Optimal Sensor Rules and Unified Fusion Rules for Multisensor Multi-hypothesis Network Decision Systems with Fading Channels

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Abstract—In this paper, we firstly present optimal sensor rules with fading channel for a given fusion rule, in which sensor observations are not necessarily independent of each other. Then as a preliminary result to solve the unified fusion rule problem for multisensor multi-hypothesis network decision systems with fading channels, we propose the unified fusion rule for a specific l sensors parallel binary Bayesian decision system under assumption that the i-th sensor is required to transmit a certain number r_i of bits via fading channel while the fusion center can receive its own observation. Since the communication pattern at every node including the fusion center in the multisensor multi-hypothesis decision network are the same as the above parallel binary Bayesian decision system, the above unified fusion rule results can be extended appropriately to more general multisensor multi-hypothesis network decision systems with fading channels, such as the tandem and tree network. More precisely speaking, for these network decision systems, a unified fusion rule is proposed. People only need to optimize sensor rules under the proposed unified fusion rule to achieve global optimal decision performance. More significantly, the unified fusion rule does not depend distributions of sensor observations, decision criterion, and the characteristics of fading channels. Finally, several numerical examples support the above analytic results and show a interesting phenomenon that the two points (0,0) and (1,1) may not be the beginning and end points of ROCs when all channels are fading channels, while in ideal channel case they are the start and end.

Index Terms—Distributed decision, optimal sensor rule, global optimization, unified fusion rule, fading channel.

I. INTRODUCTION

With the fast development of the networking, wireless communications, microprocessors, wireless sensor network (WSN) have become a significant area, therefore, the studies about fading channels are becoming more and more attractive, as evidenced by recent publication such as [1]-[10]. In [1], Thomopoulos and Zhang applied Neyman-Pearson criterion to design local decision rules in the presence of channel errors. They considered the binary symmetric channels to model the transmission of local decisions to a fusion center. Then, in [2], the person by person optimization was used to determine the likelihood ratio thresholds for both the local sensors and the fusion center. Duman and Salehi extended the result in [2] to multiple sensors model in [3]. After those works, many investigations with regard to fading channels have been made recently. For example, in [6], Chen et al. extended the classical parallel fusion structure by incorporating the fading channel layer that is omnipresent in wireless sensor network, and derived the likelihood ratio based fusion rule given fixed local decision devices. Under the conditional independence assumption, under a given fusion rule, among multiple sensor observations, Chen et al. presented that the optimal local decisions that minimize the error probability at the fusion center amount to a likelihood-ratio test given a particular constrain on the fusion rule [7]. Then, Kashyap et al. significantly improved the Chen’s result in [8]. Niu et al. proposed three other sub-optimal fusion rules, which called a two-stage approach using the Chair-Varshney fusion rule, a maximal ratio combiner fusion statistic, and an equal gain combiner fusion statistic, with only requirement of the knowledge of channel statistics [9].

Although WSNs with fading channels have been researched over the last few years, the unified fusion rules with fading channel for multi-hypothesis multisensor network decision systems remain hard tasks. The networks for parallel, tandem and hybrid have been previously addressed in [11] and [13], and have been more investigated in [14] and [15]. After that, the multi-hypothesis multisensor network decision systems attract much research interest. In ideal channel case, Zhu et al. proposed the unified fusion rule in the distributed multi-hypothesis multisensor parallel network decision systems, and then extended them to the tandem network and hybrid network decision systems [17]. Those unified fusion rules have most general form and are independent of the statistical characteristics of observations and decision criteria. However, above results are proposed under the assumption that the transmission channels are ideal. It may not be realistic for many WSNs where the transmitted information has to endure both channel fading and noise/interference.

In this paper, we first present the optimal sensor rule with fading channels for any given fusion rule, in which sensor observations are not necessarily independent of each other, the optimal sensor rules under the assumption of observation...
The fading channels can be described as follows: 

\[ r_i \text{ denotes the received } I_i \text{ bit by the fusion center/next sensor through fading channel, } i = 1, 2, \ldots, l. \]  

The fading channels can be described as follows: 

\[ P(I_j^0 = 0 | H_k) = P(I_{ij} = 0 | I_j = 1), \quad P(I_j^0 = 1 | I_j = 0), \quad i \leq l, \quad j = 1, \ldots, r_i, \]  

where \( P_{ij}^{0} \) describes the probability of a transmission error that the fusion center/next sensor receives 0 while the \( i \)th sensor rule is \( I_{ij} = 1 \), and \( P_{ij}^{0} \) denotes the probability of another transmission error.

