Sliding Mode Control of Magnetic Levitation System Using Radial Basis Function Neural Networks

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***Abstract*— This paper has developed a sliding mode controller (SMC) based on a radial basis function model for control of Magnetic levitation system. Adaptive neural networks controllers need plant's Jacobain, but here this problem solved by sliding surface and generalized learning rule in case to eliminate Jacobain problem. The simulation results show that this method is feasible and more effective for Magnetic levitation system control.**

***Keywords*: Radial basis function, Sliding mode, Magnetic levitation system, Sliding surface**

I. INTRODUCTION

Magnetic levitation systems have practical uses in many industrial systems such as in high-speed maglev passenger trains, levitation of wind tunnel models, vibration isolation of sensitive machinery, levitation of molten metal in induction furnaces, and levitation of metal slabs during manufacturing. The maglev systems can be classified as attractive systems or repulsive systems based on the source of levitation forces. These kinds of systems are usually open-loop unstable and are described by highly nonlinear differential equations which present additional difficulties in controlling these systems. Therefore, it is an important task to construct high-performance feedback controllers for regulating the position of the levitated object.

In recent years, a lot of works have been reported in the literature for controlling magnetic levitation systems. The feedback linearization technique has been used to design control laws for magnetic levitation systems [1-3]. Other types of nonlinear controllers based on nonlinear methods have been reported in the literature [4-6]. Control laws based on phase space [7], linear controller design [8], and neural network techniques [9] have also been used to control magnetic levitation systems. One of the first applications of SMC to magnetic levitation systems was carried out by Cho et al. [10]. Chen et al. [11] designed an adaptive sliding mode controller for a rather different type of magnetic levitation systems called dual-axis maglev positioning system. N. AL-Muthairi [12] designed static and dynamic sliding mode controller for the magnetic levitation system.

In this paper, we consider a magnetic levitation system and propose a radial basis function (RBF) sliding mode controller for magnetic levitation system. The proposed approach combines the advantages of the adaptive, neural network and sliding mode control strategies.

The rest of the paper is organized as follows. Section II contains the mathematical model of the magnetic levitation system. Section III deals with the RBF-Sliding mode controller in detail. Sections IV discusses the simulation results of the proposed control schemes. Finally, the conclusion is given in Section V.

II. MATHEMATICAL MODEL OF THE SYSTEM

Fig. 1 is a diagram of the magnetic levitation system. Note that only the vertical motion is considered. The dynamic model of the system can be written as [13]:

$$
\begin{aligned}
&\frac{dp}{dt} = v \\
&Ri + \frac{d(L(p)i)}{dt} = e \\
&m\frac{dv}{dt} = mg_c - C\left(\frac{i}{p}\right)^2
\end{aligned}
\tag{1}
$$

Where p denotes the ball's position, v is the ball's velocity, i is the current in the coil of the electromagnet, e is the applied voltage, R is the coil's resistance, L is the coil's inductance, g_c is the gravitational constant, C is the magnetic force constant and m is the mass of the levitated ball.

The inductance L is a nonlinear function of ball's position p .The approximation of L is:

$$L(p) = L_1 + \frac{2C}{p} \quad (2)$$

Where L_1 is a parameter of the system.

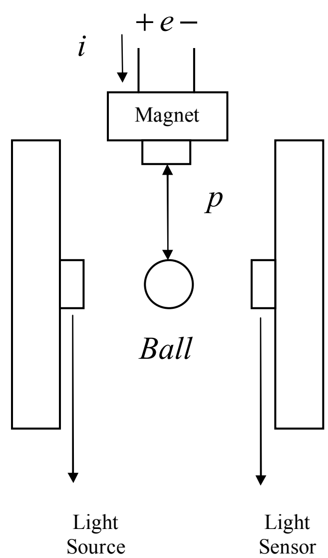

Fig. 1 Diagram of the Magnetic levitation system

Let the states and control input be chosen such $x_1 = p, x_2 = v, x_3 = i, u = e$ .Thus, the state-space model of the magnetic levitation system can be written as:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 \\ \frac{dx_3}{dt} &= -\frac{R}{L}x_3 + \frac{2C}{L}\left(\frac{x_2 x_3}{x_1^2}\right) + \frac{1}{L}u \end{aligned} \quad (3)$$

The parameters of the magnetic levitation system are as follows [13]. The coil's resistance $R = 28.7\Omega$, the inductance $L_1 = 0.65H$, the gravitational constant $g_c = 9.81$ $mili\sec onds^{-2}$, the magnetic force constant $C = 1.24 \times 10^{-4}$, and the mass of the ball $m = 11.87g$ and $x_{1d} = 0.01$ the desired value of x_1.

