# Logical Control for a Class of Nonlinear Systems

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**Abstract—In this paper, a logical control algorithm for a class of single-input-single-output (SISO) affine nonlinear systems with known parameters is approached: first, an inside-loop nonlinear feedback control is built, and a pseudo linear system is established based on the inverse system method. Second, an outside-loop control net——logical controller to make the system approximate stable is designed. Beside, the control result of the considered nonlinear system is much better than normal logical controllers by adjusting the parameters of logical controller given out in the paper.**

***Keywords*—affine nonlinear systems; logical control; input-output error**

## I. INTRODUCTION

The nonlinear control based on the feedback linearization [1] (chaos systems, fuzzy systems, and adaptive controllers) has received much attention recently [2, 3, 4]. In such schemes, the stability of the closed-looped system is established according to Lyapunov's theory or the universal approximation theorem [8]. A tracking control method [2] based on nonlinear feedback control theory is developed to synthesize chaos from n-dimensional systems ( $n \geq 3$ ) in a unified form. The paper [3] focuses on the construction of a fuzzy adaptive output feedback control based on any observer (high-gain (HG) 11 observers, sliding mode (like) observer, etc.) for a class of single-input–single-output (SISO) uncertain or ill-defined affine nonlinear systems. However, we need lots of mathematical operator and theoretic suppose for dealing with the nonlinear systems.

In recent years, a new method of control begins to be popular in China. It is logical control [5]. And it has received some application [6, 7]. In fact, logical control has a lot of similarities with fuzzy control. Both of them provide a systematic and efficient framework to incorporate linguistic information from human expert. The fuzzy control has more complicated mathematical method than logical control. Compared with fuzzy control, logical control is easy to comprehend. Here is the merit of logical control. However, the pure logical control is only a changeable proportional controller [6], it will cause the micro oscillating problems. In this paper, we design a logical control for a class of SISO affine nonlinear systems with known parameters based on exact feedback linearization. Meanwhile, a loop of adjusting parameters is designed to solve the micro oscillating problems.

Unlike the above contribution, in this paper, a stepped-up logical control for a class of nonlinear systems is designed. There are three main contributions that are worth to be emphasized. Firstly, there are still no reports of the logical control with generic nonlinear systems. In the literature [7], the design methods of nonlinear system deal with the simplest nonlinear problem. Secondly, the filter loop introduced in literature [8] is recommended as the adjusting parameter of the logical controller to approach the better results than general logical controller. Finally, a simulation based on our analysis is addressed in section Ⅳ.

## II. PROBLEM DESCRIPTIONS

Consider SISO system [1]

$$\begin{aligned} \overline{x} &= f(\overline{x}) + g(\overline{x})u \\ y &= h(\overline{x}) \end{aligned} \tag{1}$$

Where state vector $\overline{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, \cdots, x_n]^T$ belonging to an open-set $K$ of $R^n$ , $u \in R$ is input vector, $y \in R$ is output vector, $f(x), g(x), h(x)$ are smooth functions defined on an open set $K$ of $R^n$ .

Therefore, the exact feedback linearization of system (1) can be described as [1]:

For a given point $x^{\circ}$ , and find a domain $U$ , the feedback $u$ is defined in the $U$

$$u = \alpha(x) + \beta(x)v \tag{2}$$

where $v$ is the input. Substituting (2) into (1),

$$\dot{x} = f(x) + g(x)\alpha(x) + g(x)\beta(x)v \tag{3}$$

And a coordinate transformation $z = \Phi(x)$ defined in $U$ makes the corresponding closed-loop system be linear and

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controllable in the new coordinate $z = \Phi(x)$ ,namely

$$\left[\frac{\partial \Phi}{\partial x}(f(x)+g(x)\alpha(x))\right]_{x=\Phi^{-1}(z)} = Az$$
$$\left[\frac{\partial \Phi}{\partial x}(g(x)\beta(x))\right]_{x=\Phi^{-1}(z)} = B \quad (4)$$

To some suitable matrix $A \in R^{n\times n}$ and vector $B \in R^n$, we have

$$rank(B \quad AB \quad \cdots \quad A^{n-1}B) = n \quad (5)$$

So, we obtain the precise linearization form (4) of the state-space of considered nonlinear systems.