### Assumption 1
The channels connecting the sensors to the fusion center/sensors are independent, i.e.,

\[ P(I_1^0, I_2^0, \ldots, I_l^0 | I_1, I_2, \ldots, I_l) = \prod_{j=1}^l P(I_j^0 | I_j). \]  

### Assumption 2
The link errors are statistically independent of hypotheses. Thus, reliability of the sensor transmission as received by the fusion center can be given by

\[
\begin{align*}
P(I_j^0 = 0 | H_k) &= P(I_{ij} = 0 | H_k)(1 - P_{ij}^{ce}) \\
P(I_j^0 = 1 | H_k) &= P(I_{ij} = 1 | H_k)(1 - P_{ij}^{ce}) + P(I_{ij} = 0 | H_k)P_{ij}^{ce}, \quad k = 0, 1.
\end{align*}
\]  

### Assumption 3
The channels connect the sensors to fusion center/sensors are independent, i.e.,

\[ P(I_1^0, I_2^0, \ldots, I_l^0 | I_1, I_2, \ldots, I_l) = \prod_{j=1}^l P(I_j^0 | I_j). \]  

### B. Necessary condition for optimal sensor rules given a fusion rule with fading channels

No matter for what sensor network decision systems (for example, see [17], [18]), an \( m \)-ary \( l \)-sensor Bayesian cost with fading channels finally can be written as the following form:

\[
C(y_1, \ldots, y_{m-1}; y_{m}, I_j^0, P^0) = \sum_{i=0}^{m-1} C_{i} P_{i} + \sum_{i=0}^{m-1} \int_{F^0 = i} \sum_{j=0, j \neq i}^{m-1} P_{ij} (C_{ij} - C_{jj}) p(y | H_j) dy_{j},
\]  

where each \( C_{ij} \) is some suitable cost coefficients, \( P_j \) is priori probability for the hypothesis \( H_j \), \( F^0 \) is a given fusion rule at the fusion center, and each \( P(F^0 = i | H_j) \) denotes the conditional probability of the event that the fusion center’s decision \( F^0 = i \) when the actual hypothesis is \( H_j \), \( i, j = 0, 1, \ldots, m-1 \).

Substituting the conditional probabilities given \( H_0, \ldots, H_{m-1} \) into Eq. (5) and simplifying, we see that

\[
\begin{align*}
P_{ij}(I_j^0) &= \begin{cases} 1 - P_{ij}^{0}, & \text{if } d_{ij} = 0 \\ P_{ij}^{0}, & \text{if } d_{ij} = 1. \end{cases}
\end{align*}
\]  

Obviously, \( P_{H_i}^0 (I_1^0, \ldots, I_l^0) \) is the summation of all for any \( j \leq r_i, \) \( i \leq l \)

\[
\begin{align*}
P_{ij}(I_j^0) &= \begin{cases} 1 - P_{ij}^{0}, & \text{if } d_{ij} = 0 \\ P_{ij}^{0}, & \text{if } d_{ij} = 1.\end{cases}
\end{align*}
\]  

Obviously, \( P_{H_i}^0 (I_1^0, \ldots, I_l^0) \) is the summation of all \( P_{ij}(I_j^0) \) where the local message \( I_{ij}^0 = d_{ij}, \ldots, I_{j1}^0 = d_{j1} \) are in the \( i \)th subset (\( H_i \) decision subset) partitioned by the fusion rule \( F^0 \). Thus,

\[
\begin{align*}
\{ (I_1^0, \ldots, I_l^0) : F^0 (I_1^0, \ldots, I_l^0) = i \} &= \{ (I_1^0, \ldots, I_l^0) : P_{H_i}^0 (I_1^0, \ldots, I_l^0) = 1 \}.
\end{align*}
\]  

Based on Assumptions 1-3 and the total probability formula, \( P_{H_i}^0 (I_1^0, \ldots, I_l^0) \) can be rewritten as
details, cf. [16]) that the following necessary condition of the
respectively. Based on above discussion, it is easy to see (for
error probabilities yielded from $P_{H_1}$ for any individual $i$, the integrand of the right hand side of the Eq. (6) can be rewritten as following various versions:

$$
\sum_{j=0,j \neq i}^{m-1} \int \left[ P_{H_i}(I_1, I_2, \ldots, I_l; P_{ce0}, P_{ce1}) \right] \prod_{j=0,j \neq i}^{m-1} P_{H_j}(C_{ij} = C_{ij}; y)
$$

where $I_{ij}/i = (I_{i1}, \ldots, I_{ij-1}, I_{ij+1}, \ldots, I_{ir})$, $P_{11}$ and $P_{12}$ are the functions independent of $I_{ij}$, $i = 1, \ldots, l, j = 1, \ldots, r$, respectively. Based on above discussion, it is easy to see (for details, cf. [16]) that the following necessary condition of the optimal local compression rules holds.

