left|\mathcal{N}_i^+\right|,\ d_i^-=\left|\mathcal{N}_i^-\right|$$

where $\left|\mathcal{N}_i^+\right|$ and $\left|\mathcal{N}_i^-\right|$ are the cardinality of $\mathcal{N}_i^+$ and $\mathcal{N}_i^-$, respectively. The weighted out-degree and in-degree of vertex i are defined by:

$$\tilde{d}_i^+=\sum_{j\in\mathcal{N}_i^+}a_{ij}\ ,\ \tilde{d}_i^-=\sum_{j\in\mathcal{N}_i^-}a_{ji} \tag{3}$$

Let $\boldsymbol{D}_+=diag(\tilde{d}_1^+,\tilde{d}_2^+,\cdots,\tilde{d}_N^+)$ and $\boldsymbol{D}_-=diag(\tilde{d}_1^-,\tilde{d}_2^-,\cdots,\tilde{d}_N^-)$ denote the weighted out-degree and in-degree matrix. The digraph $\mathcal{G}$ is *topologically balanced* if $d_i^+=d_i^-$ and *weight-balanced* if $\tilde{d}_i^+=\tilde{d}_i^-$, $\forall i\in\mathcal{I}$.

Then the weighted Laplacian matrix of $\mathcal{G}$ is defined by

$$\boldsymbol{L}=\boldsymbol{D}_+-\boldsymbol{A} \tag{4}$$

Note that $\boldsymbol{L1}=0$, i.e. $\boldsymbol{L}$ has a zero eigenvalue corresponding to a right eigenvector $\boldsymbol{1}$, and that $\boldsymbol{L}$ is symmetric iff $\mathcal{G}$ is undirected or bi-directed. As $\mathcal{G}$ is weight-balanced, $\boldsymbol{1}^T\boldsymbol{L}=0$.

We assume the directed graph $\mathcal{G}$ is simple. A *path* in $\mathcal{G}$ is an ordered sequence of distinct vertices such that every pair of consecutive vertices is a edge of $\mathcal{G}$. If any two vertices are connected by a path, $\mathcal{G}$ is *strongly connected.* If a vertex of $\mathcal{G}$ can be reached from any other node by traversing a path, we call it a *sink* of $\mathcal{G}$.

B. Proposed Decentralized Control Strategy

The design objective is to drive the orientations of all the robots to a common value. According to the kinematics (1), the updates of θ_i is only manipulated by ω_i that is calculated using a control law. Hence we consider a subsystem of (1):

$$\dot{\theta}_i=\omega_i \qquad \theta_i\in(-\pi,\pi],\ i\in\mathcal{I} \tag{5}$$

Now, following [9] and [13], we apply the linear consensus algorithm over networks of single integrators to the design of decentralized strategies. The algorithm is given by

$$\omega_i=\sum_{j=1}^{N}a_{ij}(\theta_j-\theta_i) \tag{6}$$

Using (5) and (6), we can write the equation which governs the evolution of θ_i such that

$$\dot{\theta}_i=\sum_{j=1}^{N}a_{ij}(\theta_j-\theta_i) \tag{7}$$

where a_{ij} is the (i,j) entry of the adjacency matrix $\boldsymbol{A}$ of the associated information flow structure $\mathcal{G}$, a weighted digraph, defined in Section II. Setting $a_{ij}\neq 0$ indicates that robot i receives information about θ_j from robot j and the difference between the orientations of the two robots is calculated. θ_i of robot i is driven toward the orientations of its neighbors defined by $\mathcal{G}$.

