Attitude Estimation for Small Helicopter Using Extended Kalman Filter

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Abstract—**Accurate attitude estimation is vital for VTOL UAV (vertical take-off and landing unmanned aerial vehicle) systems, including small helicopter. The main focus of present research is to compile the principles of navigation and Kalman filtering, and their application and implementation towards UAV systems. This paper introduces a method based on EKF (Extended Kalman filter) to estimate small helicopter's attitude with little drift and low noise, which uses strapdown attitude as state update, and treats the attitude estimated by the bi-vector method as measurement update. By evaluating the EKF method with small helicopter's real ground and flight sensor data, it shows good performance: stable, small error. So it is suitable for small helicopter attitude estimation application.**

Keywords—**Attitude Estimation, Extended Kalman filter, Mini Helicopter**

I. INTRODUCTION

During the last decade, research activities for VTOL (vertical take-off and landing) aircraft has grown considerably, especially on the design of navigation and autonomous control systems. Compared to Fixed-wing aircraft, small helicopter, able to perform vertical takeoff and landing, has been recognized as more attractive tool for various military and industrial operations, especially in urban and forest environments. Small helicopters that are capable of autonomous flight have been considered in recent years. Significant research interest has been directed to the development of autonomous scale model helicopter [1] [2]. A prototype of autonomous helicopter called "iFLY" (intelligent flying object) has been designed by Robotics Institute of Beihang University as in Fig. 1. The accurate navigation and autonomous flight control have been explored.

Figure 1. Autonomous helicopter prototype - iFLY

Accurate attitude estimation is vital for VTOL UAV systems. However, for the heavy noise and great drifts characteristics of MEMS sensors, which are widely used in VTOL UAV because of their low weight, small size and low power consumption, getting accurate attitude is difficult. A suitable approach is merging of various sources of sensor data in an optimal way to get the best estimate for the current attitude. The main focus of present research is to compile the principles of navigation and Kalman filtering, and their application and implementation towards UAV systems [2] [6] [7].

For small aircrafts, there are several main ways to estimate attitude, they are listed as follows: 1) based on gyro sensors, integrating the angular rates to compute the attitude, called strapdown attitude method; 2) based on accelerators and magnetic filed sensor, using the projection of the gravity vector and geomagnetism vector in body frame to solve the attitude, called bi-vector attitude method [3]; 3) based on GPS sensor, using GPS carrier phase to determine attitude [4]; 4) based on image sensors, using vision guided methods to estimate attitude [5].

GPS method and vision method mentioned above are relative new, less mature, and need more compute recourse. The problem comes with strapdown method is that the error will accumulate due to MEMS gyros' drift. This error enlarges quickly because of the MEMS sensors' great drift. So it prevents strapdown method to be used in long endurance application. In the opposite way, the bi-vector method without integrating eliminates long term accumulating error, but the noise of the signals because of vibration of the helicopter is too great that estimated attitude derived from this method are unlikely to be directly used in the precise helicopter attitude control.

Many studies show that using Kalman filter to fuse merits of the strapdown and bi-vector method is an efficient way to overcome their disadvantages at the same time. But the filters designed for other application either using whole navigation states witch need huge compute resource [6] or using navigation state errors as Kalman filter states witch will possibly cause divergence in case of growing navigation state error [2][7]. This paper introduces a modified method based on EKF (Extended Kalman filter) to estimate attitude with little drift and low noise especially for small helicopter. By

evaluating the method with small helicopter's real ground and flight sensor data, it shows good performance: relatively smaller error, suitable for small helicopter attitude estimation application.

II. FRAMEWORK OF EKF METHOD

Kalman filtering proposed by R. E. Kalman in 1960 represents one of the key filtering methods for integrated navigation systems. Extended Kalman filter modified from standard Kalman filter is a popular and efficient nonlinear filter [8], and is suitable for small helicopter's attitude estimation; in witch system equation and measurement equation both are usually nonlinear.

