A Principal-Agent Model on Reverse Supply Chain under Asymmetric Collection Cost Information

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***Abstract*—In this paper, we develop a supply chain model to study the contract design problem for a remanufacturer who delegates the collection of used products to a collector and the collector's cost is a private information which is opaque to the remanufacturer. We apply the incentive theory to this model and characterize the optimal contract. We find that the contracts are quite different with different values of two system parameters: the probability that the collector is of high efficiency and the salvage value of the end-of-life product not to be remanufactured. The information rent and the value of cost information for the remanufacturer are also studied in different cases and managerial insights are explored.**

***Index Terms*—Principal-agent model, Reverse logistics; Game theory; Information asymmetry**

I. INTRODUCTION

Remanufacturing requires that used products are acquired from the end-users so that the value-added operations can be processed and the products(or parts from them) can be resold in the market. Nowadays one of the most important trends in this area is to outsource used products collection to a specialized collector or provider.

For instance, ReCellular, a cellular handsets remanufacturing company, procures used phones from a number of suppliers, including brokers who buy used phones in bulk from the users and then resell them to companies like ReCellular. The company never accepts individual returns because the channel returns from the consumer have too high a cost to be effective [1]. Another example lies in the automotive industry, as Karakyali et. al. [2] indicates. We also notice that the collectors usually develop a market-driven system which relies on financial incentives (actually cash paid in the above cases) to motivate end-users to return their products to the collector [3].

However, there are challenges for the remanufacturer or OEM in buying used products from the collector. One problem is the cost information asymmetry. The collector may hold private information about his internal collection and processing costs, which is opaque to the remanufacturer. Such asymmetric cost information brings about additional difficulty to the collection outsourcing and contracting with the collector.

Motivated by these facts, this paper applies the principal-agent paradigm to the above scenario, incorporating both acquisition price decision of the collector and selling price decision of the remanufacturer. The collector is regarded as an agent, acquiring used products by offering a price to the final users. The remanufacturer is a principal who proposes to buy used products from the collector, decides the quantity of used products to be processed and specifies the selling price for these remanufactured products. Except for the acquisition price the collector pays to the final users, the collector also has a variable cost for each used product, which in our paper is referred to as "collection cost" of the collector, e.g., transportation cost, disassembling cost and stock cost etc.

We focus on the case where the collection cost paid by the collector is his own private information. Specifically, the collector can be of high efficiency (low collection cost) or low efficiency (high collection cost) and the remanufacturer just knows the probability distribution of this cost. In this setting, we design an optimal truth-telling contract for the remanufacurer, which is characterized by the value of two key parameters: the probability that the collector is efficient and the salvage value of the end-of-life product not to be remanufactured. The information rent is also obtained in different cases of the two key parameters. We also study the case of symmetric collection cost information as a benchmark, thus the value of information for the remanufacturer can be specified.

The reverse logistics management is drawing attention among academic researchers due to growing popularity of remanufacturing practices. However, the research focusing on used product acquisition and the reverse supply chain management is very limited. Guide et al. [4] is the first paper to investigate product acquisition management in a remanufacturing environment based on a survey of North American remanufacturing firms. [1] considers the quantity and quality of return flows could be controlled by the acquisition price to maximize the profit. Karakayali et al. [2] develops similar models to investigate on the channel preference of the OEM. Bakal et al. [5] investigates a remanufacturing system in which the supply of cores, the remanufacturing yield and demand of remanufactured products are both price-sensitive. However, all the previous works assume the perfect information sharing. So we will consider a more realistic case of the incomplete collection cost knowledge for the remanufacturer in the paper.

Another revelent stream of research focuses on eliminating obstacles of supply chain coordination caused by asymmetry information and designing supply chain coordination mechanism. Cachon and Lariviere attempt to exchange a fixed payment as agreed in contract for information in order to realize supply chain coordination under asymmetry information [6]. Also, Lau A. H. and Lau H. S. [7] explore inventory policies of supply chain with asymmetry market demand information. Other related papers include information screening models that probe into inventory policy, quantity rebate and contract designing with asymmetry cost information [8] [9], [10]. However, to our best knowledge, our paper is the first one to incorporate the cost information asymmetry into the remanufacturing industry and reverse supply chain scenario under a market-driven collection channel.

