Online Multivariable Identification of a Nonlinear Distillation Column using an Adaptive Takagi-Sugeno Fuzzy Model

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***Abstract*: In this paper, an online multivariable identification approach has been developed based on an adaptive Takagi-Sugeno fuzzy model. The approach utilizes an evolving rule-base structure and model parameters to adapt the identified fuzzy model to process dynamic changes. Two new schemes have been proposed to improve the rule-base structure evolution. The first scheme smoothes the rule generation in the initial uncertain commissioning period of the identification. The second scheme diagnoses the inactive generated rules by examining their past activation record to delete them, leading to a more compact and efficient rule-base. A weighted recursive least squares (WRLS) algorithm is employed to estimate the rules consequent parameters. The proposed identification approach has been evaluated by a nonlinear distillation column benchmark to demonstrate its effectiveness to identify compact and accurate multivariable fuzzy models.**

***Keywords:* Multivariable identification, Takagi-Sugeno fuzzy model, Distillation column.**

I. INTRODUCTION

Many industrial processes are nonlinear multivariable systems with multiple inputs and multiple outputs, having complicated cross-coupling characteristics. Modeling of such complex, often poorly understood processes, is a substantial and crucial task and by no means routine. The conventional techniques of modeling, based on the so-called first principles models, are difficult or even impossible to apply for these practical engineering problems. A promising alternative is to employ data-driven identification approaches that rely only on processes input-output experimental data.

Fuzzy rule-based modeling approaches have gained significant impetus due to their inherent flexibility to construct process model from experimental data, heuristic rules, or a combination of both. Among the different fuzzy methods, the Takagi–sugeno (TS) modeling techniques have attracted most attention [1] because of its computational and interpretation efficiency. The TS fuzzy model consists of if-then rules with fuzzy antecedents and mathematical functions in the consequent parts.

Thus, the task of TS fuzzy model identification is to determine both the nonlinear parameters of the antecedent membership functions (i.e., model structure) and the linear parameters of the rule consequents (i.e., parameters).

There are two general approaches to identify both the TS fuzzy model structure and parameters. The classical approach employs the human experts to formulate this required knowledge. However, this approach is often inefficient because human cannot sense all the information underlying details. The recent tendency in the fuzzy model research community is focused on the data-driven techniques in which the non-linear dynamic fuzzy models can be learned from input-output measurement data without human involvement.

On-line learning of the TS fuzzy models requires a recursive identification approach for both the model structure and the consequent parameter estimation. Because, the whole set of input-output data is not available at the start of the training procedure.

[2] presents an online identification approach in which the model structure and parameters evolve gradually, without a priori information, starting from the first input-output data sample. This interesting evolving Takagi-Sugeno (eTS) identification feature makes the approach an effective modeling mechanism to adapt itself with process time-varying dynamics due to aging, wearing, change in operation mode and environmental conditions. So far, little attention has been devoted to the fuzzy model identification of multi-input, multi-output (MIMO) industrial processes, from input-output data. In this paper, an online fuzzy model identification for MIMO processes will be presented based on the modification of the approach given in [2].

II. MIMO TS FUZZY MODEL

The TS fuzzy model consists of a set of if-then rules with fuzzy antecedents and crisp mathematical functions in the consequent part. The TS fuzzy rules are usually defined as:

R^i :

$IF\ x_1\ is\ A_1^i\ AND\ x_2\ is\ A_2^i\ AND ... AND\ x_n\ is\ A_n^i$

$THEN$

$$y^i = a_0^i + a_1^i x_1 + ... + a_n^i x_n = x_e^T \pi^i \qquad ; i = 1,..., M \tag{1}$$

Where n is the number of input variables, $x = [x_1, x_2,..., x_n]^T$ represents input vector containing all the inputs of the fuzzy

