Analytical Design of Decoupling Control for Variable-Air-Volume Air-conditioning System

Jiangjiang Wang, Chunfa Zhang and Youyin Jing
School of Energy and Power Engineering, North China Electric Power University Baoding, Hebei Province, 071003, China
Jiangjiang3330@sina.com

Abstract—Variable-air-volume (VAV) air-conditioning system has the feature of multi-variable, intense coupling and nonlinear. The coupling may directly affect the property and the stability of VAV system. In this paper, a theoretical model of VAV air-conditioning system by combining the theoretical analysis with the data analysis of experiment is established, which is the four input-output transfer function matrix. Based on the model, a new analytical design method of decoupling controller matrix is proposed in terms of the standard internal model control structure for VAV air-conditioning system. Its advantage is that absolute decoupling for the nominal responses of system outputs can be implemented, and moreover, the controller parameters can be tuned on-line in a monotonous manner to cope with the plant unmodelled dynamics, so that significant decoupling regulation can be achieved. The simulation curves of the decoupling control system show the availability of the method.

Keywords—variable-air-volume (VAV) system, air-conditioning system, decoupling control, internal model control

I. INTRODUCTION

Variable-air-volume (VAV) air-conditioning system has an advantage that its energy-saving in low load is better than the constant air volume (CAV) air-conditioning system [1,2]. At present VAV air-conditioning system is gradually the main steam of the whole air conditioning system in china and abroad because of its large energy-saving. However, VAV system has the feature of multi-variable, intense coupling and nonlinear. When the several control loops in the control system are working together, there is intense coupling among them. The loops influence as well as interfere each other. The coupling may directly affect the property and the stability of VAV air-conditioning system. So the design, performance and management of VAV system are more difficult than CAV system and the stable control of VAV system is focused in particular.

Some researchers have studied the optimal control strategy of VAV system to obtain better performance or save energy. Combining PID with fuzzy theory, a fuzzy self-tuning PID controller was applied to the VAV air-conditioning system in [3]. Considered the optimizing indices, which included comfort index and energy consumption index, the former adopting PMV and the latter being energy consumption of fans and chilled water pumps, an artificial neural network (ANN) was structured and trained in adopting predictive rolling optimization algorithm in [4]. The trained network acted as an optimizing feedback controller to obtain the optimal solution of VAV system. Two on-line optimal control strategies were proposed to save energy consumption of multi-zone VAV air-conditioning system in [5]. One is on-line optimal reset of supply air temperature set-point, which uses the signal of local controller of VAV damper as index of indoor relative load; another is on-line optimal on/off control of CAV boxes which are designed for external zones because of solar heat gains. The two strategies were validated by simulation tests under kinds of weather conditions.

Additionally, some researchers were interested in the decoupling control in the VAV air-conditioning system. A decoupling controller designed in the way of feed forward compensation was used to a constant temperature and humidity air-conditioning system to decouple in [6].

In the intelligent control and optimal field, some researchers have studied the decoupling control. The highly coupling system for VAV air-conditioning system has been decoupled by using self-adaptive neuron decoupling control technology in [7]. A nonlinear non-interacting control system for temperature and relative humidity is modeled in a thermal-space conditioned by a VAV system and using a multivariable cascade control with two loops decouples the temperature and relative humidity in [8]. The inner-loop is the non-interacting control law used for decoupling, and the outer-loop is a PD controller used for stabilization and control.

A dynamic model for a transducer-electromotor-pressure loop and a fresh air valve-CO₂ volume content loop was established in order to eliminate the coupling of each loop in a VAV system and effectively control the system in [9]. The decoupling control technology based on the PID ANN was used to eliminate the coupling between the two loops.

According to the conditions that the air handling units(AHU) has coupling variable and is difficult to operate stably, based on analyzing and comparing many schemes about decoupling control of multivariable system, a α th-order inverse system based on ANN to decouple and control the plant is presented and applied to the VAV system in [10].

In this paper, a theoretical model of VAV air-conditioning system by combining the theoretical analysis with the data analysis of experiment is established. Based on the model, a
new analytical design method of decoupling controller matrix is proposed in terms of the standard internal model control structure for VAV air-conditioning system. Finally, the simulation shows the availability of the method.

II. MATH MODEL OF VAV AIR-CONDITIONING SYSTEM

The VAV air-conditioning control system is shown in Figure 1.