The paper is organized as follows. In §2 we present the model and the contract design problem for remanufacturer. In §3 we analyze the optimal contract under different parameter circumstances, investigate the information rent and value of information, and explore some managerial insights. Finally, conclusion is summarized and the future research work is pointed out in §4.

II. Model Description and Formulation

Consider the following problem: a remanufacturer outsources to a collector the collection activity of end-of-life products that can be remanufactured and resold in the remanufactured product market. The collector acquires the used products from end users and pays for them with unit acquisition price f, which is the collector's decision variable. We model the supply of used products as a deterministic, linear function of the acquisition price, so we have $q(f) = \alpha + \beta f$, where $\alpha, \beta > 0$. Similar linear functions are used in the literature for the analysis of similar business decisions [1] [2] [5]. Moreover, unit collection cost c is also spent by the collector in collecting and handling each used products.

We suppose that c is the collector's private information that cannot be observed by the remanufacturer. For the remanufacturer, she only knows that the collection cost of the collector is one of two types, $(\underline{c}, \overline{c})$, and the respective probability is v and $1 - v$. Suppose $\underline{c} < \overline{c}$ and we say the collector is efficient if his collection cost is $\underline{c}$ and the collector is inefficient if his collection cost is $\overline{c}$. And we define $\Delta c = \overline{c} - \underline{c}$.

The used products can be processed by the remanufacturer and sold in the remanufactured product market. The unit remanufacturing cost is c_R. We model the demand of the remanufactured products as a deterministic linear function of selling price p, thus we have $d(p) = a - bp$, where $a, b > 0$ and $p \geq 0$. Such demand function is widely adopted in the marketing and operations research literature, e.g. [11] [12]. If a used product is chosen not to be remanufactured, the remanufacturer can obtain a unit salvage revenue s (s can also be negative when it costs money to dispose these returned products).

The remanufacturer proposes a take-it-or-leave-it contract $\{(\underline{t}, \underline{q}),(\overline{t}, \overline{q})\}$, expecting the efficient collector to choose $(\underline{t}, \underline{q})$ and the inefficient collector to choose $(\overline{t}, \overline{q})$. That is, the collector chooses to collect $\underline{q}$ units of used products to obtain a payoff $\underline{t}$ from the remanufacturer, or to collect $\overline{q}$ units of used products for a payoff $\overline{t}$, which announces his collection cost type according to the Revelation Principle [13]. The collector specifies the acquisition price to acquire the used products based on the chosen contract. After receiving the used products, the remanufacturer processes a portion of them and decides the price p in the remanufactured product market. The used products that are not remanufactured are salvaged with unit value s. The reverse supply chain structure and related material and financial flows are depicted in Figure 1.

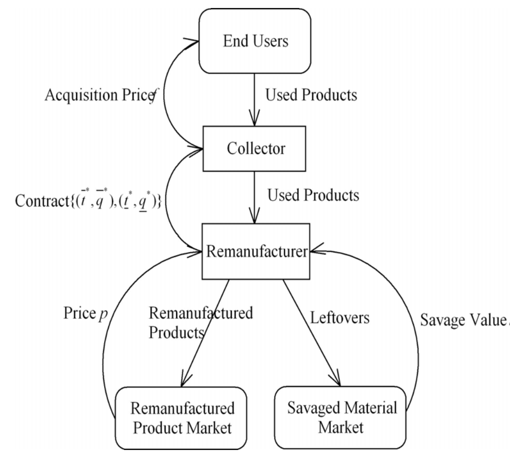

Fig. 1. The reverse supply chain structure and the material and financial flows

Note that $(f + c)q(f)$ is the total cost of the collector for collecting used products with the quantity of $q(f)$. So the total cost for a collector of type $c(c \in \underline{c}, \overline{c})$ to obtain q units of used products is described as follows,

$$C(q, c) = \frac{1}{\beta}q^2 - (\frac{\alpha}{\beta} - c)q \tag{1}$$

Obviously $C(q, c)$ is an increasing function with respect to c.