III. RBF SLIDING MODE CONTROLLER

A. Radial basis function neural networks

In this study, we use a type of neural networks which is called the radial basis function (RBF) networks [14]. These networks have the advantage of being much simpler than the perceptrons while keeping the major property of universal approximation of functions [15]. RBF networks are embedded in a two layer neural networks, where each hidden unit implements a radial activated function. The output units implemented a weighted sum of hidden unit outputs. The input into an RBF network is nonlinear while the output is linear. Their excellent approximation capabilities have been studied in [16]. The output of the first layer for a RBF network is:

$$\phi_i(x) = \exp\left(-\frac{\|x - c_i\|^2}{2\sigma_i^2}\right), \quad i = 1,2,\cdots,n \quad (4)$$

The output of the linear layer is

$$y_i = f(x) = \sum_{i=1}^{n} w_{ji}\phi_i(x) = w_j^T\phi, \quad j = 1,2,\cdots,m \quad (5)$$

Where $x \in R^n$ and $y \in R^m$ are input vector and output vector of the network, respectively, and $\phi = [\phi_1, \cdots, \phi_n]^T$ is the hidden output vector. n is the number of hidden neurons, $W_j = [w_{j1}, \cdots, w_{jn}]^T$ is the weights vector of the network, parameters c_i and σ_i are centers and radii of the basis functions, respectively. The adjustable parameters of RBF networks are W, c_i and σ_i . Since the network's output is linear in the weights, these weights can be established by least-square methods. The adaptation of the RBF parameters c_i and σ_i is a non-linear optimization problem that can be solved by gradient-descent method.

B. RBF sliding mode controller

Sliding mode control (SMC) is a variable structure control utilizing a high-speed switching control law to drive a system state trajectory onto a specified and user chosen surface, so called sliding surface, and to maintain the system state trajectory on the sliding surface at subsequent times [17]. In this paper, the sliding surface on the phase plane can be defined as:

$$\begin{aligned} e_1 &= x_1 - x_{1d} \\ S &= \left(\frac{d}{dt} + \lambda\right)^n e_1 \end{aligned} \quad (6)$$

In case n=2:

$$S = \ddot{e}_1 + \lambda_1\dot{e}_1 + \lambda_2 e_1 \quad (7)$$

The sliding variable, S will be used as the single-input signal for establishing a RBF neural network model to calculate the control law, *u.* Then for the single-input and single-output case in this paper, the output of the controller based on RBF networks is:

$$u=\sum_{i=1}^{n} w_i \exp\left(-\frac{\left\|S-c_i\right\|^2}{2\sigma_i}\right)=W^T\Phi \tag{8}$$

Where n is the number of hidden layer neurons and u is the final closed-loop control input signal. In order to combine the advantages of sliding mode and adaptive control schemes into the RBFNN, an adaptive rule is introduced to adjust the weightings between hidden and output layers.
Based on the Lyapunov theorem, the sliding surface reaching condition is:

$$V=\frac{1}{2}S^2 \Rightarrow \quad \dot{V}=S\dot{S}\langle 0 \tag{9}$$

If a control input u can be chosen to satisfy this reaching condition, the control system will converge to the origin of the phase plane. Adaptive law is used to adjust the weightings for searching the optimal weighting values and obtaining the stable convergence property. The adaptive law is derived from the steep descent rule to minimize the value of $S\dot{S}\langle 0$ with respect to W. Then the updated equation of the weighting parameters is:

$$W_{new}=W_{old}-\eta \frac{\partial S\dot{S}}{\partial W}\Big|_{W=W_{old}} \tag{10}$$

or

$$W_{new}=W_{old}-\eta S \frac{\partial \dot{S}}{\partial W}\Big|_{W=W_{old}} \tag{11}$$

$$\frac{\partial \dot{S}}{\partial W}=\frac{\partial \dot{S}}{\partial u}\times\frac{\partial u}{\partial W} \tag{12}$$

And from equation (7) we have:

$$\dot{S}=\ddot{e}_1+\lambda_1\ddot{e}_1+\lambda_2\dot{e}_1 \Rightarrow \quad \dot{S}=\dddot{x}_1+\lambda_1\ddot{x}_1+\lambda_2\dot{x}_1 \tag{13}$$

form equations (3) we can find that:

$$\frac{\partial \dot{x}_1}{\partial u}=0, \quad \frac{\partial \ddot{x}_1}{\partial u}=0 \tag{14}$$

$$\frac{\partial \dddot{x}_1}{\partial u}\neq 0$$

$$\Rightarrow \quad \frac{\partial \dot{s}}{\partial u}=\frac{\partial \dddot{x}_1}{\partial u} \tag{15}$$

