

# Planar Visual Metrology using Partition-based Camera Calibration

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**Abstract**—In visual metrology the real imaging procedure is a complicate nonlinear system. Even after rectification of the radial distortion of the lens, it is hard to achieve high accuracy with single group of planar extrinsic parameters. According to the characteristics of the radial distortion of lens, a partition-based camera extrinsic calibration method for planar visual measurement is proposed. The image plane is partitioned into 9 subspaces according to the lens radial distortion, and in each subspace one group of extrinsic parameters of objects plane is calibrated separately. Although there is only one objects plane where the visual measurement is to be done, the groups of extrinsic parameters calibrated in each subspace are different because of the nonlinear in real system. In the subspace that the image points belong to, the corresponding group of extrinsic parameters of the subspace is adopted to reconstruct the 3D positions of the objects in the camera coordinate system. Finally experimental results show that the partition-based calibration method performs well in visual metrology. Compared with the method which uses single group of extrinsic parameters the proposed method can achieve higher accuracy in measuring distance of planar objects from their perspective images.

**Keywords**—visual metrology, camera calibration, partition-based subspaces

## I. INTRODUCTION

Visual metrology is a simple and non-contact measuring method. It directly acquires the geometrical information of objects from their perspective images. Planar visual metrology uses a single camera configuration and is able to make measurements of objects plane from their single perspective image. By adding extra coplanar constrains, the 3D information on the plane can be reconstructed from the perspective image points. As an image-based metrical method, planar visual metrology plays an important role in industrial machine vision inspection application and robotic vision etc [1][2]. Although some metrical information can be obtained by using uncalibrated camera [3], calibration is essential for visual metrology. In order to obtain high measurement accuracy camera and objects plane must be calibrated carefully to get camera intrinsic and extrinsic parameters [4][5]. Reference [6] reviewed some of the most used calibrating techniques and evaluated their accuracy. Zhang's method is a flexible new technique for camera calibration, and is widely adopted in laboratory [7]. To estimate the camera parameters Zhang uses a linear model to get an initial guess and then takes radial

distortion of lens into account, and then uses maximum-likelihood estimation to obtain camera intrinsic and extrinsic parameters. Because of the use of linear model for initial guess, there are system errors in Zhang's method especially when short focus lens and single group of planar extrinsic parameters used. And it is hard to achieve high measuring accuracy in practice.

For the purpose of high accuracy a planar visual metrology with partition-based camera extrinsic calibration is proposed. According to the characteristic of radial distortions of lens the image plane of camera is divided into 9 subspaces, and on each subspace the objects plane's extrinsic parameter with respect to camera is calibrated separately. Although there is only one objects plane where metrology is to be done, the groups of extrinsic parameters calibrated in each subspace are moderately different because of the nonlinear in real system.

## II. VISUAL METROLOGY FOR OBJECTS ON PLANE

The camera model for perspective images of objects plane is well known. In this section we describe the camera model and visual metrology for planar objects from their perspective images.

### A. Camera model and parameters calibration

The world objects plane  $O_w X_w Y_w$  in  $\{W\}$  world coordinate system is projected to image plane  $O_I X_I Y_I$  in  $\{I\}$  image frame coordinate system, as shown in Fig. 1.

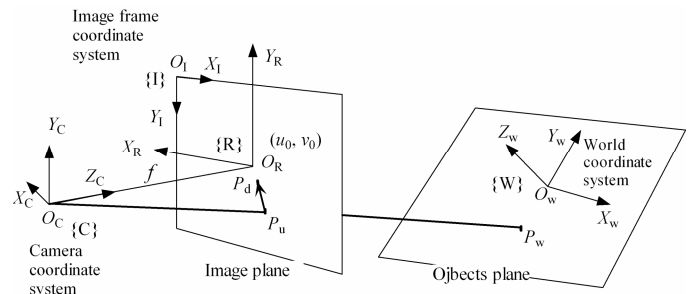


Figure 1. Plane perspective projection model

A given point  $P_w$  on the objects plane, expressed with respect to the metric world coordinate system i.e.  ${}^W P_w = ({}^W X_w, {}^W Y_w, {}^W Z_w = 0)$ , is imaging on the image plane and the

corresponding 2D image point  $\mathbf{P}_d$  in pixels with respect to the image frame coordinate system is obtained, i.e.  ${}^I\mathbf{P}_d = ({}^IX_d, {}^IY_d)$ .