When the remanufacturer receives the used products with quantity q, he specifies the selling price p to maximize her profit with the following decision,

$$\begin{array}{ll} \pi(q) & = \max_p \{(p - c_R) \min[q, a - bp] + s[q - (a - bp)]^+\} \\ s.t. & p \geq 0 \end{array} \tag{2}$$

Note that the parameters a, b, α, β are determined by the condition of end-of-life product market and remanuafactured product market, which can be observed by the remanufacturer. We have the following assumptions throughout the paper:

(i) $a > b(c_R + \overline{c})$. We assume the remanufactured product market is sufficiently large to ensure the price $p \geq 0$ always holds.

(ii) $a > b|(c_R + s)|$, i.e., $-\frac{a}{b} - c_R < s < \frac{a}{b} - c_R$. We assume the salvage value (or dispose cost) of unprocessed end-of-life product is not extremely large to avoid some trivial and bizarre cases, which is not likely to happen in reality.

(iii) $\alpha \le \underline{c}\beta$. This assumption ensures that $\frac{\partial C}{\partial q} \ge 0$ for any $q \in [0, +\infty]$, which is reasonable for the market-driven collection channel considered in this paper.

The remanufacturer's problem is to maximize his expected profit subject to the participation and the incentive constraints of the collector, as follows,

$$\begin{array}{lll} \max\limits_{\{(\underline{t},\underline{q}),(\overline{t},\overline{q})\}} & & v(\pi(\underline{q})-\underline{t})+(1-v)(\pi(\overline{q})-\overline{t}) \\ s.t. & (a) & \overline{t}-C(\overline{q},\overline{c}) \ge 0 \\ & (b) & \underline{t}-C(\underline{q},\underline{c}) \ge 0 \\ & (c) & \overline{t}-C(\overline{q},\overline{c}) \ge \underline{t}-C(\underline{q},\overline{c}) \\ & (d) & \underline{t}-C(\underline{q},\underline{c}) \ge \overline{t}-C(\overline{q},\underline{c}) \end{array} \quad (3)$$

The collector must receive at least its reservation utility, which is in our model supposed to be zero, in order to accept the contract offered by the principal. This is shown by the participation constraints (a) and (b). The incentive compatibility constraints (c) and (d) insure that under this contract the optimal choice of the collector of type $\underline{c}$ ($\overline{c}$) is to collect quantity $\underline{q}$ ($\overline{q}$) accordingly.

III. Anylasis and the Optimal Contract

A. The Optimal Contract

First we solve the remanufacturer's pricing problem under given quantity q of the used products. After a mathematical transformation we have:

$$\begin{array}{ll} \pi(q) & = \max_p (a-bp)(p-c_R-s)+sq \\ s.t. & a-bp \le q \end{array} \quad (4)$$

We derive the following result.

Lemma 1 (1) $\pi(q)$ is a differentiable and convex function with respect to q.

(2) When $q > \frac{a-b(c_R+s)}{2}$, we have the optimal price $p^* = \frac{a+b(c_R+s)}{2b}$, and $\pi(q) = \frac{[a-b(c_R+s)]^2}{4b} + sq$. When $0 \le q \le \frac{a-b(c_R+s)}{2}$, we have the optimal price $p^* = \frac{a-q}{b}$, and $\pi(q) = \frac{(a-bc_R-q)q}{b}$.

Proof. All the proofs are omitted due to the paper length limitation.

We notice that when q is smaller the remanfuacturer's profit is increasing at a larger rate and after the point $q = \frac{a-b(c_R+s)}{2}$ the marginal profit of the remanufacturer is a constant s.

