Super Resolution Based on Gradient Field

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***Abstract*—In this work, we present a novel method for high resolution image generation from a single low resolution image. The proposed algorithm begins by interpolating the gradient field of low resolution image to obtain one finer gradient field based on local binary pattern feature. Then it recurs to the finer gradient field term and constraint set to construct one reconstruction energy function. By minimizing such energy equation using gradient descent method, a high resolution image can be obtained. We have applied the proposed method to both synthetic data and real image data and comparison results with bicubic interpolation method show that our method is feasible and promising in single image super resolution area.**

I. Introduction

Super resolution technology aims to reconstruct a sharp high resolution (HR) from one or more low resolution (LR) images. super resolution is an ill-posed inverse problem because of an insufficient number of LR images and ill-condistioned blur operators, Some surveys [1]–[3] on this work discuss the strengths and weakness of current popular method. Just the shortcomings in current methods drive many research workers to improve super-resolution quality. Therefore, the exploration in this field is still an active area of research.

Since Tsai and Huang [4] first introduced the concept of super resolution from multiple frames, a wide range of very different approaches have been proposed, on the whole, which can be categorized into three groups as follows:

(i)Interpolation technology based on multiple images. This method first estimates the motion information between different frames to obtain HR image pixels at nonuniform sampling positions, then guesses the pixels values at pole positions, and at last HR image is obtained via deblur and denoise technology. The classical method includes generalized interpolation method [5], wavelet interpolation method [6], [7], triangular mesh method [8]. Candocia [5] proposed a generalized interpolation method, which begins by learning a convolution kernel matrix,and estimates the HR image by convoluting kernel matrix and LR image. The main shortcoming is very difficult to sharp HR image because it couldn't bring additional high frequency information.

(ii)Approaches based on reconstruction. These approaches recur to image priors to obtain sharp HR image under regularization constraints. The representative methods include convex-based methos [9], method based on Bayesian estimation [10], [11], edge preservation approach [12]. Freeman [10] introduced MAP(Maximum A Posterior) to represent global structure information and proposed a speedup one-pass algorithm.

(iii)Learning-based methods. These methods first learn a finer details (high frequency signal) at HR images that corresponds to different image regions at LR images in a large train set, then find the best match and the corresponding scene one in the training set for every LR patch, and Combine high frequency details and LR image to obtain sharp HR image. Learning-based method has a big advantage to capture more details with high frequency and maybe obtains better results in super resolution application. Reference [13]–[15] are included in this group.

Our main contribution in this paper is to propose one reconstruction energy framework based on gradient field.we first interpolate the gradient field of low resolution image to obtain one finer gradient field based on local binary pattern feature, then recur to the finer gradient field term and constraint set to construct one reconstruction energy function, and last obtain a sharp HR image by minimizing such energy using iteration method. Besides, we also first introduce super resolution into vector field and propose an interpolation method for gradient field based on local binary pattern feature.

Because super resolution can overcome the inherent resolution limitation of the imaging system and improve the performance of most digital image processing application, it has a broad application such as face recognition, medical image, remote sensing image [15], motion estimations, supervisory system, and so on.

The paper is organized as follows: The motivation is shown in Section 2. Section 3 introduces local binary pattern feature extraction operator and our proposed gradient field interpolation method. In Section 4, the whole algorithm framework is described in detail. Section 5 presents some experimental results. Finally, Section 6 ends with a brief discussion.