Also the factor a_{ij} represent the contribution of θ_j to the control input ω_i. We assume that the robots in the out-neighborhood of robot i have same impacts on it and define a_{ij} by

$$a_{ij}=\begin{cases}\dfrac{1}{\left|\mathcal{N}_i^+\right|} & j\in\mathcal{N}_i^+\\ 0 & j\notin\mathcal{N}_i^+\end{cases} \tag{8}$$

Finally, we rewrite (7) as

$$\dot{\theta}_i=\sum_{j\in\mathcal{N}_i^+}\frac{1}{\left|\mathcal{N}_i^+\right|}(\theta_j-\theta_i) \qquad \forall i\in\mathcal{I} \tag{9}$$

1) Convergence analysis

Here N subsystems of (5) are coupled by the information flow among the robots to form a networked system. Using (3), (4) and (7), we derive this closed-loop linear system

$$\dot{\boldsymbol{\theta}}=-\boldsymbol{L\theta} \tag{10}$$

where $\boldsymbol{\theta}=[\theta_1,\theta_2,\cdots,\theta_N]^T$ and $\boldsymbol{L}$ is graph Laplacian of $\mathcal{G}$.

From the viewpoint of control theory, the convergence of states of system (10) is determined by the eigenvalue of matrix $\boldsymbol{L}$. In terms of its definition, $\boldsymbol{L}$ has a zero eigenvalue and $\boldsymbol{1}$ is the associated right eigenvector. From Gerschgorin's disk theorem [14], the nonzero eigenvalues of $-\boldsymbol{L}$ have negative real part. Therefore, the equilibrium of linear system (10) lies in the null space of $\boldsymbol{L}$ denoted by $null(\boldsymbol{L})$. In the literature of matrix theory, if $\boldsymbol{L}$ has a simple eigenvalue at zero, $null(\boldsymbol{L})=span\{\boldsymbol{1}\}$ such that $\theta\to\alpha\boldsymbol{1}$ where α is a constant. In other words, if $\boldsymbol{L}$ has only one zero eigenvalue, system (10) can reach consensus with value α.

The multiplicity of 0 eigenvalue is related to the characteristics of the structure of $\mathcal{G}$. The work in [8][9][10] has discussed sufficient and necessary conditions for consensus problem over directed or undirected graph by using tools from spectral graph theory. It's easy to extend the results to the case of weighted digraph:

Theorem 3.1 (for consensus) Let $\mathcal{G}$ be a weighted digraph, System (10) asymptotically reaches consensus iff $\mathcal{G}$ has a sink;

Theorem 3.2 (for average consensus) If a weighted digraph $\mathcal{G}$ is strongly connected, system (10) reaches average consensus asymptotically iff $\mathcal{G}$ is weight-balanced.

Theorem 3.1 and 3.2 denote that the convergence of system (10) to a common value can be guaranteed by the design of the information flow topology. Note that a topologically balanced graph is not necessarily weight-balanced.

The final equilibrium can be calculated from the initial orientations of all the robots in the network. The method is reported as follows.

Lemma 3.1 ([9], Corollary 2) Let $\mathcal{G}$ be a strongly connected digraph. Suppose $\boldsymbol{L}$ has a nonnegative left eigenvector $\gamma=[\gamma_1,\cdots,\gamma_N]^T$ associated with *zero eigenvalue* that satisfies $\sum_i\gamma_i>0$. System (10) reaches a consensus with the common value

$$\alpha = \frac{\sum_i \gamma_i \theta_i(0)}{\sum_i \gamma_i}$$

Proposition 3.1 If a weighted digraph $\mathcal{G}$ is strongly connected and topologically balanced, system (10) with weights defined by (8) can reach consensus. The equilibrium is

$$\alpha = \frac{\sum_i \left|\mathcal{N}_i^+\right| \theta_i(0)}{\sum_i \left|\mathcal{N}_i^+\right|} \tag{11}$$

Proof. Let $\boldsymbol{d} = [d_1^+, \cdots, d_N^+]^T = [\left|\mathcal{N}_1^+\right|, \cdots, \left|\mathcal{N}_N^+\right|]^T$. By definition, $\boldsymbol{d}^T \boldsymbol{L} = 0$. Clearly, $\boldsymbol{d}$ is a left eigenvector of $\boldsymbol{L}$ with positive elements. Based on Lemma 3.1, the equilibrium is the weighted average of $\theta_i(0)$ s. ■