**Theorem 2.1.** For distributed $m$-ary multisensor decision system employing the fusion rule $P^0$. The set of optimal sensor rules $(I_{11}, \ldots, I_{1r}; \ldots; I_{l1}, \ldots, I_{lr})$ which minimizes the cost functional of Eq. (6) must satisfy following integral equations

$$
I_{ij}(y_l) = \int \left[ \prod_{i=1}^{m-1} P_{H_i}(I_1, I_2, \ldots, I_l; P_{ce0}, P_{ce1}; y) \right] dy_2 \cdots dy_l
$$

where $I[\cdot]$ is defined by

$$
I[x] = \begin{cases} 
1, & \text{if } x > 0 \\
0, & \text{if } x \leq 0. 
\end{cases}
$$

**Remark 2.2.** In the ideal channel case, the final decision rule is a deterministic decision rule for the observation space, $P_{H_0}$ is actually the indicate function of the final $H_0$ decision region in the observation space, therefore, its values are either of 0 and 1 other than that in [18]. In this paper, the final decision rule for the observation space is a randomized fusion rule, i.e., the observation space cannot be partitioned into disjoint decision regions, therefore, $P^0_{H_0}$ is not the indicate function of the final $H_0$ decision region anymore, and it can be some value in the interval $[0, 1]$. Now, since, using Theorem 2.1, the corresponding arguments on the algorithm convergence in the ideal channel case (see [16], [18]) can be used for the fading channel case without essential difficulty.

**C. A special case**

In [7], Chen et al. proposed optimal sensor rules under the assumption of conditional independence between sensor observations, based on the results of above subsection, we find out that optimal sensor rules in [7] is a particular instance of our results, furthermore, we present a more general optimal sensor rules, in which we remove the inequality assumption therein.

For the parallel binary Bayesian decision system with fading channels, under the assumption of conditionally independent observations, Eq. (9) can be rewritten as follows:

$$
P_{11} \prod_{i=1}^{l} p(y_l | H_1) - B_p p(y_l | H_0) 
$$

Therefore, the Eq. (10) can be rewritten as follows:

$$
I_{i}(y_l) = I[A_i p(y_l | H_1) - B_i p(y_l | H_0)]
$$

where $A_i$ and $B_i$ defined as follows respectively:

$$
A_i = a \int P_{11} \prod_{n \neq i} p(y_n | H_1) dy_1 \cdots dy_{i-1} dy_{i+1} \cdots dy_l
$$

$$
B_i = b \int P_{11} \prod_{n \neq i} p(y_n | H_0) dy_1 \cdots dy_{i-1} dy_{i+1} \cdots dy_l.
$$

Using Theorem 2.1, one can easily derive the similar result described in Theorem 1 [7] by Eq. (13), that is to say, under a given fusion rule, optimal sensor rules for the conventional LRT and the reverse LRT can be written as follows:

$$
P(U_k = 1 | x_k) = \begin{cases} 
1, & \frac{P(x_k | H_2)}{P(x_k | H_0)} \geq \eta_k \\
0, & \text{otherwise}.
\end{cases}
$$

or

$$
P(U_k = 1 | x_k) = \begin{cases} 
0, & \frac{P(x_k | H_2)}{P(x_k | H_0)} \geq \eta_k \\
1, & \text{otherwise}.
\end{cases}
$$

where the optimal threshold $\eta_k$ is a proper number one needs to search for in interval $[0, \infty)$.

Since one does not know $A_k$ and $B_k$ before finding optimal sensor rules, the above two kinds of likelihood ratio tests are both required to search for the optimal thresholds in practical applications, Eq. (16) has the same formulation as the result in [7], expect using “reverse” likelihood ration test and removing the inequality assumption therein.
III. THE UNIFIED FUSION RULE IN PARALLEL BINARY
BAYESIAN DECISION SYSTEM WITH FADING CHANNELS

In this section, we propose the unified fusion rule for a
specific l-sensor parallel binary Bayesian decision system
under the assumption that the jth sensor is required to transmit
a certain number $r_j$ of bits via fading channel while the fusion
center can receive its own observation $y_0$. It can be seen that
each node in sensor networks has this kind of information
compression patterns since we suppose the node can also
observe data.