II. Motivation

An image gradient field is a vector field of the corresponding image, which can be noted by $v = (x, y, v_x(x, y), v_y(x, y))$, where $v_x(x, y)$,$v_y(x, y)$ are the first partial derivatives of the image along horizontal and vertical direction respectively. It provides two pieces of image information, one of which is the magnitude of gradient that tells

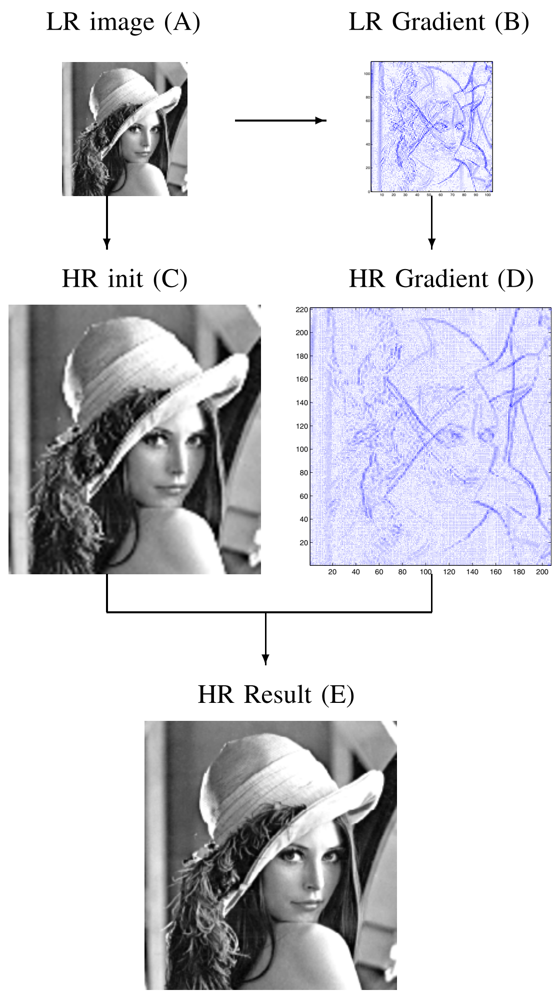

Fig. 1. The overview of our algorithm. Given a LR image (A), first calculate the gradient field of (A) and interpolate the gradient (B) to obtain HR gradient field (D) based on LBP feature. At last, Solve an energy function iteratively with initialized HR image (C) from LR image by nearest interpolation method, and the HR result (E) can be obtained with ease.

us how quickly the image is changing and the other of which is the direction of gradient tells us the direction in which the image is changing most rapidly. An edge in the original image would correspond to a longer vector in the gradient field. A gradient field describes the smoothness and variability of an image. In order to obtain a promising HR gradient field, we use LBP feature to mine the LR gradient field information.

Fig.(1) shows our proposed algorithm overview: given a LR image, first calculate the gradient field of (A) and interpolate the gradient field (B) to obtain HR gradient field (D) based on local binary pattern feature. At last, Solve an energy function iteratively with initialized HR image (C) from LR image by nearest interpolation method, and the HR result (E) can be obtained with ease. The two steps from (B) to (D) and from (C,D) to (E) are very significant and are our main task in this paper.

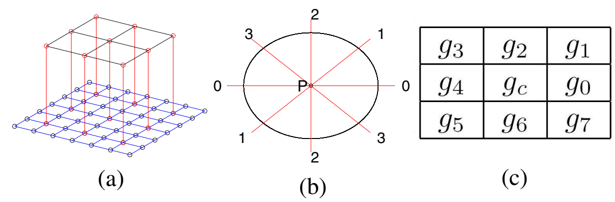

Fig. 2. Panel (a) shows simple interpolation from LR image (the upper layer with 9 pixels) to HR image (the lower layer with 49 pixels),we copy 9 red pixels in LR image to 9 red pixels in HR image directly. Late we will use this 9 red pixels as constraint set C in Eq.(1) Panel (b) describes 4 directions of pixel point P, which are noted by $0, 1, 2, 3$ respectively. Panel (c) represents 8 neighbors $g_0 - g_7$ in 3×3 patch of pixel g_c.