2) Strategies with velocity constraints

To avoid wheel slide, limits of velocity are always adopted. The control input ω_i calculated from (6) may exceed the admitted maximum of rotational velocity. Hence, considering velocity constraints we design a more practical strategy for parallel motion as follows.

$$\varphi_i : \begin{cases} \omega_i = \sum_{j \in \mathcal{N}_i^+} \frac{k}{\left|\mathcal{N}_i^+\right|}(\theta_j - \theta_i) & \text{if } \left|\mathcal{N}_i^+\right| \neq 0 \quad (12.a) \\ \omega_i = 0 & \text{if } \left|\mathcal{N}_i^+\right| = 0 \quad (12.b) \\ v_i = v_{\max} & \text{if } \left|\omega_i\right| < \omega_{\max} \quad (12.c) \\ v_i = 0 & \text{if } \left|\omega_i\right| > \omega_{\max} \quad (12.d) \end{cases}$$

where $v_{\max}$ is the maximum of v_i, $\omega_{\max}$ the maximum of ω_i as the robot moves with translational velocity $v_{\max}$ and k is a proportional factor.

According to (12), two steps are executed on the local controller of a robot: i) if robot i has at least one out-neighbor, the consensus algorithm (12.a) is executed and control input ω_i is calculated. Otherwise it keeps moving in original direction. ii) if the calculated ω_i exceeds $\omega_{\max}$, the robot stops and regulates its direction with ω_i. Otherwise robot i moves with $(v_{\max}, \omega_i)$ as control input.

The factor k is used to adjust the algorithm (12.a) in order to make the calculated result executed by low-level PID controller without actuator saturation.

Proposition 3.2 (Bound of k) Let L_w be the distance between two wheels, R_w the radius of wheel and $m_{\max}$ is the maximum of wheel speed in unit of round per second. If

$$m_{\max} \geq \frac{L}{2R_w}$$

let $k = 1$, otherwise, let

$$k \leq \frac{2R_w m_{\max}}{L}$$

such that the result of algorithm(12.a) can be executed.

Proof. Let m_1 and m_2 denote the speed of left and right wheels of a robot, respectively. Let ω_m be the maximal turning rate when robot i rotates with $v_i = 0$. The relation between wheel speed and robot velocity is given by

$$v_i = \frac{2\pi R_w(m_1 + m_2)}{2}, \quad \omega_i = \frac{2\pi R_w(m_2 - m_1)}{L} \tag{13}$$

Obviously, $v_i = 0$ and $\omega_i = \omega_m$ when $m_1 = -m_2 = -m_{\max}$. Thus

$$\omega_m = \frac{4\pi R_w m_{\max}}{L} \tag{14}$$

For $\theta_i \in (-\pi, \pi]$, according to algorithm (12.a) , we attain the bound of the calculated and desired ω_i:

$$\left|\omega_i\right| = \left|\sum_{j \in \mathcal{N}_i^+} \frac{k(\theta_j - \theta_i)}{\left|\mathcal{N}_i^+\right|}\right| \leq k \left|\max_{j \in \mathcal{N}_i^+}(\theta_j - \theta_i)\right| \leq 2\pi k \tag{15}$$

If $\omega_m \geq 2\pi$, i.e.

$$m_{\max} \geq \frac{L}{2R_w} \tag{16}$$

$\left|\omega_i\right| \leq 2\pi \leq \omega_m$ when $k = 1$.
Otherwise, when

$$k \leq \frac{2R_w m_{\max}}{L} \tag{17}$$

using (14),(15) and (17), we get

$$\left|\omega_i\right| \leq 2\pi k \leq \frac{4\pi R_w m_{\max}}{L} = \omega_m$$

Both of the case (16) and (17) denotes that the result of algorithm (12.a) can be executed without exceeding ω_m. ■

C. Topology and Mode of Parallel Motion

Theorem 3.1 and 3.2 and Proposition 3.1 demonstrate that the convergence of system behavior and the final equilibrium are closely related to the topology of arbitrary information flow structure $\mathcal{G}$. In other words, different network topology can generate various kinds of parallel motion. Here we investigate two types of parallel motion, leadless mode and leader-follower mode, and design methodology of associated topologies.