As mentioned above, this paper designed a modified EKF attitude estimation method based on strapdown and bi-vector methods. It uses strapdown attitude as state update, and treats the attitude estimated by the bi-vector method as measurement update. This is a relative loose architecture, which fuses three types of attitude sources. It shows clear physical means and makes easily modifying the power of the attitude estimated by bi-vector method. The framework is shown below:
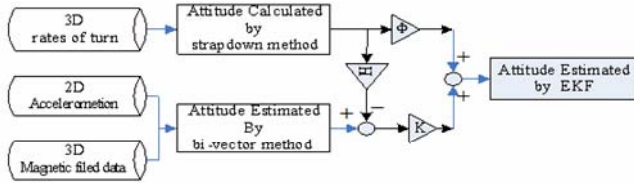

Figure 2. Framework of EKF attitude estimate method

III. ESTABLISH EKF MODEL

A. Attitude Representation Using Euler Angles and Quaternion

For aircrafts, there are two main ways to represent the attitude: Euler angles and quaternion. Euler angles have clear physical means but singularity in some cases, while quaternion has the opposite characteristics. This paper uses quaternion in attitude estimation process and Euler angles for output.
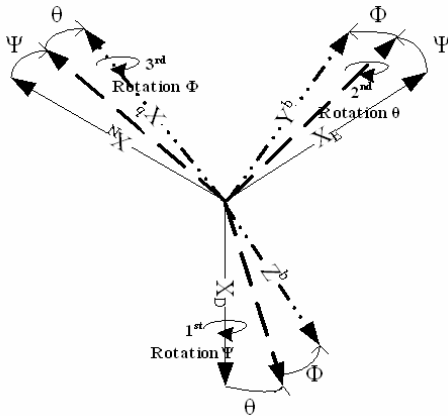

Figure 3. Euler angles between navigation and body frame

Assume Fn { X_N, X_E, X_D } as the navigation frame, Fb { X_b, Y_b, Z_b } as the body-fixed frame, and { φ, θ, ψ } are the roll, pitch, yaw angle respectively as in Fig. 3. The coordinate transformation matrix from Fb to Fn is illustrated as (1).

$$L_b^n = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\varphi s\theta c\psi - c\varphi s\psi & s\varphi s\theta s\psi - c\varphi c\psi & s\varphi c\theta \\ c\varphi s\theta c\psi + s\varphi s\psi & c\varphi s\theta s\psi + s\varphi c\psi & c\varphi c\theta \end{bmatrix} \quad (1)$$

where "s, c" means "sin, cos" for short.

The relationship between Euler angles and quaternion which represents the same attitude can be defined as in (2).

$$\begin{cases} \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} a\tan(2(q_2q_3 + q_0q_1)/[1-2(q_1^2 + q_2^2)]) \\ a\sin(-2(q_3q_1 - q_0q_2)) \\ a\tan(2(q_1q_2 + q_0q_3)/[1-2(q_2^2 + q_3^2)]) \end{bmatrix} \\ \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos\frac{\varphi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\varphi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\ \sin\frac{\varphi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} - \cos\frac{\varphi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\varphi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\varphi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\varphi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} - \sin\frac{\varphi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} \end{bmatrix} \end{cases} \quad (2)$$

B. Establish Discrete System Process Equation using strapdown method

Considered gyros bias drift, the quaternion and three gyro biases are chosen as state variables. So the state vector is (3).

$$X(k) = \begin{bmatrix} q_0(k) & q_1(k) & q_2(k) & q_3(k) & b_0(k) & b_1(k) & b_2(k) \end{bmatrix}^T \quad (3)$$

The attitude of the helicopter Q(k) in t=k*dt can be considered as an attitude with a small successive attitude variance dq(k) after the attitude Q(k-1) in t=(k-1)*dt, that is (4).

$$Q(k) = Q(k-1) \circ dq(k) \quad (4)$$

If the sample period dt is small enough, the Euler angles variance can be considered small enough. Based on (2), the attitude variance dq(k) between t=(k-1)*dt to t=k*dt can be described as (5).