The control system includes the temperature, humidity and press sensors, the VAV controller, the variable frequency controller of fan, humidifier and its control valve, heat exchanger and its control valve and the air valve, etc. In the VAV air-conditioning control system:

the input vector: \( U = [G_w, G_a, d_o, n_{fan}] \); 
the output vector: \( Y = [t_s, t_r, d_o, p_r] \); 
so the transfer function matrix is shown as follows:

\[
G(s) = [G_{ij}(s)]
\]

where \( G_w, G_a, d_o \) are respectively the water flow, air flow, moisture content in the supply air and the rational seep of fan; \( t_s, t_r, d_o, p_r \) are respectively the supply air temperature, the air temperature in the room, the moisture content in the room and the static pressure of supply air; \( i=1,2,3,4; \ j=1,2,3,4 \) and \( G(s) \) is the matrix that has 4 rows and 4 columns. The transfer function matrix is established in the following part.

A. Control of Supply Air Temperature

Based on the law conservation of energy, the thermal balance equation [11] is shown as follows:

\[
M_h C_a \frac{dt_a}{dt} = G_a C_a (t_{a,in} - t_a) - G_a C_a (t_{a,out} - t_{w,in}) \tag{2}
\]

where, \( M_h \) is the quality of heat exchanger, (kg); \( G_a \) and \( G_w \) are the respectively supply air flows and cold water (or hot water) flows, (kg/s); \( C_a, C_w \) and \( C_h \) are the respectively specific heat capacities of air, water and heat exchanger, (kJ/kg · ℃); \( t_{a,in} \) is the temperature of air intake to heat exchanger, (℃); \( t_a \) is the temperature of supply air to room, (℃); \( t_h \) is the surface temperature of heat exchanger, (℃); \( t_{a,out} \) and \( t_{w,out} \) are the respectively the temperature of supply water and back-water to heat exchanger, (℃).

The temperature of air intake to heat exchanger is shown as follows:

\[
t_{a,in} = \frac{G_{a,f} t_{a,f} + G_{a,r} t_{a,r}}{G_a} \tag{3}
\]

where \( G_{a,f} \) and \( G_{a,r} \) are the respectively flows of fresh air and return air, (kg/s); \( t_{a,f} \) and \( t_{a,r} \) are the respectively temperature of fresh air and return air, (℃).

Here assume that the surface temperature of heat exchanger is equal to the temperature after heat exchanger and \( t_h = t_r \).

Put (3) into (2), it can be expressed:

\[
M_h C_h \frac{dt_a}{dt} + t_a = \frac{G_{a,f} t_{a,f}}{G_a} + \frac{G_{a,r} t_{a,r}}{G_a} + \frac{G_a - G_{a,f}}{G_a} t_{a,r} - \frac{G_a C_w}{G_a C_a} (t_{w,out} - t_{w,in}) \tag{4}
\]

Based on (4), the transfer function of the output \( t_a \) and the control variable \( G_w \), is \( G_{11}(s) = \frac{b_{11}}{T_{11} s + 1} \), where \( T_{11} = \frac{M_h C_h}{G_a C_a} \) and \( b_{11} = \frac{C_w}{G_a C_a} (t_{w,out} - t_{w,in}) \).

Due to the nonlinear of \( t_a \) and \( G_w \), it is necessary to linearize approximately near work point in Taylor series expansion. The transfer function of the output \( t_a \) and the control variable \( G_w \), is \( G_{12}(s) = \frac{b_{12}}{T_{12} s + 1} \), where \( T_{12} = T_{11} \) and

\[
b_{12} = \frac{1}{G_a} \left[ -G_{a,f} G_{a,f} t_{a,f} + G_{a,f} G_{a,r} t_{a,r} + \frac{G_a C_w}{C_a} (t_{w,out} - t_{w,in}) \right].
\]
The transfer function of $t_t$ and $d_t$ is $G_{13}(s)=0$.
The transfer function of $t_t$ and $n_{fan}$ is $G_{14}(s)=0$.

B. Control of Air Temperature in Room

The thermal balance equation [11] is shown as follows:

$$ M_t C_a \frac{dt_t}{dt} = G_a C_a (t_t - t_a) + \frac{(t_a - t_r)}{R} + Q_{room} \tag{5} $$

where, $M_t$ is the air quality of air-conditioning room, (kg); $C_a$ is the specific heat capacities of air in air-conditioning room, (kJ/kg·℃); $t_a$ is the outdoor air temperature and is also the temperature of fresh air to AHU, (℃); $t_r$ is the supply air temperature to room, (℃); $t_t$ is the air temperature of room, (℃); $R$ is the thermal resistance of maintenance structure, (kw/℃); $Q_{room}$ is the dissipated heat from people, equipments, etc, to room, (kw).