Leaderless mode denotes that the final direction $\bar{\theta}$ is not decided by some robot but by the weighted average of θ_i s. We define the topology of $\mathcal{G}$ by a *Hamilton cycle* [15] (See Fig. 3(a)) to generate this motion mode. Obviously, $\mathcal{G}$ is strongly connected, i.e. $\forall i, j \in \mathcal{I}$ there is a directed path from i to j, and satisfies

$$\left|\mathcal{N}_i^-\right| = \left|\mathcal{N}_i^+\right| = 1, \ \forall i \in \mathcal{I}$$

Thus $\mathcal{G}$ is topologically balanced and weight-balanced with weights defined by (8). According to Theorem 3.2, system (10) can reach average consensus with the equilibrium

$$\bar{\theta} = \frac{1}{N} \sum_{i=1}^{N} \theta_i(0)$$

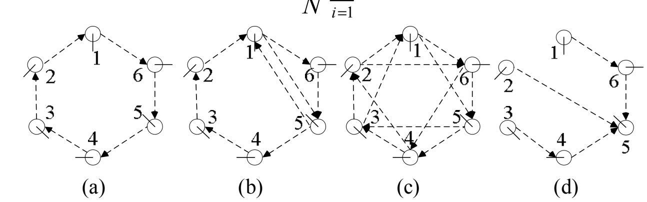


Fig. 3 Three types of information flow topology. (a) and (c) are weight-balanced, (b) is topologically balanced but not weight-balanced and (d) is a rooted spanning tree.

Obviously, any vertex in a Hamilton cycle is a sink. Therefore, adding edges to a Hamilton cycle, e.g. the graph shown in Fig 3(b) is derived from that of Fig 3(a) by adding edges $(1,5)$ and $(5,1)$, will still get leaderless mode generated. However, $\mathcal{G}$ in Fig 3(b) is not weight-balanced even if topologically balanced such that it's consensus but not average consensus that is reached. If $\mathcal{G}$ designed by this method is regular (See Fig. 3(c)), i.e.

$$\left|\mathcal{N}_i^-\right| = \left|\mathcal{N}_i^+\right| \qquad \forall i \in \mathcal{I}$$
$$\left|\mathcal{N}_i^+\right| = \left|\mathcal{N}_j^+\right| \qquad \forall i, j \in \mathcal{I}$$

$\mathcal{G}$ is weight-balanced, which guarantees the generation of leaderless mode with average consensus reached.

Leader-follower Mode denotes that a robot l is defined as leader and the others as follower such that the equilibrium $\bar{\theta} = \theta_l$. Note that not all followers have knowledge of the state of leader. Our design methodology for this mode of motion: Let the leader, robot l, be a sink without any out-neighbor, that is, $\left|\mathcal{N}_l^+\right| = 0$ and any other robot can reach l. An instance of such kind of network topology is shown in Fig. 3(d).

IV. Simulated Experiments

A. Simulated Testbed

The motion of a MRN is simulated by a program built in MATLAB running on a PC. This software is composed of three parts (See Fig. 4): a console for setting parameters and manipulating experiments, a solver for numerical simulation and a recorder for drawing trajectories and animation.

In this testbed, a MRN has 6 robots with $v_{\max} = 0.5\,\mathrm{m/s}$, $\omega_{\max} = 0.5\,\mathrm{rad/s}$ and $L_w = 0.3\mathrm{m}$. We assume that no obstacle exists. The standard 4-order Runge-Kutta method is adopted to resolve the differential equations of kinematics (1). This simulated MRN is synchronous for all the robots calculate their desired velocity in the same time.

B. Experiments

The initial configurations of this MRN are shown in Fig. 5. The average orientation can be calculated: $\tilde{\theta} = 0.104\mathrm{rad}$. We set the simulation step 0.01s and the duration 18s. The control law (12) is applied to the simulated MRN. The three topologies in Fig. 3, (a), (b) and (d) are adopted in experiments with results shown in Fig. 6 (a), (b) and (c) respectively.