III. Gradient Field Interpolation

A. Local binary pattern operator

T.Ojila [16] proposed the local binary pattern (LBP). which is a texture analysis operator as a gray-scale invariant texture measure and is derived from a general definition of texture in a local neighborhood. The LBP operator can be seen as a unifying approach to the traditionally divergent statistical and structural models of texture analysis. The basic idea of LBP is to construct a binary code that describes the local texture pattern, which is built by thresholding a neighbourhood by the gray value of its center.

Because the LBP operator shows excellent results in terms of accuracy and computational complexity in many empirical studies, it has been made into a really powerful measure of image texture. Many of the applications in the literature involve industrial inspection, classification of 3D textured surfaces, facial expression recognition [17], content-based retrieval, and detecting moving objects, and recognition of dynamic textures [18].

B. Interpolate Gradient Field

We introduce super resolution(or interpolation) into vector filed. The intensity image is continuous on most space domain,on the contrary, the magnitude image corresponding to gradient field is discontinuous on most space domain. Our interpolation approach of gradient field based on LBP feature can be described as follows,

1) Obtain LBP value of every pixel by computing coocurring differences in 3×3 patch in magnitude image of gradient field based on LBP feature operator.
2) For every pixel, find the most similar pixel through comparing the corresponding LBP values of its 8-neighbors, the smaller the difference of two pixel LBP values is, the more similar the two pixels are. At last obtain a local relationship matrix L, the element $l_{i,j}$ of which represents the relative position of the most similar pixel of pixel (i, j), e.g. for the following patch, if g_4 is the most similar pixel to pixel g_c, then $l_{i_c,j_c} = 0$ (where (i_c, j_c) is coordinate of pixel g_c and $0, 1, 2, 3$ are shown in Fig.(2) panel (b).
3) Interpolate the HR gradient image via the above relationship matrix. Here we recur to this relationship matrix as a confidence level measure. The higher the confidence

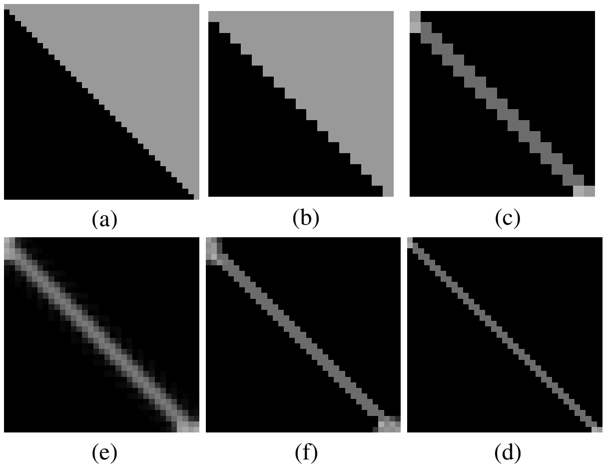

Fig. 3. (a) HR image, (b) LR image, (c) LR gradient magnitude, (d)HR gradient magnitude, (e)Interpolation of gradient magnitude via bilinear, (f)Interpolation of gradient magnitude based on LBP feature

level is, the more accurate the pixel value is, which is interpolated. This interpolation detail will be shown in the following three steps:

- Simply copy some pixels in LR image to HR image.Aa shown in Fig.(2) panel (a), we copy pixels in LR image to the corresponding positions in HR image directly.
- Process pixels with high conference level of local neighbors. For unknown pixel g_c, we search the following 3×3 patch shown in Fig.(2) panel(c) and compare the LBP values of its 8-neighbors $g_0 - g_7$. There are four cases: (1) if g_0 and g_4 are both known, and their corresponding elements values in relationship matrix are both 0, i.e. they are both in horizontal direction, then g_c can be recovered using the average of g_0 and g_4. (2) if g_1 and g_5 are both known, and they have the same direction noted by 1, then g_c can be recovered by the average of g_1 and g_5. (3) if g_2 and g_6 are both known, and they have the same value noted by 2 in relationship matrix, then g_c can be reconstructed by the average of g_2 and g_6. (4) if g_3 and g_7 are both known, and they have the same direction value 3, then g_c can be recovered by the average of g_3 and g_7.
- Fill other pole pixels in HR image via bilinear interpolation method.