Now we direct our attention to derive the optimal contract for the remanufacturer, maximizing his expected profit while inducing the collector to confess his true type. This problem is solved by Proposition 1-3 and we use the following notations:

$$\underline{q}_1 = \frac{\frac{\alpha}{\beta}+\frac{a}{b}-\underline{c}-c_R}{2(\frac{1}{\beta}+\frac{1}{b})} \quad (5)$$

$$\overline{q}_1 = \frac{\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R-\frac{v}{1-v}\Delta c}{2(\frac{1}{\beta}+\frac{1}{b})} \quad (6)$$

$$\underline{q}_2 = \frac{\alpha+(s-\underline{c})\beta}{2} \quad (7)$$

$$\overline{q}_2 = \frac{\alpha+(s-\overline{c}-\frac{v}{1-v}\Delta c)\beta}{2} \quad (8)$$

Proposition 1 When $s < \frac{[a-bc_R]-[\alpha-\beta\underline{c}]}{\beta+b}$:

1) If $v < \frac{\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R}{\Delta c+(\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R)}$, then the optimal contract for the remanufacturer is $\{(\underline{t}^*,\underline{q}^*),(\overline{t}^*,\overline{q}^*)\}$, in which

$$\begin{cases} \underline{q}^* & = \underline{q}_1 \\ \overline{q}^* & = \overline{q}_1 \end{cases} \quad (9)$$

and

$$\begin{cases} \underline{t}^* & = \frac{1}{\beta}\underline{q}_1^2-(\frac{\alpha}{\beta}-\underline{c})\underline{q}_1+\Delta c\overline{q}_1 \\ \overline{t}^* & = \frac{1}{\beta}\overline{q}_1^2-(\frac{\alpha}{\beta}-\overline{c})\overline{q}_1 \end{cases} \quad (10)$$

2) If $v \ge \frac{\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R}{\Delta c+(\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R)}$, then the optimal contract for the remanufacturer is $\{(\underline{t}^*,\underline{q}^*),(\overline{t}^*,\overline{q}^*)\}$, in which

$$\begin{cases} \underline{q}^* & = \underline{q}_1 \\ \overline{q}^* & = 0 \end{cases} \quad (11)$$

and

$$\begin{cases} \underline{t}^* & = \frac{1}{\beta}\underline{q}^{*2}-(\frac{\alpha}{\beta}-\underline{c})\underline{q}^* \\ \overline{t}^* & = 0 \end{cases} \quad (12)$$

Proposition 2 When $\frac{[a-bc_R]-[\alpha-\beta\underline{c}]}{\beta+b} \le s \le \frac{[a-bc_R]-[\alpha-\beta(\overline{c}+\frac{v}{1-v}\Delta c)]}{\beta+b}$:

1) If $v < \frac{\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R}{\Delta c+(\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R)}$, then the optimal contract for the remanufacturer is $\{(\underline{t}^*,\underline{q}^*),(\overline{t}^*,\overline{q}^*)\}$, in which

$$\begin{cases} \underline{q}^* & = \underline{q}_2 \\ \overline{q}^* & = \overline{q}_1 \end{cases} \quad (13)$$

and

$$\begin{cases} \underline{t}^* & = \frac{1}{\beta}\underline{q}_2^2-(\frac{\alpha}{\beta}-\underline{c})\underline{q}_2+\Delta c\overline{q}_1 \\ \overline{t}^* & = \frac{1}{\beta}\overline{q}_1^2-(\frac{\alpha}{\beta}-\overline{c})\overline{q}_1 \end{cases} \quad (14)$$

2) If $v \ge \frac{\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R}{\Delta c+(\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R)}$, then the optimal contract for the remanufacturer is $\{(\underline{t}^*,\underline{q}^*),(\overline{t}^*,\overline{q}^*)\}$, in which

$$\begin{cases} \underline{q}^* & = \underline{q}_2 \\ \overline{q}^* & = 0 \end{cases} \quad (15)$$

and

$$\begin{cases} \underline{t}^* & = \frac{1}{\beta}\underline{q}_2^2-(\frac{\alpha}{\beta}-\underline{c})\underline{q}_2 \\ \overline{t}^* & = 0 \end{cases} \quad (16)$$