Lemma 1. The precise linearization problem of state-space is solvable, if and only if when $x^\circ$ exists a domain $U$ and a real function $\lambda(x)$ that defined in $U$ , and the system (6) has relation degree= $n$ in $x^\circ$ .

$$\begin{aligned} \dot{x} &= f(x)+g(x)u \\ y &= \lambda(x) \end{aligned} \quad (6)$$

On the basis of feedback-linearization, we can introduce the theory of linear control systems and design controller such as PID, adaptive and fuzzy control algorithm shown in literature [3]. In this paper, we will design a new logical controller based on the precise linearization.

Logic control generally refers to the systems' condition model that expressed by a group of concepts and corresponding operation, the decision-making, in order to meet the required performance. Such a process is known as logic control. The theoretical basis is the pan-Boolean algebra. The control method is judged by the reflect deviation composed by state-variable and the trend of variable systems' operating conditions, according to different conditions output under designed control variable. The viewpoint of control is viewed as the process of compensating and consuming energy.

## III. CONTROLLER DESIGN AND STABILITY ANALYSIS

Review the process for precision linearization at first [1].

### A. Precise linearization

For system (1), assume the related degree is $r = n$ at $x^\circ$ . Based on the definition of related degree, we can let

$$\begin{aligned} \phi_1(x) &= h(x) \\ \phi_2(x) &= L_f h(x) \\ &\cdots \\ \phi_n(x) &= L_f^{n-1} h(x) \end{aligned} \quad (7)$$

Easily find the description of that system in the new coordinate $z_i = \phi_i(x), 1 \le i \le n$ :

For $z_1, \cdots, z_r$ , satisfied:

$$\begin{aligned} \dot{z}_1(t) &= \frac{dz_1}{dt} = \frac{\partial \phi_1}{\partial x}\frac{dx}{dt} = \frac{\partial h}{\partial x}\frac{dx}{dt} \\ &= L_f h(x(t)) = \phi_2(x(t)) = z_2(t) \\ &\cdots \\ \dot{z}_{n-1}(t) &= \frac{dz_{n-1}}{dt} = \frac{\partial \phi_{n-1}}{\partial x}\frac{dx}{dt} = \frac{\partial (L_f^{n-2}h)}{\partial x}\frac{dx}{dt} \\ &= L_f^{n-1} h(x(t)) = \phi_n(x(t)) = z_n(t) \end{aligned} \quad (8)$$

for $z_n$ , we have

$$\dot{z}_n = \frac{dz_n}{dt} = L_f^n h(x(t)) + L_g L_f^{n-1} h(x(t)) u(t) \quad (9)$$

let

$$\begin{aligned} m(z) &= L_g L_f^{n-1} h(\Phi^{-1}(z)) \\ n(z) &= L_f^n h(\Phi^{-1}(z)) \end{aligned} \quad (10)$$

The description of system as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= n(z) + m(z)u \end{aligned} \quad (11)$$

Where $z = (z_1, \cdots, z_n)^T$ Based on the definition of relative degree, at $z^\circ = \Phi(x^\circ)$ , $m(z^\circ) \ne 0$ .so coefficient $m(z)$ is not equal to 0 at neighborhood $z^\circ$ . So, choose the state feedback control laws as follows:

$$u = \frac{1}{m(z)}(-n(z) + v) \quad (12)$$

Then we get the (13) by substituting (12) into (11)

$$\dot{z} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} z + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \quad (13)$$

And we can summarize the precise linearization ( $r = n$ ) as Figure.1.

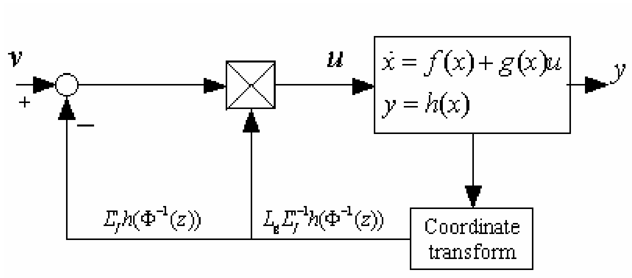


Figure 1. The scheme of precise linearization ( $r = n$ )

*B. The basic theory of logic control*

The logic controller according to error $e$ and the error rate of change $\dot{e}$ make corresponding control action based on different conditions, so as to achieve real-time tracking control. The basic logic controller, the error e can be divided into positive, zero, negative. When the error is positive, that is $e > 0$ , the value of output $y(t)$ is less than the set value $r(t)$ and the system is at shortage energy condition, we should set up a larger control role to supply energy. When the error is zero, that $e = 0$ , the value of output $y(t)$ is equal the set value $r(t)$ , and we can assume that the energy is at balance state, we do not need doing anything. When the error is negative, that $e < 0$ , the value of output $y(t)$ is larger than the set value $r(t)$ and the system is at excessive energy state, we can inhibit excessive energy according to inhibiting action. Similarly, we can use the same method to deal with the error rate of change in various situations. We can get nine different working conditions, and according to different working conditions we can have corresponding control quantity（TABLE Ⅰ）.