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model. $x_e = [1 \quad x^T]^T$ is the extended input vector which accommodates the free bias coefficient a_0^i. $\pi^i = [a_0^i, a_1^i, ..., a_n^i]^T$ indicates the column vector of parameters for the consequent part of the ith rule. y^i is the output of the ith local linear sub-models. A_i^j denotes the antecedent fuzzy set of the jth input (j=1,…,n) in the ith rule. For a given input, x, the output of TS model is computed by weighted averaging of individual rules contribution:

$$y = \frac{\sum_{i=1}^{M} y^i \alpha^i}{\sum_{i=1}^{M} \alpha^i} \tag{2}$$

Where:

$$\alpha^i = \prod_{j=1}^{n} \mu_{A_j^i}(x_j) \tag{3}$$

is the degree of activation or fulfillment of the ith rule, defined as the Cartesian product of respective fuzzy sets for this rule. μ represents the following Gaussian-like antecedent fuzzy set by which the activation degree of each rule is determined:

$$\mu_i = e^{-\alpha \left\| x_j - x_i^* \right\|^2} \qquad ;i = 1,...,M \tag{4}$$

Where x^{i*} denotes the center (focal point) of the ith rule and $\alpha = 4/r^2$ defines the influence zone of the rule which can be tuned by r as a leverage for a trade-off between the model complexity and precision.

Now, consider a MIMO process with n_i input and n_o outputs. The process model can be represented by a collection of coupled multi-input, single-output (MISO) discrete-time fuzzy models. The Nonlinear AutoRegressive with eXogenous input (NARX) model is frequently used with many nonlinear identification methods. The resulting fuzzy NARX model establishes a nonlinear functional relation (f (.)) between the past input-output and the predicted output, as:

$$y(k+1) = f(y(k),...,y(k-n+1),u(k),...,u(k-m+1)) \tag{5}$$

Where n and m are the number of delays in output and input, respectively.

The MISO fuzzy models are assumed to be of the input-output NARX type, represented by:

$$y_l(k+1) = f_l(z_l(k)) \qquad ; l=1,\ldots, n_o \tag{6}$$

Where the regression vector $z_l(k)$ is given by:

$$\begin{aligned} z_l(k) = [&\{y_1(k)\}_0^{n_{yl_1}}, ..., \{y_{n0}(k)\}_0^{n_{yln_0}}, \\ &\{u_1(k)\}_{n_{dl_1}}^{n_{ul_1}}, ..., \{u_{n_i}(k)\}_{n_{dln_i}}^{n_{uln_i}}] \end{aligned} \tag{7}$$

Where the notation {.} represents an ordered sequence of delayed samples of a signal, defined as:

$$\{y(k)\}_{n_1}^{n_2} \underline{\underline{\Delta}} [y(k-n_1), y(k-n_1-1), ..., y(k-n_2)] \qquad ; n_2 \geq n_1 \tag{8}$$

n_y and n_u denote matrices with the number of delays in each output and input, respectively, and n_d is the matrix with the numbers of pure delays from each input to the output. n_y is an $n_0 \times n_0$ matrix, and n_u, n_d are $n_0 \times n_i$ matrices. f_l are rule-based fuzzy models of the TS type. With the antecedent in the conjunctive form, the rules are:

$$\begin{aligned} &R_l^i : if\,(x_{l_1} is A_{l_1}^i) and ... and\,(x_{l_n} is A_{l_n}^i), Then \\ &y_l^i(k+1) = \beta_l^i y(k) + \gamma_l^i u(k) + \theta_l^i \qquad ; i = 1,2,..., m_l \end{aligned} \tag{9}$$

Where x_{l_i} is an element from the regression vector (7), A_l^i is the antecedent fuzzy set of the ith rule, β_l^i and γ_l^i are polynomials in q^{-1}, i.e. the backward shift operator $(q^{-1}y(k) = y(k-1))$, $U \in R^{n_i}$ and $U \in R^{n_o}$ are input and output vectors, respectively and θ_l^i is the bias term. m_l denotes the number of rules in the lth model.

III. ONLINE MIMO FUZZY MODEL IDENTIFICATION BASED ON AN EVOLVING STRATEGY

The MIMO fuzzy model identification procedure based on the evolving strategy consists of two distinct steps. In the first, fuzzy rules and fuzzy sets are determined by an online potential clustering approach [2]. In the second step, called as on-line adaptation of the TS fuzzy model, the consequence's parameters of the generated fuzzy rules are adapted by a weighted recursive least squares (WRLS) method.