Proposition 3 When $s > \frac{[a-bc_R]-[\alpha-\beta(\overline{c}+\frac{v}{1-v}\Delta c)]}{\beta+b}$, the optimal contract for the remanufacturer is $\{(\underline{t}^*,\underline{q}^*),(\overline{t}^*,\overline{q}^*)\}$, in which

$$\begin{cases} \underline{q}^* & = \underline{q}_2 \\ \overline{q}^* & = \overline{q}_2 \end{cases} \quad (17)$$

and

$$\begin{cases} \underline{t}^* & = \frac{1}{\beta}\underline{q}_2^2-(\frac{\alpha}{\beta}-\underline{c})\underline{q}_2^+\Delta c\overline{q}_2 \\ \overline{t}^* & = \frac{1}{\beta}\overline{q}_2^2-(\frac{\alpha}{\beta}-\overline{c})\overline{q}_2 \end{cases} \quad (18)$$

The results are interesting since the optimal contract is characterized by the value of two key parameters: s and v. In the following we denote:

$$s_1 = \frac{[a-bc_R]-[\alpha-\beta\underline{c}]}{\beta+b} \quad (19)$$

$$s_2 = \frac{[a - bc_R] - [\alpha - \beta(\bar{c} + \frac{v}{1-v}\Delta c)]}{\beta + b} \tag{20}$$

$$v_0 = \frac{\frac{\alpha}{\beta} + \frac{a}{b} - \bar{c} - c_R}{\Delta c + (\frac{\alpha}{\beta} + \frac{a}{b} - \bar{c} - c_R)} \tag{21}$$

Note $s_2 > s_1 > 0$ and $1 > v_0 > 0$. s_1 and v are independent constants completely determined by the market condition parameters a, b, α, β and cost parameters $\underline{c}, \bar{c}, c_R$. However, s_2 relies on the value of v, although $s_2 > s_1$ is always true.

We specify five cases:

1) Case SSSE(Small Salvage value, Small probability of Efficient collector) is defined as the case when $s < s_1$ and $v < v_0$, which corresponds with case 1 in Proposition 1.

2) Case SSLE(Small Salvage value, Large probability of Efficient collector) is defined as the case when $s < s_1$ and $v \geq v_0$, which corresponds with case 2 in Proposition 1.

3) Case MSSE(Middle Salvage value, Small Probability of Efficient collector) is defined as the case when $s_1 \leq s \leq s_2$ and $v < v_0$, which corresponds with case 1 in Proposition 2.

4) Case MSLE(Middle Salvage value, Large probability of Efficient collector) is defined as the case when $s_1 \leq s \leq s_2$ and $v \geq v_0$, which corresponds with case 2 in Proposition 2.

5) Case LS(Large Salvage value) is defined as the case when $s > s_2$, which corresponds with Proposition 3.

The contract is selected by the remanufacturer, according to a combination of the possibility that the collector is efficient and the salvage value of the end-of-life product if it is not remanufactured. Figure 2 shows the segmentation for different cases and contract types.

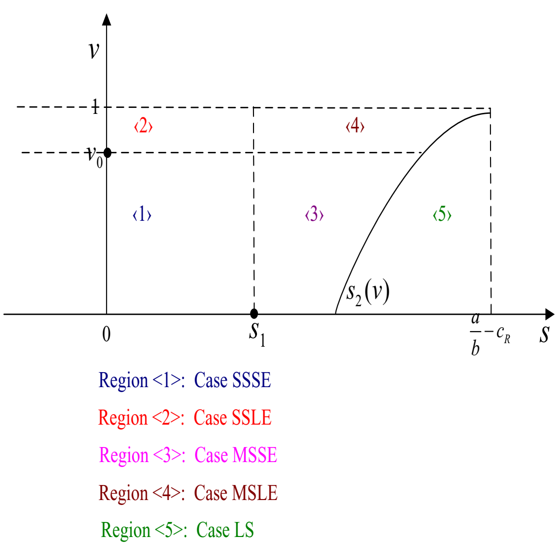

Fig. 2. Region segmentation on contract types

B. Information Rent

The information rent refers to the cost that the principal has to pay for screening the type of the collector due to information asymmetry, which is also the profit (the part of revenue exceeds the reservation utility) of efficient agent due to his higher efficiency(or equivalently, lower cost). Note that the profit of the inefficient agent is always zero. We recommend [14] to the reader for a detailed knowledge of this issue.