Fig.(3) shows the results of interpolation gradient filed using bicubic interpolation method and our proposed gradient interpolation based on LBP feature. Panel (d), panel (e) and panel (f) are bicubic, our method and ground truth respectively. The ground truth of gradient field has a thin oblique line. By comparing the results among three panel, the performance of our method is excellent in terms of high accuracy.

IV. Reconstruction Energy Function

We recur to gradient field and constraint set to construct our reconstruction energy function. Firstly we should construct one constraint set C consisting of pixels in LR image, which are copied to HR image directly. The constraint set is used to maintain high resolution detail in HR image. For Fig.(2) panel (a), nine red pixels in upper layer (LR image) is copied to the lower layer (HR image),then the size of the corresponding constraint set is nine. Secondly we interpolate gradient field of LR image via our method in the above section to obtain HR gradient field V. At last we can define our reconstruction energy function via HR gradient field and constraint set.The energy function value represents the error between reconstructed HR image and ground truth approximately.

A. Energy Function Definition

Our energy function could be defined as:

$$\begin{aligned} E(u) &= \frac{\mu}{2}\int_{\Omega} ||\nabla u(x) - V(x)||^2 dx \\ &+ \frac{\lambda}{2}\sum_{i\in C}\int_{\Omega}\phi(p_i)|u(x) - f(x)|^2 dx \end{aligned} \tag{1}$$

$$where \quad \phi(p_i) = \delta(||x - p_i||) = \begin{cases} 1, & x = p_i \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where f is HR image by simple copy from LR image,intensities of other pixels except simple corresponding pixels are zero, and u is HR image. We use i as the index of constraints set C, p_i represents the position of pixel i, and u_i and f_i are corresponding pixel values.

The first term describes that the reconstructed HR image is the same smoothness as ground truth if the HR gradient field V is good enough. The second term restricts the corresponding pixel intensities approximately equal.

By calculus of variation, the first derivative of function $E(u)$ in Eq.(1) can be written as

$$\frac{\partial E}{\partial u} = -\mu(\triangle u - div(V)) + \lambda\sum_{i\in C}\phi(p_i)(u - f)$$

where $\triangle$ is Laplacian operator and div is divergence operator. Therefore, the image u that minimize the energy Eq.(1) satisfies the Euler-Lagrange equation $\frac{\partial E}{\partial u} = 0$. We use steepest descent method to minimize the functional and obtain the following gradient flow:

$$\frac{\partial u}{\partial t} = \mu(\triangle u - div(V)) - \lambda\sum_{i\in C}\phi(p_i)(u - f) \tag{3}$$

So we obtain iteration formulation as follows,

$$\begin{cases} u_{t+1} = u_t + \tau * \frac{\partial u}{\partial t} \\ u_0 \;\;=\;\; f_0 \end{cases} \tag{4}$$

where τ is step length of iteration. In order to maintain stable image u evolution, our parameters must satisfy $\tau\mu < \frac{1}{4}$ and $\tau\lambda < 1$, and if this condition is not satisfied, the iteration may be not convergent.

B. Algorithm Framework

Given one LR image, we aim to construct one HR image. The proposed framework for super resolution is mainly composed of four stages as follows,

1) **Calculating the gradient filed.** Obtain the gradient field $v = (x, y, v_x(x, y), v_y(x, y))$ of given LR image f by Sobel operator, which has many distinct advantages, because Sobel operator is less sensitive to isolated high intensity point variations by averaging points over a larger area.
2) **Interpolating LR gradient field.** Do interpolation on gradient field V via bilinear interpolation by our method shown in the former section.
3) **Constructing energy function.** Define an energy function Eq.(1) with constructed HR gradient field and constraint set.
4) **Solving the energy equation.** Obtain the sharp HR image by solving this energy function using Eq.(4).