Equation (5) is formed to the below expression:

$$ \frac{M_t C_a}{G_a C_a + \frac{1}{R}} \frac{dt_t}{dt} + t_t = \frac{G_a C_a}{G_a C_a + \frac{1}{R}} t_a + \frac{1}{R} t_r + Q_{room} \tag{6} $$

Based on (6), the transfer function of the output $t_t$ and $t_r$ is $\frac{b_2}{T_{22}s+1}$, where $T_{22} = \frac{M_t C_a}{G_a C_a + \frac{1}{R}}$ and $b_2 = \frac{G_a C_a}{G_a C_a + \frac{1}{R}}$. Because of $G_{11}(s) = \frac{t_t}{G_a} = \frac{b_1}{T_{11}s+1}$, the transfer function of $t_t$ and $G_{13}$ is $G_{13}(s) = \frac{b_1 b_2}{(T_{11}s+1)(T_{22}s+1)}$.

The transfer function of the output $t_t$ and the control variable $G_a$ includes two parts. The first part is the transfer function of $t_t$ and $G_a$ that (7) is linearized near the work point in Taylor series expansion and $\frac{b_2}{T_{22}s+1}$, where $T_{22}=T_{t_r}$ and $b_2 = \frac{1}{(G_a C_a + \frac{1}{R})^2} [Q_{room} + \frac{(t_a - t_r)}{R}] C_a$. The second part is the transfer function of $t_t$ and $t_r$ in (7) The $t_t$ and $G_a$ is $G_{13}(s)$and so the second part of transfer function of $t_t$ and $G_a$ is $\frac{t_t}{G_a} = \frac{t_r}{G_a} = \frac{b_1}{T_{11}s+1} \frac{b_2}{T_{22}s+1}$. The total transfer function is $G_{22}(s) = \frac{b_2 T_{22}s + (b_2 + b_1) b_1}{(T_{22}s+1)(T_{11}s+1)}$.

The transfer function of $t_t$ and $d_t$ is $G_{23}(s)=0$.
The transfer function of $t_t$ and $n_{fan}$ is $G_{24}(s)=0$.

C. Control of Moisture Content in Room

The moisture balance equation [11] is shown as follows:

$$ M_t \frac{dd_t}{dt} = G_s (d_t - d_r) + D_n $$

where, $D_n$ is the dissipated moisture from people, equipments, etc, to room, (g/s).

Based on (7), the transfer function of $d_t$ and $G_a$ is $G_{13}(s)=0$.

$d_t$ and $G_a$ is also nonlinear and it is linearized near the work point in Taylor series expansion. The transfer function of $d_t$ and $G_a$ is $G_{13}(s) = \frac{b_{32}}{T_{32}s+1}$, where $T_{33}=T_{32}$ and $b_{32}=\frac{D_n}{G_a^2}$.

The transfer function of $d_t$ and $n_{fan}$ is $G_{34}(s)=0$.

D. Control of Static Pressure in Duct

The static pressure in duct is related to the air flow and the rational speed of fan, which is nonlinear. Here the linearization is treated to the function.

Assume that the work point is $p_{st}$, $n_{fan0}$ and $G_{lab}$, the Taylor series near the work point [12] is shown in (8):

$$ p_s = f(n_{fan0}, G_{lab}) + \left[ (\frac{df}{dn_{fan}})_{n_{fan}=n_{fan0}} \right] (n_{fan} - n_{fan0}) + \left[ (\frac{df}{dG_a})_{G_a=G_{lab}} \right] (G_a - G_{lab}) + \frac{1}{2} \left[ \cdots \right] \tag{8} $$

The two-order and multi-order polynomials are ignored here and get the below expression:

$$ p_s - p_{st} = k_1 (n_{fan} - n_{fan0}) + k_2 (G_a - G_{lab}) \tag{9} $$

Where $k_1 = \left( \frac{df}{dn_{fan}} \right)_{n_{fan}=n_{fan0}}$, and $k_2 = \left( \frac{df}{dG_a} \right)_{G_a=G_{lab}}$.

The transfer function of $p_s$ and $G_a$, is $G_{41}(s)=0$.

The transfer function of $p_s$ and $n_{fan}$ is $G_{42}(s)=0$.

The transfer function of $p_s$ and $d_t$ is $G_{43}(s)=0$.