According to the optimal contract (Proposition 1-3) and the definition of the information rent, we obtain:

Proposition 4 The information rent is $\Delta c \cdot \bar{q}^*$. More specifically,

(1) When $s \leq s_2$ and $v < v_0$ (Case SSSE and Case MSSE), the information rent is $\Delta c \cdot \bar{q}_1$;

(2) When $s \leq s_2$ and $v \geq v_0$ (Case SSLE and Case MSLE), the information rent is 0;

(3) When $s > s_2$ (Case LS), the information rent is $\Delta c \cdot \bar{q}_2$.

We also have the following corollaries about the sensitivity on the information rent.

Corollary 1 Given other parameters are fixed, the information rent is strictly decreasing and convex function of v when $v \leq \max\{v_0, \frac{m}{m+1}\}$. When $v > \max\{v_0, \frac{m}{m+1}\}$ the information rent is 0. Here $m = \frac{\alpha - \beta\bar{c} - (a - bc_R) + (b+\beta)s}{\beta \Delta c}$.

The information rent is depicted in Fig. 3-5 for differen range of the salvage value s (We define $\tilde{s} = \frac{\frac{a}{b}\beta - \beta c_R + a - bc_R}{\beta + b}$):

Corollary 2 Given other parameters are fixed, the information rent is a piecewise linear function of s. When $0 < v < v_0$, the information rent is a positive constant in $(-\infty, s_2(v))$, and afterwards increasing linearly at rate $\beta\Delta c$. When $v_0 \leq v < 1$, the information rent is 0 in $(-\infty, s_2(v))$, and afterwards increasing linearly at rate $\beta\Delta c$.

The information rent is depicted in Fig. 6-7 for differen range of salvage value s.

We summarize the above results and obtain several important managerial insights:

1) For the returned product with a large salvage value, or if it is not so sure that the collector is efficient, the remanufacturer will propose a contract that the more efficient collector will obtain a positive information rent.

2) For the returned product with little or negative salvage value and if it is almost sure that the collector is efficient, e.g., there are all over efficient collectors everywhere, a "close inefficient agent" contract will be proposed. The inefficient collector will not obtain an offer and the information rent for an efficient collector is zero, i.e., the collector just obtains his reservation value no matter whether she is efficient or not.

3) The efficient collector prefers to collect a used product with high salvage value. She also has an incentive to make the remanufacturer who hires her to believe the number of efficient collectors is scarce. In the above two cases, the efficient collector can obtain more information rent.

4) It is easy to draw some other conclusions from the perspective to the efficient collector's best interest. For example, the efficient collector prefers a larger market size and less price sensitivity for the remanufactured products. Therefore the collector also benefits from the advertising of the remanufacturer or the government who encourages consumers to buy remanfacturable products for the sake of environmental protection.

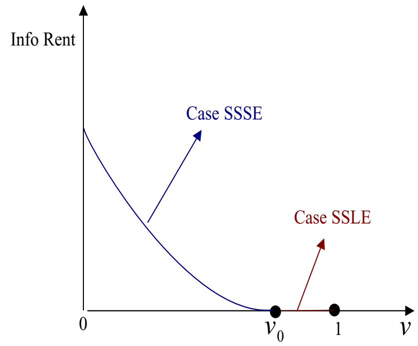

Fig. 3. Information rent and efficient collector probability v when $s < s_1$

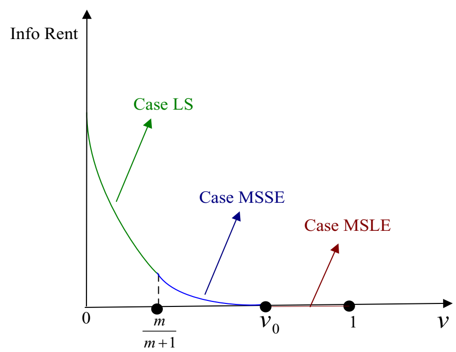

Fig. 4. Information rent and efficient collector probability v when $s_1 \leq s \leq \min\{s_1, \widetilde{s}\}$