C. Implement Detail

In practical, we use the following regularized $\phi_\epsilon(p_i)$ instead of $\phi(p_i)$ in Eq.(1). This trick can reduce the sensibility to noise and improve the robustness of our algorithm. In this work, we use regularized ϕ_ϵ with $\epsilon = 1.5$ for all experiments.

$$\begin{aligned}\phi_\epsilon(p_i) &= \delta_\epsilon(||x - p_i||) \\ &= \begin{cases} 0, & ||x - p_i|| > \epsilon \\ \frac{1}{\pi}\frac{\epsilon}{\epsilon^2 + ||x - p_i||^2}, & ||x - p_i|| \le \epsilon \end{cases}\end{aligned}$$

In the process of implementation, we use image (C) in Fig.(1) as initial HR image (HR image from LR image via nearest interpolation method). We set parameters $\mu = 0.2$, $\lambda = 0.8$ and step $\tau = 0.5$ for the whole experiments and stop iteration process when the energy function value can't be descend.

V. Experimental Results

In this section we present several experimental results on real LR images. Based on these results, our method is feasible and promising in super resolution.

Fig.4 provides results of applying bicubic interpolation and our proposed method on four LR images. The bicubic interpolation is used for comparison. ALL input LR images in panel (a) are scaled to from 80×80 to 150×150, output of Bicubic interpolation is shown in panel (b), panel (c) presents result of our proposed method, panel (d) is ground truth. The top row is an example for toy data and the others are for real image named as tire, dandelion and house respectively.

In order to evaluate the performance of our approach, we use mean distance, variation of distance between our HR result and ground truth and compare the two marks with bicubic interpolation method. Mean distance is $dmean = mean(||u - I||)$ and variation distance is $dvar = var(||u - I||)$, where u and I are HR result and ground truth respectively. Tab.I shows the comparison result on one toy data (shown in Fig.(4) top row) ten real LR images. ALL input LR images except that images in Fig.(4) panel (a) are not shown due to the limitation of space and can be found in Matlab software subdirectory.

TABLE I
THE COMPARISON OF OUR METHOD WITH BICUBIC METHOD.

Examples	**Bicubic Method**		**Our Method**	
	dmean	dvar	dmean	dvar
toydata	0.0213422	8.39746e-03	0.0157527	7.778519e-03
tire	0.0178807	1.16797e-03	0.0157719	1.08815e-03
dandelion	0.009267	4.63694e-04	0.00644903	2.63627e-04
house	0.00973819	7.58689e-04	0.00566133	3.14702e-04
cameraman	0.0235712	2.94464e-03	0.022717	2.86396e-03
boston	0.0401837	3.96172e-03	0.0362497	3.6516e-03
m83	0.104365	2.78294e-02	0.0935972	2.39435e-02
spine	0.00161276	1.90789e-05	0.00135199	1.71717e-05
trees	0.0181201	9.84716e-04	0.0164802	9.64353e-04
kids	0.0712954	1.85471e-04	0.0635396	1.64373e-04
forest	0.0312854	5.88342e-03	0.0288427	5.72715e-03

VI. Conclusion

In this paper, we propose a method for super resolution from a single image. we first interpolate the gradient field of low resolution image to obtain one finer gradient field based on LBP feature, then recur to the finer gradient field and constraint set to construct one reconstruction energy function, and last obtain a sharp HR image by minimizing such energy equation using gradient descent method.Besides, we also first introduce super resolution into vector field and propose an interpolation method for gradient field based on local binary pattern feature. The above experimental results show that our method is feasible and promising in single image super resolution.