$$dq(k) = \begin{bmatrix} dq_0(k) \\ dq_1(k) \\ dq_2(k) \\ dq_3(k) \end{bmatrix} = \begin{bmatrix} 1 \\ [p(k)/2]dt \\ [w(k)/2]dt \\ [r(k)/2]dt \end{bmatrix} \quad (5)$$

where p(k), w(k), r(k) are the turn rates of roll, pitch and yaw in time t=k*dt separately. So equation (4) can be written as in (6).

$$Q(k) = \begin{bmatrix} 1 & -[p(k)/2]dt & -[w(k)/2]dt & -[r(k)/2]dt \\ [p(k)/2]dt & 1 & [r(k)/2]dt & -[w(k)/2]dt \\ [w(k)/2]dt & -[r(k)/2]dt & 1 & [p(k)/2]dt \\ [r(k)/2]dt & [w(k)/2]dt & -[p(k)/2]dt & 1 \end{bmatrix} Q(k-1) \quad (6)$$

As mentioned above, the gyros have bias drift. In order to eliminate the error caused by this, the gyro bias errors models

have to be considered. Constant bias with white noise combined error model (7) is one simple but efficient model as observed from the raw gyro data measured in real flight shown in Fig. 4.

$$w_b(t) = c + w(t) \qquad (7)$$

where c is a random constant and w(t) is white noise. So the turn rates bias of roll, pitch, yaw in time t=k*dt is (8).

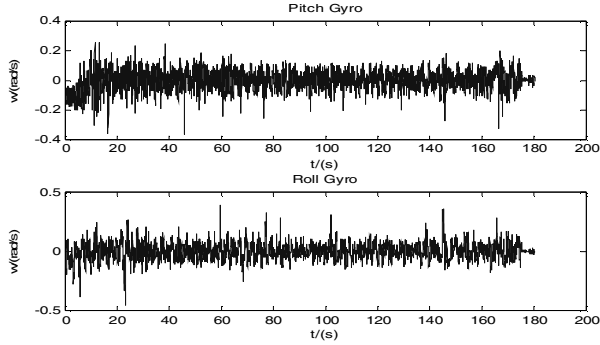$$B(k) = \begin{bmatrix} b_0(k)) \\ b_1(k) \\ b_2(k) \end{bmatrix} = \begin{bmatrix} b_0(k-1) \\ b_1(k-1) \\ b_2(k-1) \end{bmatrix} \qquad (8)$$

Figure 4. Pitch and roll gyros raw data in hovering

Now, the system state process equation is established as in (9), the vector state contained both attitude quaternion and gyro bias.

$$X(k) = \begin{bmatrix} Q(k) \\ B(k) \end{bmatrix} = f[X(k-1), k-1] + W(k)$$

$$= \begin{bmatrix} Q(k-1) \circ dq(k) \\ B(k) \end{bmatrix} + W(k) \qquad (9)$$

where W(k) is white noise sequence.

Obviously the process equation is nonlinear. The filter estimation can be linearized around the current estimate attitude $\hat{X}(k)$ using the partial derivatives of process function based on Taylor series. The new process equation is below.

$$X(k) = \Phi(k, k-1)X(k-1) + W(k) + \phi(k-1) \qquad (10)$$

where

$$\Phi(k, k-1) = \frac{\partial f[X(k-1), k-1]}{\partial X(k-1)} \Big|_{\hat{X}(k-1)}$$

$$\phi(k-1) = f\left[\hat{X}(k-1), k-1\right] - \frac{\partial f}{\partial X(k-1)}\Big|_{\hat{X}(k-1)} \hat{X}(k-1)$$

C. Establish Descrete System Measurement Equation Using Bi-vector Method

The Euler angles are chosen as measurement variables. The measurement vector is (11).