Fig. 6 shows that any of the three topologies can result in the robots moving in the same direction while the final states are different from each other. When the weight-balanced topology in Fig. 3(a) is applied, not only leaderless mode of parallel motion but also average consensus is reached as shown in Fig. 6(a). Fig. 6 (b) demonstrates that the topology in Fig.3 (b), topologically balanced but not weight-balanced, guarantees the emergence of group behavior of leaderless parallel motion but $\bar{\theta} \neq 0.104\mathrm{rad}$. Fig. 6 (c) shows that the orientations of the others converge to that of leader robot 5 defined as the unique sink with no outneighbors in Fig. 3 (d).

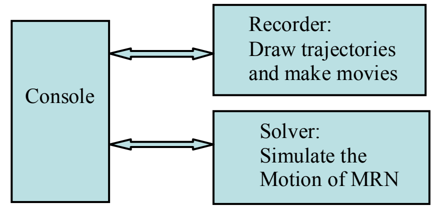


Fig. 4 Three parts of simulation software

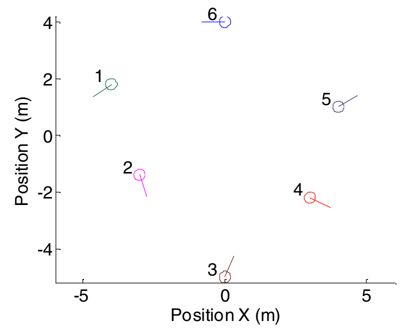


Fig. 5 Initial configuration of MRN

Note that many robots, e.g. No. 1, 2, 3, 6 in Fig. 6(a), rotate at their initial positions until the control input $\omega_i \leq \omega_{\max}$ which calculated by equation (10.a). These results verify the effectiveness of the coordination strategy and the correctness of the design methodology of information flow topologies.

Fig. 7 demonstrates the results of applying the control laws with three different sampling rates $\tau = 200\mathrm{ms}, 600\mathrm{ms}, 850\mathrm{ms}$ under the same topology of Fig. 3(a). Clearly, as the sampling interval increases, the settling time of orientation trajectories increases. It means that the longer the robots wait for the data packets about states of their neighbors, the worse the system performance is.

V. Conclusion

We propose a decentralized control strategy for a group of wheeled mobile robots to move in the same direction with local information exchange and without priori knowledge about the final state. The description of a differential-driven MRN is reported including individual kinematics, information flow structure among robots and the object group behavior. With this framework one can naturally extend consensus algorithm to design the coordinated control rules. While speed constraints exist, the proposed consensus-based strategy must be corrected by multiplying by a factor which is related to basic parameters of the wheel and chassis. Two kinds of mode of parallel motion can be generated when different information flow topologies adopted, which are verified in those simulated experiments. Also the influence of sampling rate on system performance is demonstrated.

The model of mobile robotic system and proposed methods can form principles for the design of wheeled multi-robot system and coordinated group behavior. In future work, we'll consider noise or disturbance from environment and obstacles in the way.