**Theorem 3.1.** The unified fusion rule for the above informa-
tion structure is $F^0(I_{11}, \ldots, I_{1r_1}, \ldots; I_{l1}, \ldots, I_{lr_l}, y_0)$, this
unified fusion rule can be equally written as following fusion
rule:

$$\begin{align}
\{I_{11}^0, \ldots, I_{1r_1}^0; \ldots; I_{l1}^0, \ldots, I_{lr_l}^0; y_0 : F^0 = 0\} \\
= \left\{ \begin{array}{l}
I_{11}^0 = 0, \ldots, I_{1l}^0 = 0; I_{01}(y_0) = 0 \\
I_{11}^0 = 1, \ldots, I_{1l}^0 = 0; I_{02}(y_0) = 0 \\
\vdots \\
I_{l1}^0 = 1, \ldots, I_{lr_l}^0 = 1; I_{02^N}(y_0) = 0 \\
\end{array} \right.,
\end{align}$$

where $N = \sum_{i=1}^{l} r_i$.

**Proof.** Consider a set of sensor rules $P_{l1}^0, \ldots, P_{lr_l}^0$, observation $y_0$, and a
general fusion rule :

$$\begin{align}
\{I_{11}, \ldots, I_{1r_1}; \ldots; I_{l1}, \ldots, I_{lr_l}; y_0 : F^0 = 0\} \\
= \left\{ \begin{array}{l}
F^0(I_{11}^0 = d_{11}^0, \ldots, I_{1l}^0 = d_{1l}^0; y_0) = 0 \\
F^0(I_{11}^0 = d_{11}^0, \ldots, I_{lr_l}^0 = d_{lr_l}^0; y_0) = 0 \\
\vdots \\
F^0(I_{l1}^0 = d_{l1}^0, \ldots, I_{lr_l}^0 = d_{lr_l}^0; y_0) = 0 \\
\end{array} \right.,
\end{align}$$

where $d_{ij}^k, j \leq r_i, i \leq l, k = 1, \ldots, L$ equal 0 or 1. Therefore, there are at most $2^N$ different groups in Eq. (18). If $L = 2^N$, we define a new set of binary compression rules as follows:

$$I_{01}(y_0) = F^0(d_{11}^0, \ldots, d_{1r_1}; \ldots; d_{l1}^0, \ldots, d_{lr_l}; y_0).$$

If $L < N$, for example, $P_{l1}^0 = 0, \ldots, P_{1r_1}^0 = 0; \ldots; P_{l1}^0 = 0, \ldots, P_{lr_l}^0 = 0; I_{02^N}(y_0) = 0$ does not appear in the rewritten Eq. (18) via Eq. (19), we can add $F^0(I_{11}^0 = 0, \ldots, I_{1l}^0 = 0; \ldots; I_{l1}^0 = 0, \ldots, I_{lr_l}^0 = 0; y_0) = 0$ into Eq. (18), but let sensor rule $I_{02^N}(y_0) \equiv 1$, in fact, $P_{l1}^0 = 0, \ldots, P_{1r_1}^0 = 0; \ldots; P_{l1}^0 = 0, \ldots, P_{lr_l}^0 = 0; I_{02^N}(y_0) = 0$ never happen. Obviously, this modification does not change the rewritten Eq. (18) at all. Thus, our fusion rule (17) with a proper set of sensor rules allow us to represent any rule of the form (18). Moreover, using $I_{01}(y_0)$ with other $2^N$ bits $P_{ij}^0$, $j \leq r_i, i \leq l$ received by the fusion center and Eq. (17) insures that the overall scheme produces the same output as the original scheme using the rule from Eq. (18). Q.E.D.

By Eq. (17), we only consider the polynomial $P_{H0}^0$ of the
local sensor rules. The polynomial $P_{H0}^0$ of Eq. (17) can be
written as

$$P_{H0}^0(I_{11}^0, \ldots, I_{1r_1}^0; \ldots; I_{l1}^0, \ldots, I_{lr_l}^0; y_0) = \sum_{k=1}^{\sum_{i=1}^{l} r_i} \prod_{i=1}^{l} I_{ij}^0 (1 - I_{0k}(y_0))),$$

where $I_{ij}^0$ is $I_{ij}$ or $1 - I_{ij}$. Thus, the remaining problem is to search for optimal sensor rules given a fusion rule which has solved before in section 2.