Changing the world coordinate system to the camera coordinate system is carried out by using a transformation which consists of a translation vector and a rotation matrix as shown in follow equation.

$$\begin{pmatrix} {}^cX_w \\ {}^cY_w \\ {}^cZ_w \end{pmatrix} = {}^cR_w \begin{pmatrix} {}^wX_w \\ {}^wY_w \\ {}^wZ_w \end{pmatrix} + {}^cT_w \quad (1)$$

${}^cR_w$  and  ${}^cT_w$  are the rotation matrix and the translation vector of world objects plane coordinate system with respect to the camera coordinate system.

Consider that any optical sensor can be modeled as a pinhole camera [5]. In camera coordinate system a given objects point  ${}^c\mathbf{P}_w$  is projected to image plane through the focal point  $O_c$  by applying the pinhole model and we obtain undistorted 2D image point  ${}^c\mathbf{P}_u$ :

$${}^cX_u = f \frac{{}^cX_w}{{}^cZ_w}, \quad {}^cY_u = f \frac{{}^cY_w}{{}^cZ_w} \quad (2)$$

where  $f$  is the focal length of the lens used in system.

To model the distortion of lenses the undistorted point  ${}^c\mathbf{P}_u$  can be transformed to the distorted point  ${}^c\mathbf{P}_d$ , by using (3):

$${}^cX_u = {}^cX_d + \delta_x, \quad {}^cY_u = {}^cY_d + \delta_y \quad (3)$$

where  $\delta_x$  and  $\delta_y$  represent the distortion involved in system. It has been proved that the first order radial distortion is sufficient in most of the applications [8]. So we have:

$$\begin{aligned} \delta_x &= k_1 {}^cX_d ({}^cX_d^2 + {}^cY_d^2), \\ \delta_y &= k_1 {}^cY_d ({}^cX_d^2 + {}^cY_d^2) \end{aligned} \quad (4)$$

where  $k_1$  is the first order coefficient of the radial distortion.

To express the point  ${}^c\mathbf{P}_d$  with respect to the computer image frame coordinate system  $\{I\}$  in pixels, a simple transformation of coordinates is applied:

$${}^IX_d = -k_u {}^cX_d + u_0, \quad {}^IY_d = -k_v {}^cY_d + v_0 \quad (5)$$

where  $k_u$  and  $k_v$  are the scale coefficients that transform the retinal coordinates to pixels with respect to the image frame coordinate system,  $u_0$  and  $v_0$  are the coordinates of the principal point in pixels.

As above mentioned  ${}^cR_w$  and  ${}^cT_w$  are called camera extrinsic parameters which relate the objects plane with camera coordinate system. Focal length  $f$ , distortion coefficient  $k_1$ , scale coefficients ( $k_u$ ,  $k_v$ ) and the principal point ( $u_0$ ,  $v_0$ ) are called camera intrinsic parameters.

Camera calibration is the process to estimate the intrinsic and extrinsic parameters from the correspondence between calibration board feature points and their image points. The images are taken by the camera from different directions. Zhang's method is a flexible camera calibration widely adopted in laboratory. It carries out a linear approximation with the aim to get an initial guess and then takes radial distortion of lenses into account, uses an iterative maximum-likelihood estimation to optimize the camera intrinsic and extrinsic parameters. In this paper we use Zhang's method to calibrate camera intrinsic and extrinsic parameters.