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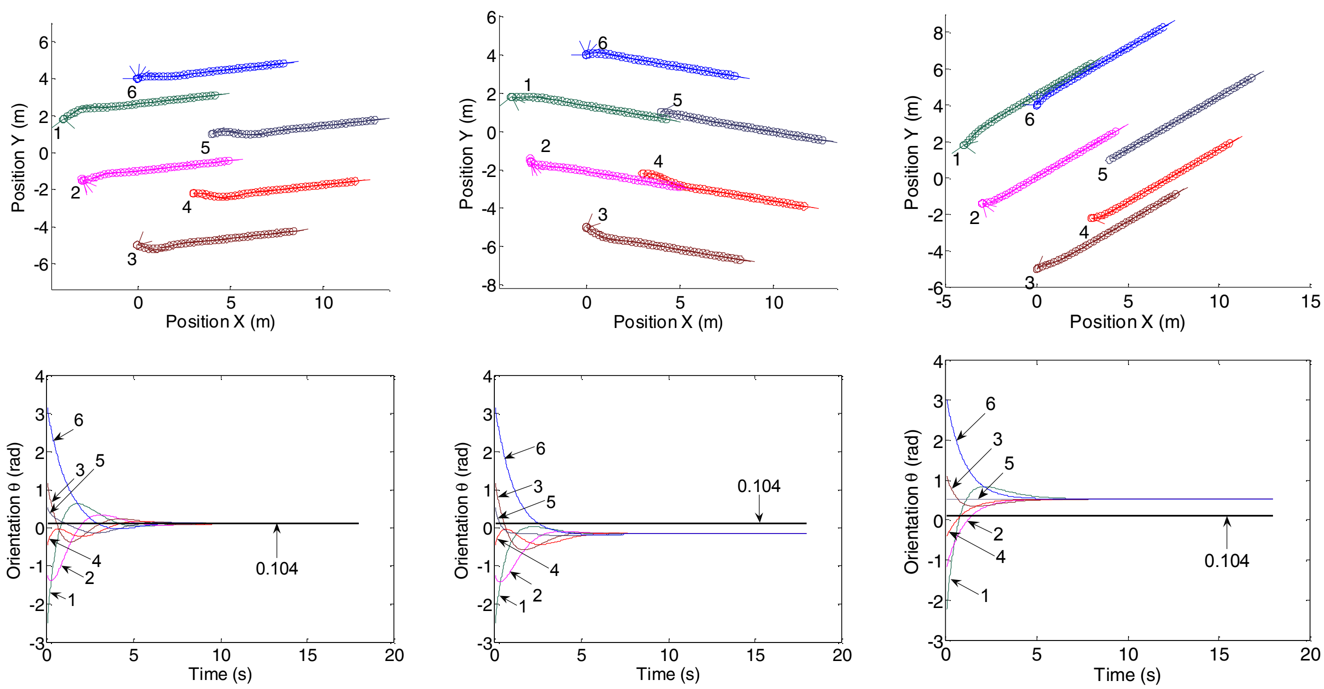
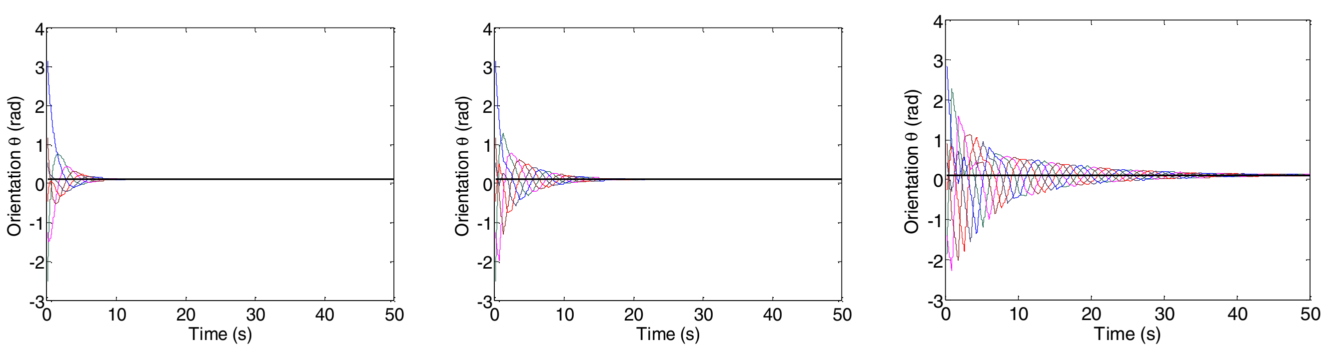


(a) (b) (c)

Fig. 6 Three modes of parallel motion are resulted from different information flow topologies. (a) Leaderless mode reaching average consensus, (b) Leaderless mode reaching consensus and (c) Leader-follower mode



(a) (b) (c)

Fig. 7 The orientation trajectories corresponding to different sampling rates (a) $\tau = 200\text{ms}$, (b) $\tau = 600\text{ms}$ and (c) $\tau = 850\text{ms}$