$$Z(k) = [\varphi(k) \quad \theta(k) \quad \psi(k)]^T \qquad (11)$$

The bi-vector mentioned above uses the projection of gravity vector and geomagnetism vector in body frame to calculate the helicopter's attitude. As derived in [3], the roll, pitch and yaw angles can be calculated from acceleration and magnetic field data as below.

$$\begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arcsin(g_x^b) \\ \arcsin(g_y^b / \cos\varphi) \\ \arctan(\dfrac{-H_y^b \cos\varphi + H_z^b \sin\varphi}{H_x^b \cos\theta + H_y^b \sin\theta \sin\varphi + H_z^b \sin\theta \cos\varphi}) \end{bmatrix} \qquad (12)$$

From equation (2), the measurement transfer function can be derived,

$$h[X(k)] = \begin{bmatrix} \arcsin[-2(q_3(k)q_1(k) - q_0(k)q_2(k))] \\ \arctan\left\{\dfrac{2(q_2(k)q_3(k) + q_0(k)q_1(k))}{1 - 2(q_1^2(k) + q_2^2(k))}/\right\} \\ \arctan\left\{\dfrac{2(q_1(k)q_2(k) + q_0(k)q_3(k))}{1 - 2(q_2^2(k) + q_3^2(k))}\right\} \end{bmatrix} \qquad (13)$$

And the measurement equation is

$$Z(k) = \begin{bmatrix} \theta_a(k) \\ \varphi_a(k) \end{bmatrix} = h[X(k)] + V(k) \qquad (14)$$

where V(k) is white noise sequence.

Similar to the process equation, the measurement equation is nonlinear, too. It can be linearized at the current estimate $\hat{X}(k, k-1)$ in the same way.

$$Z(k) = H(k)X(k) + V(k) + y_k \qquad (15)$$

where

$$H(k) = \frac{\partial h[X(k)]}{\partial X(k)}\Big|_{\hat{X}(k,k-1)}$$

$$y(k) = h\left[\hat{X}k, k-1\right] - \frac{\partial h}{\partial X}\Big|_{\hat{X}(k-1)} \hat{X}(k, k-1)$$

D. EKF Time and Measurement Update Equation

The specific equations for the time and measurement updates are presented below.

The time update equations are (16) and (17).

$$\hat{X}(k, k-1) = f\left[\hat{X}(k-1), k-1\right] \qquad (16)$$

$$P(k, k-1) = \Phi(k, k-1)P(k-1)\Phi^T(k, k-1) + Q(k-1) \qquad (17)$$

The measurement update equations are (18), (19) and (20).

$$K(k) = P(k, k-1)H^T(k)\left[H(k)P(k, k-1)H^T(k) + R(k)\right]^{-1} \qquad (18)$$

$$\hat{X}(k) = \hat{X}(k, k-1) + K(k)\left[Z(k) - h[\hat{X}(k, k-1), k]\right] \qquad (19)$$

$$P(k) = [I - K(k)H(k)]P(k, k-1) \qquad (20)$$

IV. EXPERIMENTS AND ATTITUDE ESTAMATION RESULTS

In order to validate the performance of the EKF attitude estimation method, some simulations using real sensor data are revealed. The real sensor data are recorded by the autopilot prototype (fig. 5), which can sample the sensor data in 100 Hz.

Figure 5. Autopilot Prototype

The autopilot is mounted on a three-degree turntable on the ground to record raw data without vibration in 100 Hz. The raw data is then simulated by the EKF filter.

Fig.6 shows the roll, pitch and yaw angles separately which are calculated by strapdown method, bi-vector method and EKF method, comparing with the real angles. It shows clearly the EKF method has best performance.
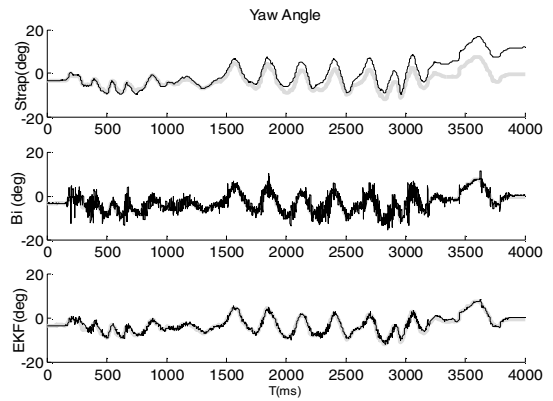