### B. Reconstruct 3D points coordinates from image perspective points

As shown in Fig. 1 each image point determines an optical ray passing through the focal point of the camera. By adding the coplanar constrain the ray intersects the objects plane with only one point. So if camera intrinsic parameters and the objects plane extrinsic parameters are known the 3D points coordinates on the objects plane can be reconstructed from their perspective images.

From the visual metrology above it is obvious that there are two facts to affect the measuring accuracy. The first one is the errors in positioning the image feature points. The second one is the errors in the camera intrinsic parameters and objects plane extrinsic parameters. The final measurement result errors are the synthesis of the two types of errors. By using a moment-based subpixel measurement technique, the location of the image feature points can achieve the accuracy of 1/10 pixel [9].

In present available planar visual metrology, in order to reconstruct objects points from image points, only one group of extrinsic parameters i.e. ( ${}^cR_w$ ,  ${}^cT_w$ ) is adopted to relate the objects plane with the camera coordinate system. So all image points are back-projected to the objects plane where Euclidean distance measurement is done. But as above mentioned in section II, because the initial guess of model parameters are based on the linear approximation Zhang's technique will introduce errors when doing camera intrinsic and extrinsic parameters estimation. So By using only one group of extrinsic parameters we can't achieve high accuracy in planar visual metrology in practice especially when short focus lenses used.

In this paper according to the principle of planar visual metrology and the characteristics of the radial distortion of lens, a partition-based camera extrinsic parameter calibration method is proposed to achieve high measuring accuracy for planar metrology.

### III. PARTITION-BASED EXTRINSIC PARAMETERS CALIBRATION

#### A. Partition image plane into subspaces

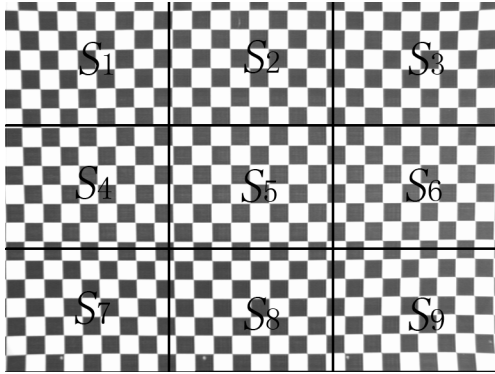


Figure 2. Image with distortion and subspace partition

Fig. 2 is a real image with distortion. From (3), it is obvious that as far away from the center of images the radial distortion is more serious especially at the four corners of the image. Although there is only one objects plane where visual metrology is going to be done, the groups of extrinsic parameters calibrated when calibration board moving to different positions on objects plane are moderately different. This is because that the real imaging system is a complex nonlinear system and the camera model in section II is an approximation of the real system.

According to the characteristic of the lens radial distortion the image plane is divided into 9 parts, i.e.  $S_1 \dots S_9$ , each part is a subspace of the image plane, as shown in Fig. 2. And in each subspace there is a group of extrinsic parameters related to the objects plane. For 9 subspaces there will be 9 groups of extrinsic parameters. So the 9 groups of extrinsic parameters on the subspaces could be calibrated separately by using Zhang's calibration technique.

#### B. Extrinsic parameters calibration in all subspaces

In order to calibrate intrinsic and extrinsic parameters of camera a high accurate dots pattern board with 100 mm  $\times$  80 mm dimensions is used, as shown in Fig. 3.

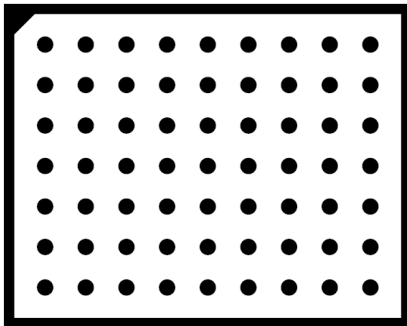


Figure 3. Calibration board with dots pattern

To get one group of extrinsic parameters on each subspace Zhang's calibration method is used. So each calibration on each subspace needs one image. By moving the calibration board on

the objects plane to different subspace positions, 9 images for subspaces calibration are obtained, as shown in Fig. 4.