The transfer function of $p_s$ and $n_{fan}$ is also zero order and so is $G_{44}(s)=b_{44}$.

Through the above analysis based on the theoretical analysis with the data analysis of experiment, the four input-output transfer function matrix $G(s)$ is shown as follows:
\[ G(s) = \begin{bmatrix} \frac{b_1}{T_{11}s+1} & \frac{b_2}{T_{12}s+1} & 0 & 0 \\ \frac{b_{12}}{(T_{11}s+1)(T_{22}s+1)} & \frac{b_{22}}{(T_{12}s+1)(T_{22}s+1)} & \frac{b_{23}}{T_{23}s+1} & 0 \\ 0 & \frac{T_{23}s+1}{T_{33}s+1} & \frac{b_{34}}{T_{34}s+1} & 0 \\ 0 & 0 & \frac{b_{42}}{T_{42}s+1} & \frac{b_{44}}{T_{44}s+1} \end{bmatrix} \] (10)

III. DECOUPLING CONTROL DESIGN

A. Decoupling Control Design

Here the analytical design of decoupling control based on the internal model control in VAV air-conditioning system is discussed. The internal model control structure is shown in Figure 2.

\[ H(s) = G(s)C(s)[I + (G(s) - G_m(s))] \] (11)

When the system is nominal and \( G(s) = G_m(s) \), the output in diagonalization forms through decoupling control is shown in (10).

\[ H(s) = G(s)C(s) = \begin{bmatrix} h_1(s) & 0 & 0 & 0 \\ 0 & h_2(s) & 0 & 0 \\ 0 & 0 & h_3(s) & 0 \\ 0 & 0 & 0 & h_4(s) \end{bmatrix} \] (12)

From (12), it is seen that the control system can be decoupled only when the transfer function matrix of control process is static nonsingular as \( \det[G(0)] \neq 0 \). To the VAV system, the determinant of matrix of control process is shown in (13):

\[ \det G(s) = G_{11}(s)G_{44}(s)[G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)] \] (13)

\[ \det G(0) = b_1b_{22}b_{34}b_{44} \] (14)

Because \( \det[G(0)] \) is not equal to zero, the system can be decoupled.

Equation (12) is inverse and the decoupling matrix \( C(s) \) is shown in (15).

\[ C(s) = G(s)^{-1}H(s) = \frac{\text{adj} G(s)}{\det G(s)}H(s) \] (15)

Where \( \text{adj} G(s) = [G(s)]^T \) is the adjoint matrix of \( G(s) \).

Thus the decoupling controller matrix can be calculated. Here the first column can be gotten as follows:

\[ C_{11}(s) = \frac{G_{11}(s)}{\det G(s)}h_1(s) = \frac{(b_{22}T_{11}s + b_{23}b_{12})(T_{11}s + 1)}{b_1b_{22}(T_{11}s + 1)}h_1(s) \] (16)

\[ C_{21}(s) = \frac{G_{21}(s)}{\det G(s)}h_2(s) = \frac{b_{22}}{b_{22}}h_2(s) \] (17)

\[ C_{31}(s) = \frac{G_{31}(s)}{\det G(s)}h_3(s) = \frac{b_{23}b_{13}(T_{11}s + 1)}{b_{22}b_{34}(T_{33}s + 1)}h_3(s) \] (18)

\[ C_{41}(s) = \frac{G_{41}(s)}{\det G(s)}h_4(s) = \frac{b_{23}b_{13}b_{44}}{b_{22}b_{34}}h_4(s) \] (19)

From the above equations it can be seen that \( G_{11}(s)/\det G(s) \) is non-regular and it is not to implement psychically. So \( h_1(s) \) must be a one-order or multi-order transfer function. Combining \( H_2 \) optimal performance specification, the expected output is shown as follows:

\[ h_1(s) = \frac{1}{\lambda s + 1} \] (20)

Where \( \lambda \) is the adjustable parameter that is used to adjust the first way’s output to satisfy the response performance specification.

The Laplace inverse transform of (20) is shown as below:

\[ y_1(t) = 1 - e^{-\lambda t} \] (21)

The above equation shows that the set-point response has no overshoot. Moreover, adjusting \( \lambda \) in a monotonous manner can satisfy the time domain response index. For example, the tuned formula of the up slope time of \( y_1(T_{11}) \), which is the time to reach 90% of final value, is shown as \( T_{11} = 2.3026 \lambda \). So the parameters of controller can be tuned according to the requirement performance.