A. Online Potential Clustering Approach

The on-line clustering procedure starts with the first data point established as the focal point of the first cluster. Following the procedure with the next data point onwards, the potential of the new

data points, $z_l(k)$, is calculated recursively as follows:

$$P_{lk}(z_l(k)) = \frac{1}{1 + \frac{1}{(k-1)} \sum_{i=1}^{k-1} \sum_{j=1}^{n_i+n_o} \left(d_{lik}^j\right)^2} \qquad ; k = 2,3,... \tag{10}$$

Where $P_{lk}(z_l(k))$ denotes the potential of the data point $z_l(k)$ calculated at time k corresponding to lth MISO model. $d_{lik}^j = z_i^j - z_k^j$ indicates projection of the distance between two data points at different sample times i, k on the axis of z_l.

This function is monotonic and inversely proportional to the distance and enables updating the coordinates of the focal points of the existing clusters by the following recursive equation:

$$P_{lk}\left(z_l^{i*}\right) = \frac{(k-1)P_{l(k-1)}\left(z_l^{i*}\right)}{k - 2 + P_{l(k-1)}\left(z_l^{i*}\right) + P_{l(k-1)}\left(z_l^{i*}\right)\sum_{j=1}^{n_i+n_o}\left(d_{lk(k-1)}^j\right)^2} \tag{11}$$

Where $P_{l_k}(z_l^{i*})$ is the potential of the existing ith rule center, which is updated for each MISO fuzzy model at time k.
The evolution of the MIMO fuzzy model structure (rule-base) is then conducted by the following two basic principles, based on the comparison of the new data potential to the updated potential of the existing rule centers. for each MISO fuzzy model:

(a)-Rule generation criterion:
If the potential of the new data point is higher than the potential of the centers of the existing clusters, i.e.,

$$P_{lk}(z_l(k)) > \max_{i=1}^{m_l} P_{lk}(z_l^{i*}) \tag{12}$$

Then, the new data point is accepted as a new center and a new rule is added to the rule-base with the following characteristics:

$$m_l = m_l + 1; z_l^{m_l*} = z_l; P_{lk}(z_l^{m_l*}) = P_{lk}(z_l) \tag{13}$$

(b)-Rule replacement criterion
If in addition to the condition expressed by
Eq.(12), the new data point is close to an old existing rule center, i.e.,

$$\frac{P_{lk}(z_l(k))}{\underset{i=1}{\overset{m_l}{Max}} P_{lk}(z_l^{i*})} - \frac{\delta_{\min}}{r} > 1 \tag{14}$$

Where $\delta_{\min} = \min_{i=1}^{m_l} \left\| z_{lk} - z_l^{i*} \right\|$. Then, the new data point z_l replaces the closest center using the following substitutions:

$$z_l^{h*} = z_l; P_{lk}(z_l^{h*}) = P_{lk}(z_l) \tag{15}$$

Where z_l^{h*} denotes the closest center.

B. Online Consequent Parameter Estimation Using a WRLS Approach

As discussed in the previous subsection, the MIMO fuzzy model structure evolves gradually. This affects all the existing data and hence the straight forward application of the WRLS is not applicable. A proper resetting of the covariance matrices and parameters initialization is needed at each time a rule is added to the rule base. The estimation of consequent parameters is conducted by the WRLS algorithm using the following recursive procedure:

1. Initialization of the algorithm with
$$\theta_{l_0} = [(\pi^1)^T \quad (\pi^2)^T ... (\pi^{m_l})^T] = 0; C_{l_0} = \Omega I$$
Where Ω is a user specified initial covariance value.
2. Form the new regressor vector φ_{lk}^T at each sample time.
3. Evaluate the kalman gain vector:
$$K_{lk} = C_{lk}\varphi_{lk} = \frac{C_{l(k-1)}\varphi_{l(k-1)}}{\lambda_{lk} + \varphi_{l(k-1)}^T C_{l(k-1)}\varphi_{l(k-1)}} \tag{16}$$
Where C_{lk} denotes the covariance matrix, and $0 < \lambda_{lk} \le 1$ is a forgetting factor.
4. Update the consequent parameters:
$$\theta_{lk} = \theta_{l(k-1)} + K_{lk}e_{lk} \tag{17}$$
Where $e_{lk} = y_{lk} - \varphi_{l(k-1)}^T \theta_{l(k-1)}$.
5. Update the covariance matrix:
$$\begin{aligned} C_{lk} &= C_{l(k-1)} - \frac{C_{l(k-1)}\varphi_{l(k-1)}^T C_{l(k-1)}}{\lambda_{lk} + \varphi_{l(k-1)}^T C_{l(k-1)}\varphi_{l(k-1)}} \\ &= [I - K_{lk}\varphi_{l(k-1)}^T] C_{l(k-1)} \;/\lambda_{lk} \end{aligned} \tag{18}$$

The execution of the WRLS algorithm continues for the next time-step from step 2 in the above procedure.