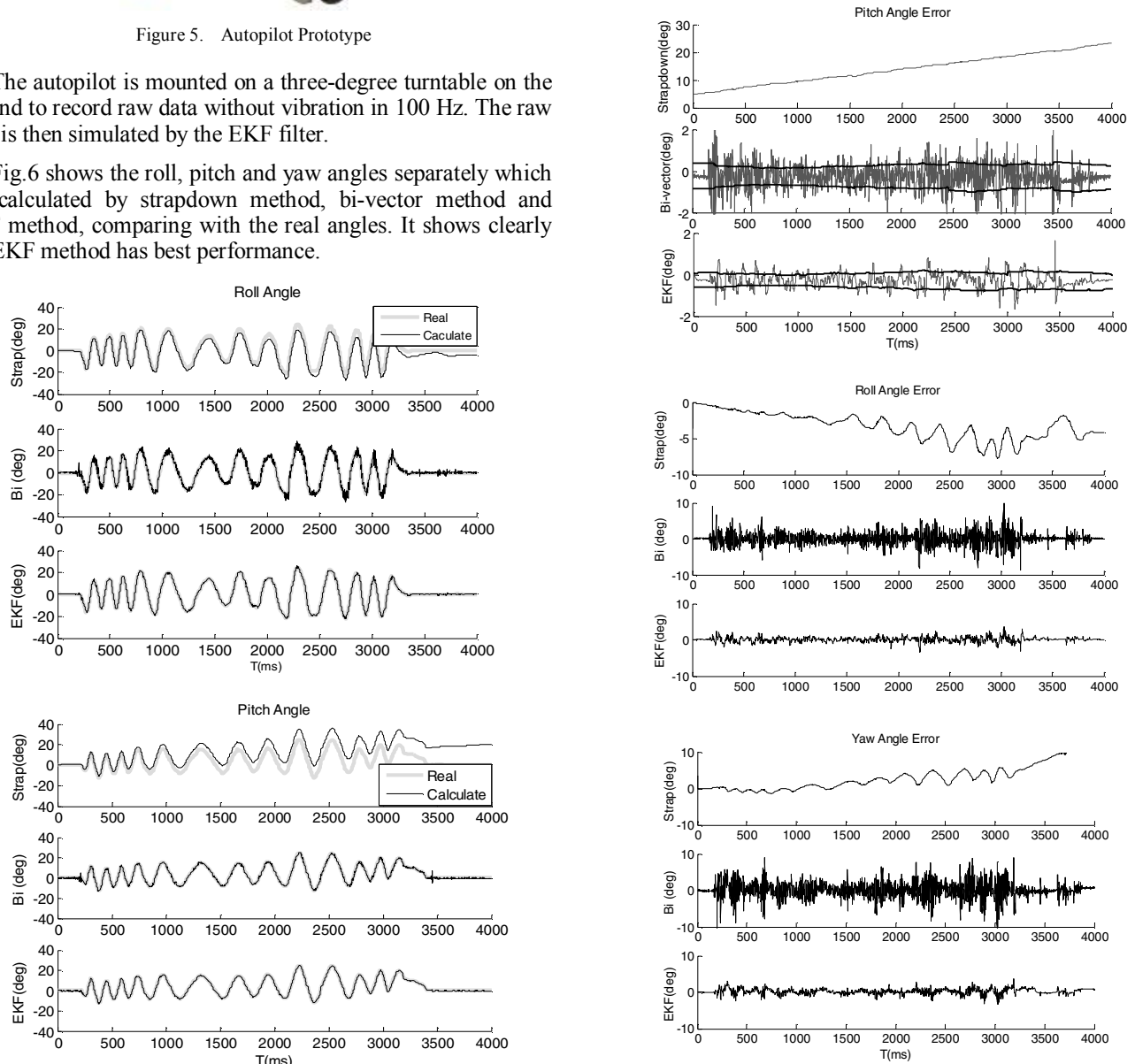Figure 6. Angle Results for ground test: Bold lines are real angles.