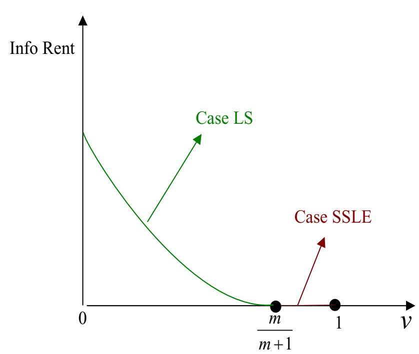

Fig. 5. Information rent and efficient collector probability v when $s > \min\{s_1, \widetilde{s}\}$

C. Value of information

In this section we focus on the value of the collector's cost information for the remanufacturer. Our main objective is to answer the question:

How much would the remanufacturer like to pay for the collector's information of the collection cost?

We first study the benchmark problem when the collector's cost is known as c by the remanufacturer without uncertainty. In this case the remanufacturer will squeeze all the channel

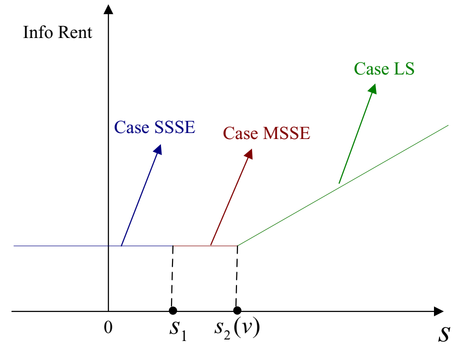

Fig. 6. Information rent and efficient collector probability v when $v < v_0$

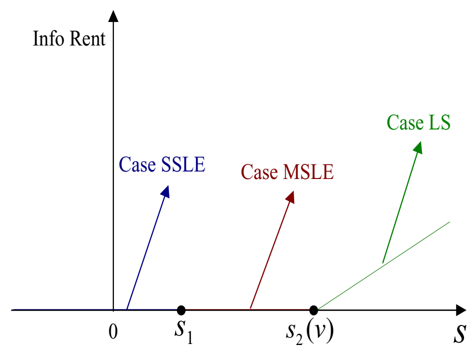

Fig. 7. Information rent and efficient collector probability v when $v \geq v_0$

profit as a principal who has full bargaining power:

$$\begin{aligned} \pi_I(c) \quad &= \max_{f,p} \quad \{(p - c_R)\min[a - bp, \alpha + \beta f] \\ &\quad -(c+f)(\alpha + \beta f)\} \end{aligned} \tag{22}$$

Then we derive the following conclusion.

Lemma 2 (1) When $s < s_1$, we have

$$\begin{cases} \pi_I(c)^* &= \frac{(\frac{\alpha}{\beta}+\frac{a}{b}-c-c_R)^2}{4(\frac{1}{\beta}+\frac{1}{b})} \\ f^* &= \frac{q^*-\alpha}{\beta} \\ p^* &= \frac{a-q^*}{b} \end{cases} \tag{23}$$

where $q^* = \frac{\frac{\alpha}{\beta}+\frac{a}{b}-c-c_R}{2(\frac{1}{\beta}+\frac{1}{b})}$.

(2) When $s \geq s_1$, we have

$$\begin{cases} \pi_I(c)^* &= \frac{[\alpha+\beta(s-c)]^2}{4\beta} + \frac{[a+b(c_R+s)]^2}{4b} \\ f^* &= \frac{q^*-\alpha}{\beta} \\ p^* &= \frac{a+b(c_R+s)}{2b} \end{cases} \tag{24}$$

where $q^* = \frac{\alpha+\beta(s-c)}{2}$.

Lemma 2 shows the solution of the maximized profit when the remanufacturer realizes exactly the collection cost c of the collector. So the information value of the collection cost can be formulated as:

$$V_I = v\pi_I(\underline{c}) + (1-v)\pi_I(\overline{c}) - \pi_N \tag{25}$$

where π_N is defined as the optimization objective of (3). The first part of (25) is the expected profit when the remanufacturer has the knowledge of the collection cost information c, while π_N is the expected profit in asymmetric information case. The difference between them is the value of information for the remanufacturer, which represents the maximum amount of money the remanufacturer would like to pay to obtain the information.