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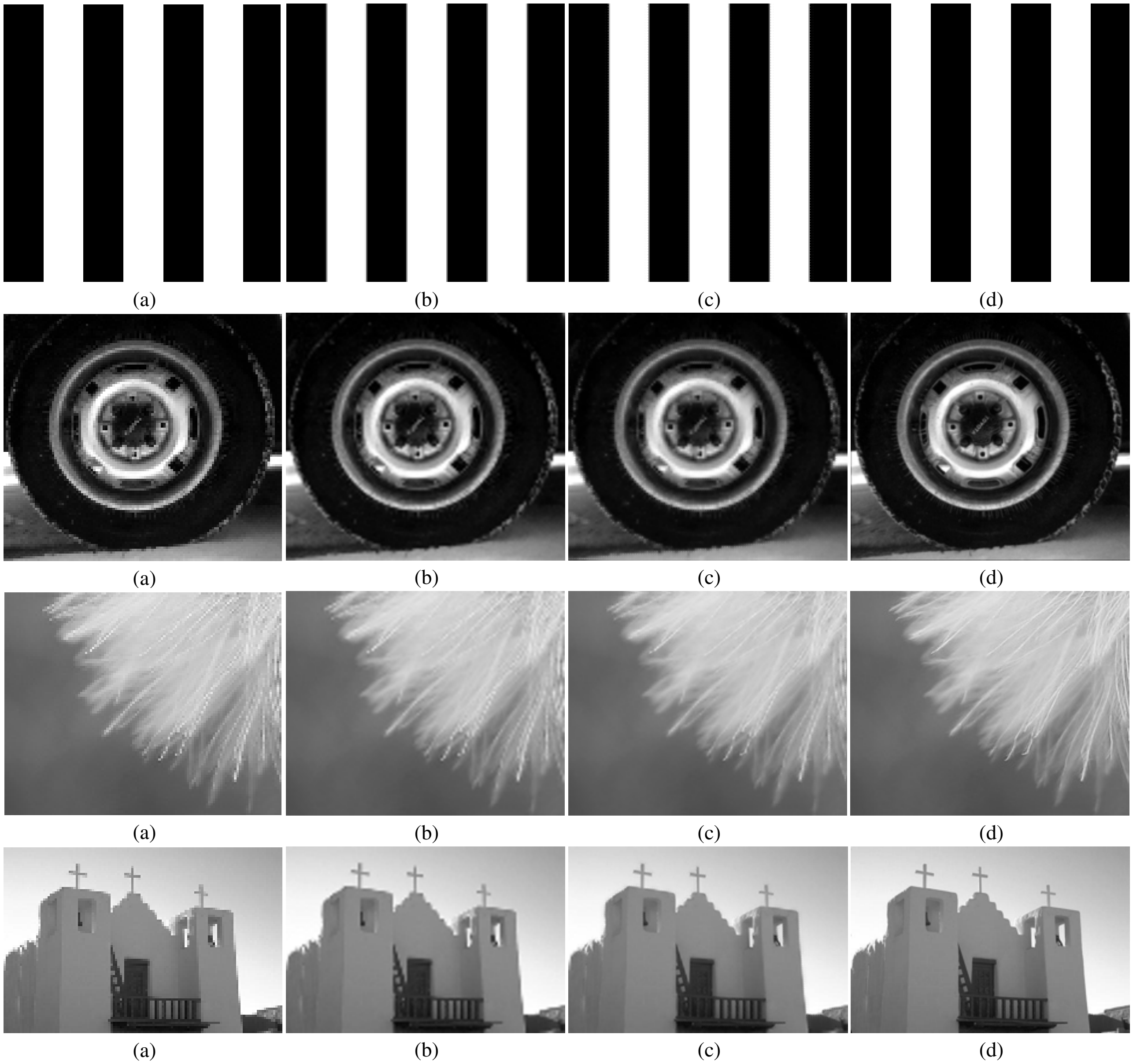

Fig. 4. Comparative results: (a) LR image, (b)Output of Bicubic interpolation, (c)Output of our proposed method, (d) ground truth. The top row is an example for toy data and the others are for real image named as tire, dandelion and house respectively. The evaluation of this three experiments are shown in Tab.I.

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