Figure 7. Angles Error Results for ground test

Fig. 7 shows the angle errors. It shows that the angles error calculated by the strapdown method is accumulated and reaches up to 10°~30° after 40 seconds. The angles calculated by bi-vector method are suffered by heavy noise. The EKF method gets the best attitude estimates, with error less than ±1°.
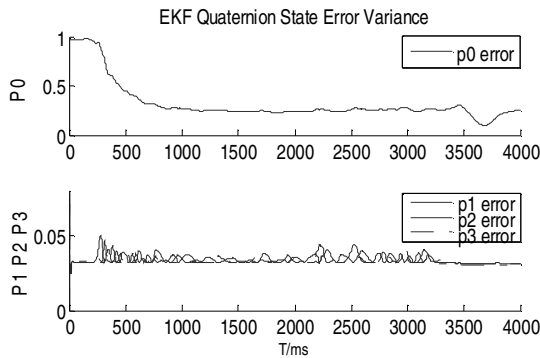

Figure 8.   EKF Quaternion State Error Variance

Fig. 8 shows the EKF quaternion state error variance convergence with time, which denotes that the filter can work in a stable state after a short time.

Next, the real flight data which are recorded from the autonomous helicopter prototype in a real hovering flight experiment are sent to simulate by the EKF attitude estimation methods. The raw gyro and accelerator data are show in Fig. 4.
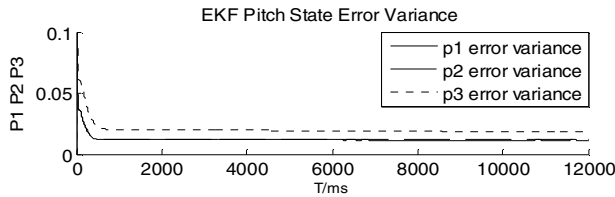

Figure 9.   EKF Pitch State Error Variance for hovering data

The EKF pitch state error variance in Fig. 9 shows that the EKF method can also work stable with real hovering flight data. Fig. 10 shows the pitch angle estimate results, while the roll and yaw angles have similar performance. It shows similar performance to results getting from the ground experiment sensor data.
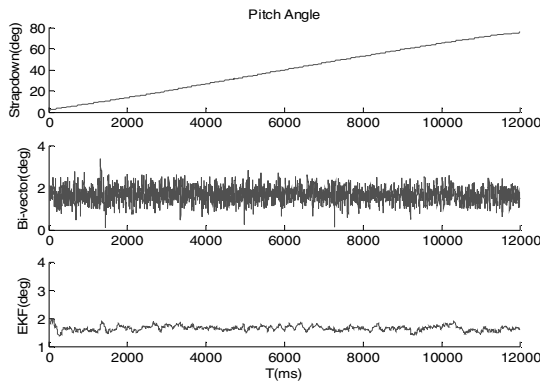

Figure 10. Hovering Pitch Angle Estimate Results.

V.   CONCLUSION

This paper introduces a small helicopter attitude estimation method based on EKF (Extended Kalman filter) to fuse merits of the strapdown and bi-vector method based on analyzing several main attitude estimate ways. It uses strapdown attitude as state update, and treats the attitude estimated by the bi-vector method as measurement update. The paper establishes the system model and linearizes the process and measurement equations based on EKF method principles to get a practical EKF attitude estimate method. The method has a relative loose architecture, which shows clear physical means and makes it easily to modify the power of the attitude estimated by strapdown method and bi-vector method. By evaluating the method with small helicopter's ground and real flight sensor data, It validates that the EKF method has good performance: stable, relatively small error, suitable for small helicopter attitude estimation application.

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