IV. NEW ADAPTIVE RULE HANDLING SCHEMES IN THE DEVELOPED ONLINE MIMO FUZZY MODEL IDENTIFICATION

In this section, two innovative schemes will be presented to improve the rule handling mechanism.

A. A New Smoothing Scheme in Rule Generation

It is natural that in the initial commissioning period of the identification algorithm, fuzzy rules have more chances to be produced with a high rate due to initial high model uncertainty and transient response fluctuations. This may lead to generation of unnecessary rules under the influence of high frequency noise. An exponential time-varying weight, $\eta(k)$, is included in Eq.(12), as follows:

$$P_{lk}(z_l(k)) > \eta_l(k) \times \underset{i=1}{\overset{m_l}{Max}} P_{lk}(z_l^{i*}) \tag{19}$$

in which:

$$\eta_l(k) = 1 + (\eta_{l0} - 1)\exp\left(-\frac{k}{\tau_l}\right); l = 1,..., n_o \tag{20}$$

Where $\eta_{l0} > 1$ and τ_l are set by the user. This scheme makes the algorithm more cautious in the initial identification phase to add extra rules. Then, as the identification progresses and more dynamic knowledge are captured, the rule generation mechanism gets back to its original lower decision level.

B. A New Rule Reduction Scheme

When the identified MIMO model is more tuned, the rule base might end up with some inactive identified rules. Therefore, an improvement can be achieved by recognizing and deleting the generated inefficient rules. In order to identify the inactive rules, the firing level λ_l^i of each already generated rule can be selected as a measure to evaluate the rule activation. Thus, one simple approach is to compare this rule activation measure with a threshold value. When its value is less than the threshold value, the corresponding rule can be deleted. This method, however, may lead to oscillation in rule creation-deletion procedure due to its instantaneous nature. A more rationalized scheme in the rule reduction is proposed which has memory to investigate the rule activation record in the past. This scheme can be formulated as follows:

$$RA_l^i(k) = (1-\gamma_l^i)RA_l^i(k-1) + \gamma_l^i\lambda_l^i \qquad ;l = 1,\ldots,n_o \tag{21}$$

Where $RA_l^i(k)$ denotes the rule activation RA of the ith rule corresponding to the lth MISO fuzzy model at time k. $0 < \gamma_l^i \le 1$ represents the related record forgetting factor and λ_l^i indicates the firing level of the ith rule. Thus, the proposed rule reduction criterion can be expressed by the following logic rule:

If $RA_l^i(k) < \xi_l$, then remove ith rule.

Where ξ_l is the threshold value. To achieve smoother rule reduction procedure, $RA_l^i(k)$ is reset to an initial value after each rule reduction task.

V. ONLINE IDENTIFICATION OF A DISTILLATION COLUMN BENCHMARK CASE STUDY

This simulation study is used to illustrate the advantages of the proposed identification method. The process to be identified is a simulated binary distillation column [3], which covers the most important effects for the dynamic of a real distillation column and is known to be strongly nonlinear.

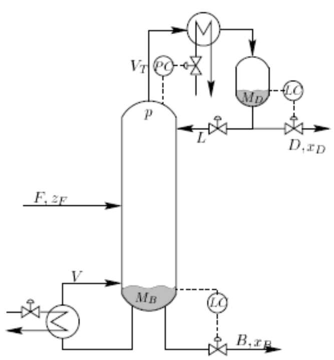

Fig.1. Distillation column

A. Binary Distillation Column Description

The process to be identified is a first-principle model of high-purity binary distillation column, depicted in Fig.1. It is usually called as "column A" in the literature which consists of 39 trays, a reboiler and a condenser. The simulation model, developed by Skogestad [4] under the assumption for (i) equilibrium on all trays, (ii) total condenser, (iii) no vapor holdup and, (iv) linearized liquid dynamics. The model is a 4×4 "open-loop" (uncontrolled column) [4] with four manipulated variables (reflux flow rate, LT, boilup flow rate, VB, distillate product flow rate, D, bottom product flow rate, B) and four uncontrolled variables(top product composition, yD, bottom product composition, xB, condenser hold up, MD, reboiler boilup, MB). Further

details of the simulation model are described in [3].