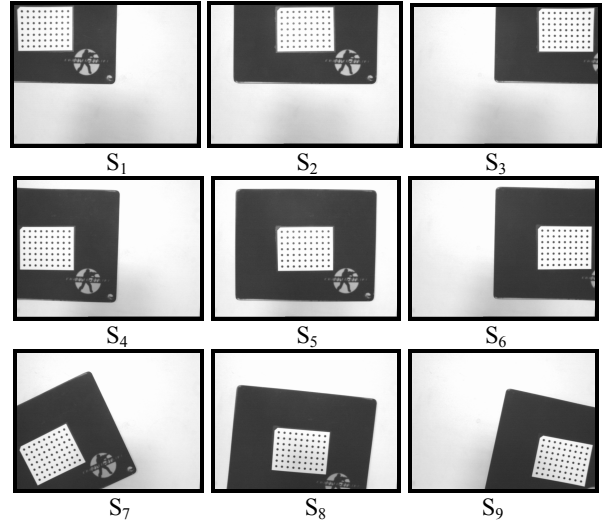


Figure 4. Images for subspace extrinsic parameters calibration

After using Zhang's technique for calibration from 9 images, 9 groups of the extrinsic parameters is obtained from the image plane subspaces, i.e.  $({}^C R_{wi}, {}^C T_{wi})$  ( $i = 1 \dots 9$ ).

### IV. PLANAR VISUAL METROLOGY WITH PARTITION-BASED METHOD

Reconstruction 3D points on the objects plan from their perspective image feature points can obtain 3D points' coordinates in objects plane coordinate system. By using partition-based method as above there are 9 groups of extrinsic parameters. So for the convenience of measurement of Euclidean distance for the objects points, all the reconstructed 3D points' coordinates from different groups of extrinsic parameters should be transformed to the same camera coordinate system  $\{C\}$ .

In the camera coordinate system  $\{C\}$  as shown in Fig. 1, the objects plane equation can be expressed as:

$$n_a {}^C X_W + n_b {}^C Y_W + n_c {}^C Z_W - d = 0 \quad (6)$$

where  $N = (n_a, n_b, n_c)^T$  is the normal vector of the objects plane,  $d$  is the distance of camera center point  $O_c$  to objects plane. According to (1),  $N$  and  $d$  meet the following equations:

$$\begin{pmatrix} n_a \\ n_b \\ n_c \end{pmatrix} = {}^C R_W \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = {}^C R_W \begin{pmatrix} {}^w X_{O_c} \\ {}^w Y_{O_c} \\ {}^w Z_{O_c} \end{pmatrix} + {}^C T_W \quad (8)$$

$${}^w Z_{O_c} = d \quad (9)$$

where  $({}^wX_{oc}, {}^wY_{oc}, {}^wZ_{oc})^T$  is the coordinates of camera center point  $O_c$  in world objects plane coordinate system.

So if the extrinsic parameters are known the objects plane equation vector  $\mathbf{L} = (n_a, n_b, n_c, -d)^T$  can be solved. Corresponding to the 9 groups of extrinsic parameters i.e.  $({}^cR_{wi}, {}^cT_{wi})(i = 1 \dots 9)$  there are 9 objects plane equation vectors  $\mathbf{L}_i$  ( $i = 1 \dots 9$ ). So according to the subspace that the perspective image feature points belong to, the corresponding plane equation vector  $\mathbf{L}$  is chosen to use for the reconstruction of the 3D objects points  $({}^cX_w, {}^cY_w, {}^cZ_w)^T$  in camera coordinate system from (2)-(5).