Finally we can find updating rule as follow:

$$W_{new}=W_{old}-\eta S\frac{-2Cx_3}{Lmx_1^2}\times\frac{\partial u}{\partial W}\Big|_{W=W_{old}} \tag{16}$$

from equation (8) we have:

$$\frac{\partial u}{\partial W}=\Phi(S) \tag{17}$$

It is clear that we do not need any identifier for magnetic levitation system.
For improve control signal, a modified RBF-sliding mode controller is now designed for the magnetic levitation system. Fig. 2 is a block diagram of the modified RBF-sliding mode controller.

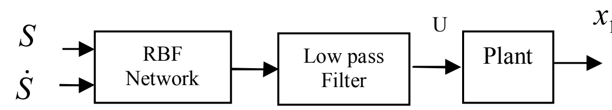

Fig. 2: Block diagram of the modified RBF-sliding mode

IV. SIMULATION RESULTS

In this section, simulation results are presented. Parameter λ_1 and λ_2 are set 61 and 930, respectively. The simulation results are shown in Figures 3 and 4. The figures show the position versus time (millisecond) and the control (the applied voltage) versus time for the system. Fig. 3 shows the RBF-Sliding mode controller of system and Fig. 4 shows the modified RBF-Sliding mode controller of magnetic levitation system. In Fig. 5, we compare these results with static (classical) sliding mode controller [12]. Our proposed method shows better performance respect to classical.

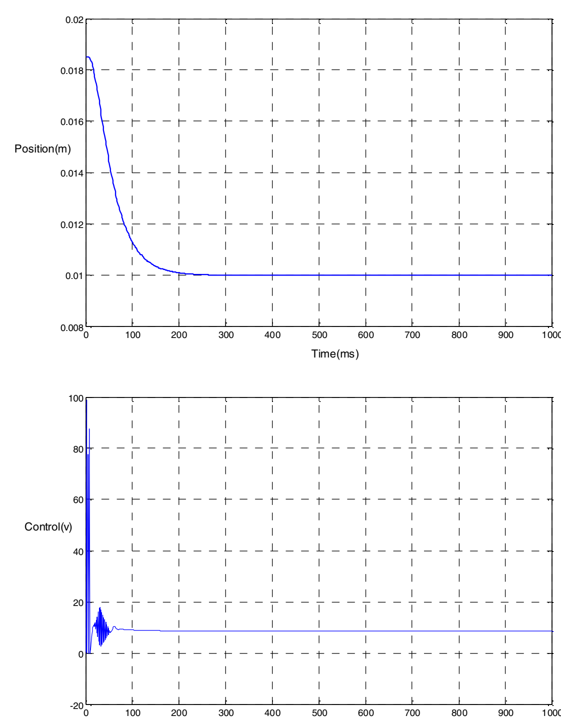

Fig. 3: RBF-Sliding mode controller

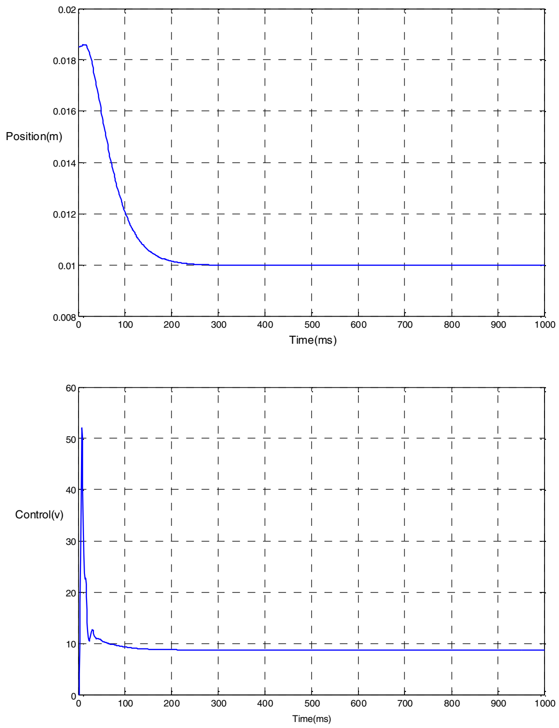

Fig. 4: Modified RBF-Sliding mode controller

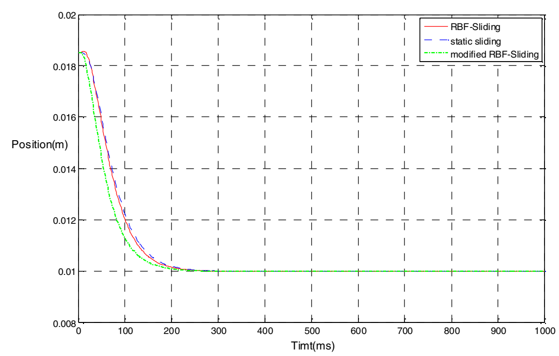

Fig. 5: Comparison results with static sliding mode controller

V. CONCLUSION

This paper introduced RBF-sliding mode method for control of magnetic levitation system which has practical uses in many industrial systems.