B. Online Identification of MIMO Fuzzy Model

The first step in the identification process is the selecting of the general NARX model structure in Eq.(6). It is assumed that the process under study can be represented with the following simple model structure:

$$\hat{y}_l(k) = f_l[y(k-1), y(k-2), u(k-1)] \qquad ;l = 1,2,3,4 \tag{22}$$

Where $y(k) = [y_1(k), y_2(k), y_3(k), y_4(k)]$ and $u(k) = [u_1(k), u_2(k), u_3(k), u_4(k)]$ are the process output and input vectors at time k, respectively, and $\hat{y}_l(k)$ represents the predicted lth fuzzy model output. Thus, the regression vector $z_l(k)$ in Eq.(7) includes 12 variables as inputs to the NARX fuzzy model which have four outputs .

The simulated distillation column is excited by four normal distributed random input changes in VB, LT, D and B around their steady-state values, given in [3]. As depicted in Figs.2-5 the excitation signals have been chosen so that they could generate rich data without disturbing the product quality. The parameters r and Ω are initialized at r=0.5 and Ω=10000.

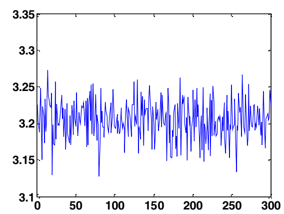

Fig.2. VB input

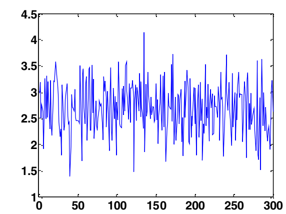

Fig.3. LT input

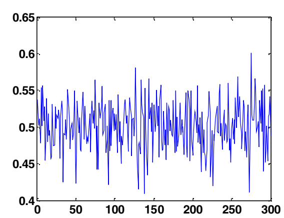

Fig.4. B input

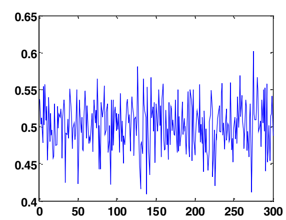

Fig.5. D input

a. Online Identification Using the Original Developed eTS Approach

Figs.6-9 illustrate the resulting distillation column outputs yD, xB, MD and MB due to input excitation signals. These figures include the estimated process outputs due to the online MIMO fuzzy model identification.

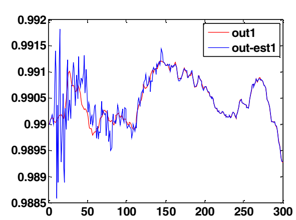

Fig.6. yD output

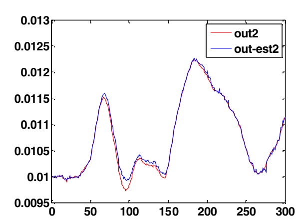

Fig.7. xB output

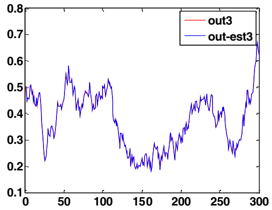

Fig.8. MD output

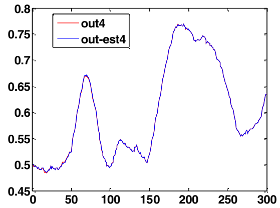

Fig. 9. MB output

As a measure of performance, the Root Mean Square Error (RMSE) is used as follows to evaluate the identified fuzzy model

$$RMSE_l = \sqrt{\frac{\sum_{i=1}^{N}(y_l(k)-\hat{y}_l(k))^2}{N}} \quad ;l=1,2,3,4 \tag{23}$$

Where N is the total number of data samples used in the identification, y_l is the actual lth output and $\hat{y}_l$ is the estimated lth output. Fig.10 depicts the evolution of the generated rules during the online identification process. To demonstrate the performance of the fuzzy model parameter estimation by the WRLS algorithm, the resulted estimated parameters are shown in Figs.11-14.