The steps for measuring the distance  $D_{ij}$  of the two points  $\mathbf{P}_i$  and  $\mathbf{P}_j$  on objects plane can be summarized as:

- 1) Obtain 10-15 images for camera intrinsic parameters calibration from noncoplanar directions, and use Zhang's technique to get the estimation of camera intrinsic parameters;
- 2) Partition image plane into 9 subspaces, i.e.  $S_1 \dots S_9$ , and use the method proposed in this paper to get 9 groups of extrinsic parameters  $({}^cR_{wi}, {}^cT_{wi})(i = 1 \dots 9)$  and 9 plane equation vectors  $\mathbf{L}_i$  ( $i = 1 \dots 9$ );
- 3) Get the image pixel coordinates  ${}^iP_i$  and  ${}^iP_j$  with subpixel accuracy using the moment-based location method, and confirm each object-point's subspace from the location of image points, i.e.  $S_i$  and  $S_j$ ;
- 4) Reconstruct the 3D object point  ${}^cP_i = ({}^cX_w, {}^cY_w, {}^cZ_w)^T$  in camera coordinate system according to camera intrinsic parameters and the corresponding plane equation vector  $\mathbf{L}_i$  using (2)-(9);
- 5) As similar as step 4, reconstruct the 3D object point  ${}^cP_j$  according to the corresponding plane equation vector  $\mathbf{L}_j$ ;
- 6) Calculate the Euclidean distance  $D_{ij} = |{}^cP_i - {}^cP_j|$ ;

## V. EXPERIMENTAL RESULTS

A large number of experimental works is carried out in order to verify the performance and compare accuracy of the method proposed in this paper.

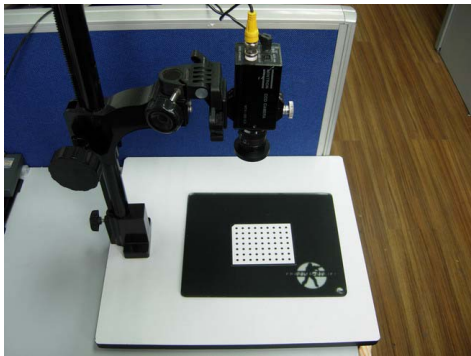


Figure 5. Experimental system setup

The set-up is shown as Fig. 5. The camera is mounted on the fixed test bench to measure the two feet of the caliper on the platform from their perspective images. The camera system is a Mintron MTV-1881EX analog CCD camera with

Computar 5mm lens. The size of the images captured by the camera is 768 pixel  $\times$  576 pixel.

By using Zhang's calibration technique the intrinsic parameters of the camera system are obtained as:

$$f = 0.00527 \text{ m}; k_l = -3052.13;$$

$$k_u = 1/(8.56001e-006) \text{ pixel/m}; k_v = 1/(8.6e-006) \text{ pixel/m};$$

$$u_0 = 378.06 \text{ pixel } v_0 = 281.04 \text{ pixel}.$$

From 9 images taken in the different subspaces, as shown in Fig. 4, 9 groups of extrinsic parameters are obtained and the 9 plane equation vectors  $\mathbf{L}_1 \dots \mathbf{L}_9$ , are as follows:

$$\mathbf{L}_1 = (-0.0593 \ -0.0600 \ 0.9964 \ -0.2636);$$

$$\mathbf{L}_2 = (-0.0004 \ -0.0156 \ 0.9999 \ -0.2556);$$

$$\mathbf{L}_3 = (0.0601 \ -0.0677 \ 0.9959 \ -0.2650);$$

$$\mathbf{L}_4 = (-0.0289 \ -0.0193 \ 0.9994 \ -0.2585);$$

$$\mathbf{L}_5 = (-0.0036 \ -0.0205 \ 0.9998 \ -0.2553);$$

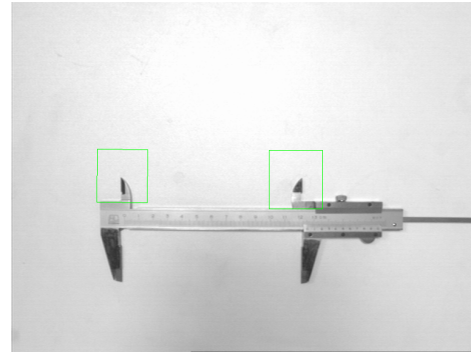
$$\mathbf{L}_6 = (-0.0036 \ -0.0205 \ 0.9998 \ -0.2585);$$