In same manner the other columns of decoupling controller can be calculated. The decoupling controller matrix is:
the perturbed system is shown in Figure 3.

When the controlled process has additive uncertainty that complicated from (11), whose stability is difficult to determine. So according to the small gain theorem [14], the necessary and sufficient condition based on the spectral radius is shown as below equation:

\[
\rho(G(s)[I + (G(s) - G_m(s))C(s)]^{-1}G(s)\Delta_m) < 1, \quad \forall \omega.
\]

The multiplicative uncertainty of process can be described to \[\prod_1 = \{G_1(s) : G_1(s) = G(s)(I + \Delta_1)\}\] or \[\prod_0 = \{G_0(s) : G_0(s) = (I + \Delta_0)G(s)\}\] , where \(\Delta_1\) and \(\Delta_0\) are both stable and regular. Here referred [16], the necessary and sufficient condition to the system in which there are multiplicative uncertainty is given as follows:

\[
\rho(G(s)[I + (G(s) - G_m(s))C(s)]^{-1}G(s)\Delta_0) < 1, \quad \forall \omega
\]

IV. SIMULATION

To certify the validation of the designed decoupling control system in VAV air-conditioning system, we choose the summer work condition to simulate.

The numerical values of the different parameters and the nominal operating conditions used in the simulations are given as follows:

- The volume of air-conditioning room is 10m long, 8m wide and 4.5m high, the air and water specific heat capacity are respectively \(C_p=1.0 \text{ kJ/kg} \cdot \text{°C}\) and \(C_w=4.18kJ/kg \cdot \text{°C}\), the air and water density is 1.2 kg/m\(^3\) and 1000 kg/m\(^3\), the heat resistance of wall is \(1/R=0.2kw/\text{°C}\), the number of air exchange frequency per hour is 8, the fresh air is about 30% of the supply air to room and the temperature error of supply cold-water and back-water to the heat exchanger in summer is \(T_{wic}-T_{woc}=\pm 5\text{°C}\).

The volume of air-conditioning room is 10m long, 8m wide and 4.5m high, the air and water specific heat capacity are respectively \(C_p=1.0 \text{ kJ/kg} \cdot \text{°C}\) and \(C_w=4.18kJ/kg \cdot \text{°C}\), the air and water density is 1.2 kg/m\(^3\) and 1000 kg/m\(^3\), the heat resistance of wall is \(1/R=0.2kw/\text{°C}\), the number of air exchange frequency per hour is 8, the fresh air is about 30% of the supply air to room and the temperature error of supply cold-water and back-water to the heat exchanger in summer is \(T_{wic}-T_{woc}=\pm 5\text{°C}\).

First the VAV air-conditions system works at the balance state, whose parameters are given as follows: \(t_s=30\text{°C} , t_t=16\text{°C} , t_{a}=24\text{°C} , G_p=1.08kg/s, G_w=0.506kg/s\), \(Q_{room}=7.44kw\), \(D_m=4.356g/\text{kg} \cdot \text{s}\), \(d_1=10.216kg/\text{kg}\), \(d_4=6.182g/\text{kg}\) and \(p_s=139Pa\).

Here the decoupling controller’s parameters, which also are the excepted output, are shown as follows: \(\lambda_1=50, \lambda_2=100, \lambda_4=50, \lambda_3=50\).

In the simulation the moisture content in the room \(d_1\) is changed to the relative humidity, which reflects directly the
humidity in the room, $h_r$. At the 0 second, 200 second, 400 second and 600 second, the step inputs are respectively entered into the system. Figure 4 shows the simulation results of nominal system response. It is seen that the system is absolute decoupled. Additionally there is no overshoot in the set-point response.

Figures 4 and 5 display the simulation results of perturbed system response. It is seen that the system can be stable. The decoupling performance has a little degradation, but the system is still decoupled.

### V. CONCLUSION

This paper presented the analytical design method of decoupling controller matrix in terms of the standard internal model control structure for VAV air-conditioning system. Its advantage is that absolute decoupling for the nominal responses of system outputs can be implemented, and moreover, the controller parameters can be tuned on-line in a monotonous manner to cope with the plant unmodelled dynamics. Simulation results were used to demonstrate the validity of the proposed controller. The decoupling behavior can be attractive for industrial process applications where independent and accurate control of temperature and humidity is desired.

### ACKNOWLEDGMENT

The authors would like to acknowledge the financial and technical support of the Key Laboratory of Condition Monitoring and Control for Power Plant Equipment of Ministry of Education, China, during the course of this research.

### REFERENCES