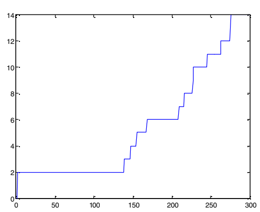
Fig.10. Evolution of the rule base

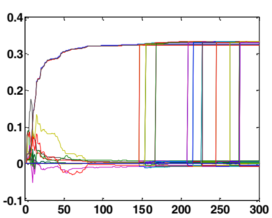
Fig.11. Estimated parameters (θ_1)

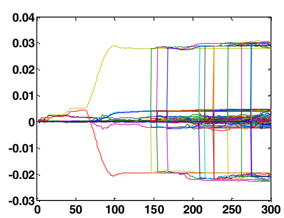
Fig.12. Estimated parameters (θ_2)

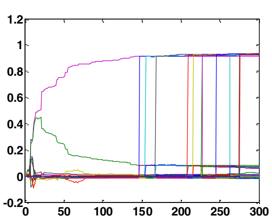
Fig. 13. Estimated parameters (θ_3)

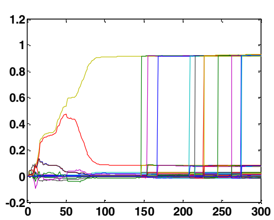
Fig.14. Estimated parameters (θ_4)

b. *Online Identification Using the Modified Developed eTS Approach*

a) Smoothing scheme in rule generation

The online fuzzy identification was repeated using the proposed smoothing scheme, given in Eq.(19-20). The tuning parameters were set to $\eta_{l0} = 1.05$ and $\tau_l = N/5$; where N denotes the number of data samples in the identification time interval. Fig.15 demonstrates the resulting time evolution of the generated rules. As depicted, the final rules have reduced from 14 to 3, which is a significant achievement.

The resulting fuzzy model accuracy has been determined in terms of the calculated RMSE measure for each process output which has been summarized in Table1.

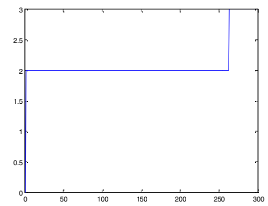
Fig.15. Evolution of the rule base

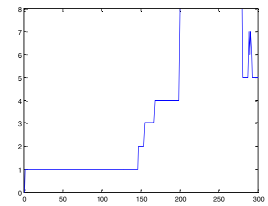
Fig. 16. Evolution of the rule base

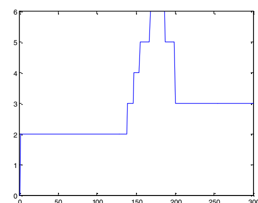
Fig.17. Evolution of the rule base

b) Inactive rule reduction scheme

The online fuzzy identification was repeated using the proposed inactive rule reduction scheme, specified by Eq.(21). The resulting time evolution of the generated rules is illustrated in fig.16. As shown, the rules have converged to five final rules.

The accuracy of the identified fuzzy MIMO model has been expressed in terms of the RMSE measures in table1.

c) The combined rule smoothing and reduction scheme

The online fuzzy identification experiment was repeated with the combined schemes. The resulting rule time evolution has been depicted in Fig.17. As shown, the number of rules has finally converged to three. The resulting model accuracy has been given in terms of the RMSE in Table1. Table1 gives a comparative result for the different online proposed fuzzy identification approaches with that of the original method [5]. The results demonstrate the superiority of the modified approaches in the number of generated rules and consequently in the speed of algorithm, while the total RMSE error has not been affected considerably.

Table 1. Number of rules and RMSE

Method	No. of Rules	RMSE
Original scheme	14	4.5742e-4
Smoothing scheme	3	5.7720e-4
rule reduction scheme	5	4.5984e-4
combined scheme	3	4.6245e-4

IV. CONCLUSIONS

In this paper, an online fuzzy model identification method has been developed for MIMO processes using an eTS approach. Two new schemes have been proposed to enhance the rule generation mechanism. The modified approach generates rules cautiously at the initial identification commissioning period. The second scheme investigates the activation record of already generated rules to recognize and delete the inactive rules. The performance of the resulting approach was demonstrated on a high-purity distillation column benchmark with 4×4 dimension. The simulation results indicate that the proposed fuzzy MIMO identification approach leads to compact accurate dynamic models.

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