TABLE I. ACTION OF BACIS LOGICAL CONTROL

| $\dot{e}$ | $e$ | | |
|---|---|---|---|
| | positive | zero | negative |
| positive | Bigger compensate | Compensate | No action |
| zero | Compensate | No action | Inhibition |
| negative | No action | Inhibition | Bigger inhibition |

For the realization of logic control quantity, at present there is only proportion control plan. So, in essence, we can call logical control as logic proportional control, the control system structure as the following (Figure.2):

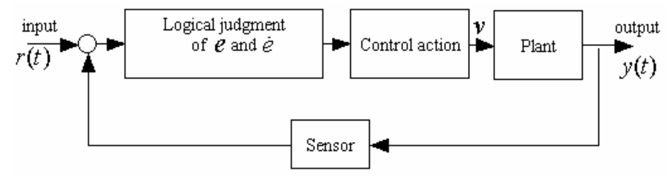


Figure 2. Scheme of the basic logic control

For general logic controller, we can look upon $v$ of (12)

$$v = K_c^t \quad (14)$$

so

$$u = \frac{1}{m(z)}(-n(z) + K_c^t) \quad (15)$$

With (12)

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= K_c^t \end{aligned} \quad (16)$$

Thus, the relation of input and output becomes a simple integral compensation. Obviously, for general affine non-linearization only integral control effect is not enough.

*C. Design of parameter self-adjusting controller*

From above, for the further steady state problem of system, we choose the local parameter self-adjusting $u_a$ as follow:

$$u_a = -\theta^T sign(e_f), \quad (17)$$

Where, $\theta^T = [\theta_1, \theta_2, \ldots \theta_n]$ is parameter adjusting vector.

Here $e_f$ is based on literature [8].

Defined a new vector, specific as follows:

$$\dot{e}_f + Ke_f = \alpha_0(\dot{e} + \alpha e) \quad (18)$$

From (18) we know

$$e_f = \left[ \frac{\alpha_0 (s+\alpha)}{(s+K)} \right] e \qquad (19)$$

Where $K$ and $\alpha_0$ are positive design parameters.

To summarize, Figure.3 shows the overall scheme of the logical control based on input and output error proposed in this paper.

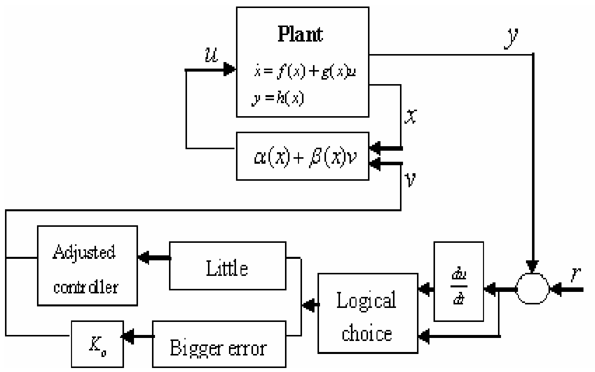


Figure 3. Overall scheme of the parameter self-adjusting logical controller

### D. *Stability analysis and proof*

For large-scale stability, we can easily see in the phase plane [5], the relation of input-output of system is under a trend that energy constantly becomes stable.

Based on the above analysis we can use the Routh-Hurwitz criterion in the process of local error adjustment. So we can get the open-loop transfer function

$$K(s) = \frac{y(s)}{e(s)} = \frac{\alpha_0 (s+\alpha)}{s(s+K)} \qquad (20)$$

And close-loop system characteristic equation

$$D(s) = s^2 + (K+\alpha_0)s + \alpha_0 \alpha \qquad (21)$$

Obviously, system is stable if and only if $\alpha_0, \alpha, K > 0$.