In this paper, a new RBF-sliding mode control method for magnetic levitation is proposed, which combines the merits of adaptive neural network and sliding mode control. Based on the Lyapunov stability theory, a RBF-sliding mode controller is designed for stabilization of magnetic levitation system to the desired point in the state space. Simulation results show that the proposed controller is able to control magnetic levitation and the chattering phenomenon of conventional switching type sliding control does not occur in this study.

REFERENCES

[1] W. Barie and J. Chiasson, *Linear and nonlinear state-space controllers for magnetic levitation,* International Journal of Systems Science 27(1996), no.11, 1153–1163.

[2] A. El Hajjaji and M. Ouladsine, *Modeling and nonlinear control of magnetic levitation systems*, IEEE Transactions on Industrial Electronics 48 (2001), no. 4, 831–838.

[3] D. L. Trumper, M. Olson, and P. K. Subrahmanyan, *Linearizing control of magnetic suspension systems*, IEEE Transactions on Control Systems Technology 5 (1997), no. 4, 427–438.

[4] S. A. Green and K. C. Craig, *Robust, design, nonlinear control of magnetic-levitation systems*, Journal of Dynamics, Measurement, and Control 120 (1998), no. 4, 488–495.

[5] C.-M. Huang, J.-Y. Yen, and M.-S. Chen, *Adaptive nonlinear control of repulsive maglev suspension systems*, Control Engineering Practice **8** (2000), no. 5, 1357–1367.

[6] Z.-J. Yang and M. Tateishi, *Robust nonlinear control of a magnetic levitation system via backstepping approach*, Proc. 37th SICE Annual Conference (SICE '98), (Chiba, Japan), July 1998,pp. 1063–1066.

[7] F. Zhao, S. C. Loh, and J. A.May, *Phase-space nonlinear control toolbox: the maglev experience*, 5th International Hybrid Systems Workshop, (Ind, September 1997) (P. Antsaklis,W. Kohn,M. Lemmon, A. Nerode, and S. Sastry, eds.), Lecture Notes in Computer Science, vol. 1567,Springer-Verlag, London, 1999, pp. 429–444.

[8] O. El Rifai and K. Youcef-Toumi, *Achievable performance and design trade-offs in magnetic levitation control*,Proc.5th International Workshop on Advanced Motion Control (AMC'98) ,(Coimbra , Portugal) , July 1998, pp. 586–591.

[9] M. Lairi and G. Bloch, *A neural network with minimal structure for maglev system modeling and control*, Proc.1999 IEEE International Symposium on Intelligent Control / Intelligent Systems and Semiotics , (Massachusetts) ,1999, pp. 40–45.

[10] D. Cho, Y. Kato, and D. Spilman, *Sliding mode and classical controllers in magnetic levitation systems*, IEEE Control Systems Magazine 13 (1993), no. 1, 42–48.

[11] M.-Y. Chen, C.-C.Wang, and L.-C. Fu, *Adaptive slidingmode controller design of a dual-axismaglev positioning system*, Proc. 2001 American Control Conference, (Virginia), vol. 5, 2001,pp. 3731–3736.

[12] N.Muthairi, M.Zribi*, Sliding Mode Control of a magnetic levitation,* Mathematical Problems in Engineering 2004:2 (2004) 93–107

[13] W. Barie and J. Chiasson, *Linear and nonlinear state-space controllers for magnetic levitation*, International Journal of Systems Science 27 (1996), no. 11, 1153–1163.

[14] Powell M. (1987), "Radial basis functions for multivariable interpolation: A review", J.C.Mason and M.G. Cox, eds, Algorithms for Approximation, pp.143-167.

[15] Poggio T., Girosi F. (1987), "Networks for approximation and learning", Proceedings of IEEE 78, pp. 1481-1497.

[16] J. Park, J. W. Sandberg, *"Universal approximation using radial basis functions network"*, Neural Computation, vol. 3, 1991, pp. 246-256.

[17] J. J. E. Slotin, W. Li, *"Applied Nonlinear Control"*, Prentice Hall, 1991.