Now we denote $\widetilde{s}_1 = \frac{[a-bc_R]-[\alpha-\beta\overline{c}]}{\beta+b}$. Note $\widetilde{s}_1 > s_1$ is a constant, further we get the following conclusion.

Proposition 5 The value of information of collector's cost is:

(1) When $s \leq \widetilde{s}_1$, and $v < v_0$ (Case SSSE, and part of Case MSSE),

$$V_I = \frac{v\Delta c[2(\frac{a}{b} + \frac{\alpha}{\beta} - c_R - \overline{c}) - \frac{v}{1-v}\Delta c]}{4(\frac{1}{\beta} + \frac{1}{b})} \tag{26}$$

When $s \leq \widetilde{s}_1$, and $v \geq v_0$ (Case SSLE, and part of Case MSLE),

$$V_I = \frac{(1-v)(\frac{\alpha}{\beta} + \frac{a}{b} - \overline{c} - c_R)^2}{4(\frac{1}{\beta} + \frac{1}{b})} \tag{27}$$

(2) When $\widetilde{s}_1 \leq s \leq s_2$, and $v < v_0$ (part of Case MSSE)

$$\begin{aligned} V_I \quad = \quad & \tfrac{1-v}{4}\Big[\tfrac{[\alpha+\beta(s-\overline{c})]^2}{\beta} + \tfrac{[a+b(c_R+s)]^2}{b} \\ & - \tfrac{[\frac{\alpha}{\beta}+\frac{a}{b}-\overline{c}-c_R-\frac{v}{1-v}\Delta c]^2}{(\frac{1}{\beta}+\frac{1}{b})}\Big] \end{aligned} \tag{28}$$

When $\widetilde{s}_1 \leq s \leq s_2$, and $v \geq v_0$ (part of Case MSLE)

$$V_I \quad = \quad \tfrac{1-v}{4}\Big[\tfrac{[\alpha+\beta(s-\overline{c})]^2}{\beta} + \tfrac{[a+b(c_R+s)]^2}{b}\Big] \tag{29}$$

(3) When $\frac{[a-bc_R]-[\alpha-\beta\underline{c}]}{\beta+b} \leq s \leq s_2$ (Case LS)

$$V_I = \frac{v\Delta c\{2[\alpha + (s-\overline{c})\beta] - \frac{v}{1-v}\beta\Delta c\}}{4} \tag{30}$$

Corollary 3 (1) V_I is a concave and unimodal (first increasing then decreasing) function with respect to v. (2) V_I is an increasing function with respect to s.

Proposition 5 gives us the solution of information value in different cases, which is of great importance to the remanufacturer's information sharing and supply chain integration decision. For example, if the expense to obtain the collector's cost information (e.g., by investigating on or collaborating with the collector) is less than the information value, it is beneficial for the remanufacturer to invest for the information sharing. Corollary 3 further explores the sensitivity of two crucial values: v and s. It is showed that the value of cost information decreases when the probability that the collector is efficient is too small or too large. It also implies that the remanufacturer has more incentive to invest to obtain the cost information for the used product with a higher salvage value.

IV. Conclusion

This paper studies a reverse supply chain model consisting of a remanufacturer and a collector, in which the remanufacturer has full bargaining power and delegates the collector to collect end-of-life products in a market-driven product return channel. A notable feature of our problem is that the remanufacturer only knows the probability distribution of the collection cost of the collector. The collector acquires the used product by providing cash to the end-users and the remanufacturer recoveries these used products and resells them in the remanufactured product market.

We utilize the principal-agent theory to analyze the contracting problem for the remanufacturer under the cost information asymmetric case, and characterize a catalogue of the optimal contracts. Which contract should be adopted depends on values of the key parameters: the probability that the collector is efficient and the salvage value of the end-of-life product unremanufactured. In each case we give the solution of information rent, value of information and the sensitivity analysis on these two key parameters, which provide us with useful managerial insights. In the future study we will extend this model to some more complicated settings, e.g. a continuous distributed collection cost, etc.

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