## IV. Simulation

Considering the following third-order nonlinear system:

$$\begin{cases} \dot{x}_1 = e^{x_2} u \\ \dot{x}_2 = x_1 + x_2^2 + e^{x_2} u \\ \dot{x}_3 = x_1 - x_2 \\ y = x_3 \end{cases} \qquad (22)$$

for the system, we have:

$$\begin{aligned} & L_g h(x) = 0, \; L_f h(x) = x_1 - x_2 \\ & L_g L_f h(x) = 0, \; L_f^2 h(x) = -x_1 - x_2^2 \\ & L_g L_f^2 h(x) = -(1+2x_2) e^{x_2} \\ & L_f^3 h(x) = -2x_2 (x_1 + x_2^2) \end{aligned} \qquad (23)$$

Obviously, for each point suits $1+2x_2 \neq 0$, the relative degree of the system is 3(equal to $n$). Such an arbitrary point in the vicinity, for example near the point $x=0$, by means of feedback control (24) and coordinate transformation (25) we can transform the original system into a linear and controllable system.

$$u = \frac{-2x_2 (x_1 + x_2^2) - v}{(1+2x_2) e^{x_2}} \qquad (24)$$

$$\begin{aligned} & z_1 = h(x) = x_3 \\ & z_2 = L_f h(x) = x_1 - x_2 \\ & z_3 = L_f^2 h(x) = -x_1 - x_2^2 \end{aligned} \qquad (25)$$

In the new coordinates, we can get

$$\dot{z} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} z + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \qquad (26)$$

Obviously, the transfer function after feedback-linearization is:

$$G(s) = \frac{1}{s} \qquad (27)$$

The simulation of the system shows that the original system changes to a one-order integral loop after precise feedback-linearization. The open-loop unit step response is shown as Figure.4. From Figure.4, the response approximate to open-loop step-response and the curve slope of the response is similar to its relative degree.

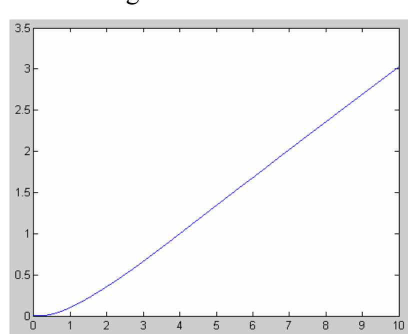


Figure 4. Open-loop step response after feedback-linearization

Preliminary design logic controller, we can get the control effect of simulation as Figure.5.

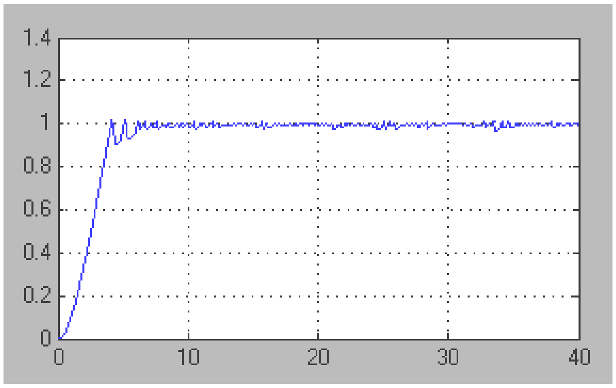


Figure 5. Simulation of basic logical controller

From above, we use the logic control to realize the steady control for affine nonlinear SISO system with known parameters after feedback-linearization. Here, we can easily see the vacuum point that because of precise linearization, and the control effect is not idealistic if only use the single proportional control.

Based on (19) , we make further simulation design for the above example, obtained the better control effect figure ( $\alpha_0 = 1, \alpha = 0.1, K = 100$ ) as Figure.6:

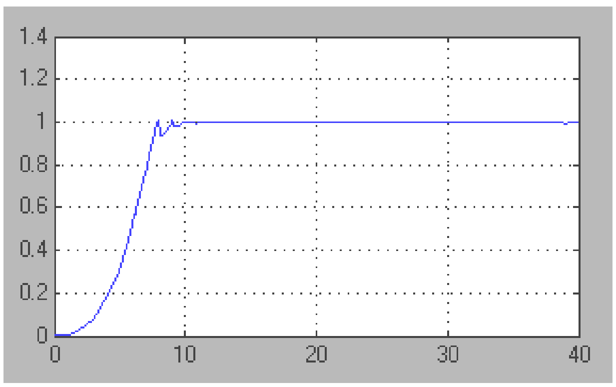


Figure 6. Simulation of stepped-up logical control

## V. CONCLUSION

Logic control is based on the Pan-Boolean algebra which has strict control mode of thinking. Here we apply it into a class of general affine nonlinear systems and prove the general simple logic control can be achieved steady-state approximation control of nonlinear systems. Then based on this logic control we added to achieve a better control effect.

On basis of this issue, we face many problems that are urgent to be solved. For example, logic control problems for affine nonlinear systems control when the general parameters of uncertain; relatively simple logic control whether can achieve the effect of chaos logic control; the parameters of logic control and so on.

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