$$\mathbf{L}_7 = (-0.0357 \ 0.0011 \ 0.9994 \ -0.2600);$$

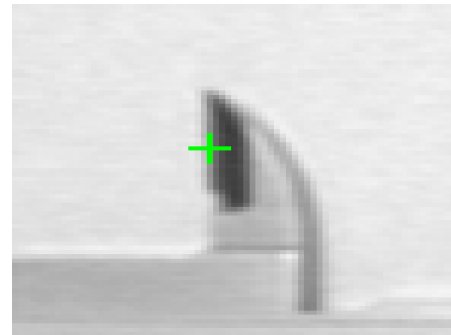
$$\mathbf{L}_8 = (-0.0013 \ -0.0282 \ 0.9996 \ -0.2550);$$

$$\mathbf{L}_9 = (0.0463 \ 0.0141 \ 0.9988 \ -0.2628);$$

A perspective image of the caliper on the platform is as shown in Fig. 6(a). By using moment-based location techniques the pixel coordinates of the two feet can be obtained with subpixel accuracy, as the cross mark shown in Fig. 6(b).



(a)



(b)

Figure 6. Measurement of caliper

(a) Caliper feet to be measured (b) Magnified image and subpixel location

So the distance of the two feet can be measured from the perspective image by using the method proposed in this paper.

The read of the caliper was referred to as the true value. A number of experiments where the caliper was put in different position on the platform were carried out. And as a comparison the distance was also measured by using the method based on single group of extrinsic parameters. Here subspace  $S_5$  is chosen. Table I lists the results of the ten groups experiments.

TABLE I. RESULTS AND ERRORS COMPARE WITH TWO METHODS

Images No.	Real Value (mm)	Partition-based method		Single group of extrinsic parameters method		Subspace
		Estimate value (mm)	Absolute value of error (mm)	Estimate value (mm)	Absolute value of error (mm)	
1	125.5	125.25	0.25	124.51	0.99	$S_4 S_5$
2	125.5	125.70	0.20	124.80	0.70	$S_7 S_8$
3	125.5	125.66	0.16	124.82	0.68	$S_1 S_5$
4	125.5	125.84	0.34	125.02	0.48	$S_9 S_2$
5	125.5	125.88	0.38	124.87	0.63	$S_7 S_2$
6	125.5	125.83	0.33	124.88	0.62	$S_8 S_3$
7	125.5	125.22	0.28	124.73	0.77	$S_4 S_2$
8	125.5	125.24	0.26	124.78	0.72	$S_8 S_6$
9	125.5	125.10	0.40	124.99	0.51	$S_8 S_6$
10	125.5	125.24	0.26	124.70	0.80	$S_5 S_8$

As table I shown, with the image resolution of 768 pixel  $\times$  576 pixel the proposed method can achieve the accuracy in 0.5 mm. And with comparison to the single group of extrinsic parameters the proposed method has fewer errors in planar visual metrology.

## VI. CONCLUSION

In order to obtain high accurate measurement for planar visual metrology a partition-based camera extrinsic calibration method is proposed. The objective is to reduce the errors

introduced by applying the camera model. By replacing single group of planar extrinsic parameters with 9 groups in image subspace the proposed method uses a partition-based nonlinear parameters model to approximate the real system. Experimental results show that the partition-based calibration method performs well in visual metrology and can achieve higher accuracy in measuring distances of planar objects from their perspective images.

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