IV. MULTISENSOR MULTI-HYPOTHESIS NETWORK
STRUCTURES WITH FADING CHANNELS

In this section, we consider more general multisensor multi-
hypothesis decision systems with fading channels: parallel and
tandem network with $m$ hypotheses, $H_0, H_1, \ldots, H_{m-1}$, and
$l$ sensors, $S_1, \ldots, S_l$ with multiple observation data $y_i, y_i$
in space $\mathbb{R}^{n} \times \cdots \times \mathbb{R}^{n}$. In addition, we assume that $m$ known
conditional pdf $p(y_1, \ldots, y_l|H_0), \ldots, (y_1, \ldots, y_l|H_{m-1})$ are
of arbitrary forms.

A. Parallel network with fading channels

In this subsection, we consider modified parallel network, which
the fusion center can also observe data, i.e. the ith sensors transmit $r_i$ bits to a fusion center $F$ through fading channels. Then, based on the received $(I_{11}^0, \ldots, I_{1r_1}^0)$ and observation data $y_i$, fusion center $F$ makes a final decision under a given fusion rule (see Fig. 1).

Thus, the above information structure can be expressed by

$$(\{y_1^{(r_1)}, y_2^{(r_2)}, \ldots, y_{l-1}^{(r_{l-1})}\} \\
\{I_{11}^0, I_{12}^0, \ldots, I_{1r_1}^0, I_{21}^0, \ldots, I_{lr_l}^0\} \\
\Rightarrow (y_i, F) \rightarrow \{m\}),$$

where $\{\cdot, \cdot, \cdot\}$ means that all sensors inside “{}” are
in parallel position without communications between sensors.

Besides, “$y_i^{(r_i)}$” expresses the sensor $S_i$ observes its own
data $y_i$ and then compresses them to $r_i$ information bits,
$i = 1, \ldots, l - 1$, and then the $r_i$ information bits are
sent over a fading channel, characterized by $\mathbf{T}_i^{r_i}$. Moreover,
$\{y_1^{(r_1)}, y_2^{(r_2)}, \ldots, y_{l-1}^{(r_{l-1})}\} \Rightarrow \{I_{11}^0, I_{12}^0, \ldots, I_{1r_1}^0\} \Rightarrow (y_i, F) \rightarrow \{m\}$” implies that all sensor in “{}” transmit
their compressed data-information bits to the fusion center $F$
through fading channel, then the fusion center makes $m$-ary
decision together with observation data $y_i$. 

![Fig. 1. The parallel Multisensor multi-hypothesis network decision system](image-url)
The fusion rule $F_{para}$ of fusion center for a parallel network is given by an $m$-valued function

$$F_{para}^{0}(I_{1}^{0}(y_{1}), \ldots, I_{l-1}^{0}(y_{l-1}); y_{l}) : \{0,1\}^{m} \rightarrow \{0,1, \ldots, m - 1\},$$

where $N = \sum_{i=1}^{l-1} r_{i}$.

B. Tandem network with fading channels

In the tandem system, the sensor network is a team of sensors in tandem, i.e., the first sensor $S_1$ compresses its observation data $y_{1}$ to $r_{1}$ information bits $I_{1} = (I_{1}^{0}(y_{1}), \ldots, I_{1}^{r_{1}}(y_{1}))$ and transmits them to the second sensor $S_2$ through fading channel, then, due to communication bandwidth limits, the second sensor $S_2$ has to compress its observation $y_{2}$ and the received message $I_{2}^{0} = (I_{2}^{01}(y_{1}), \ldots, I_{2}^{0r_{1}}(y_{1}))$ to $s_{2}$ bits $(T_{2}^{l}(y_{2}), \ldots, T_{2}^{r_{2}}(y_{2}))$ and transmits them to the next sensor $S_3$ through fading channel, and so on. This procedure is repeated until the $(l - 1)$th sensor $S_{l-1}$. The last sensor $S_{l}$ also uses its observation $y_{l}$ together with the receive message $(T_{l-1}^{01}, \ldots, T_{l-1}^{r_{s_{l-1}}})$ to make a final $m$-ary decision under a given final fusion rule $F_{fan}$, therefore, in this model, the sensor $S_{1}$ and decision center are built in the same station, (see Fig. 2).

Therefore, similarly to the notation of the information structure in parallel networks, denote the information structure for the tandem network simply by

$$(y_{1})^{(r_{1})} \rightarrow (s_{1}) \rightarrow (y_{2})^{(s_{2})} \rightarrow \ldots \rightarrow (y_{l-1})^{(s_{l-1})} \rightarrow (y_{l}: F) \rightarrow (m),$$

where $r_{1} = s_{1}, T_{i}^{l} = (T_{i}^{01}, \ldots, T_{i}^{r_{s_{i}}})$ and $\mu^{(s_{i-1})} y_{i}^{r_{i}}(s_{i})$, $s_{i}$, $s_{i}$, $s_{i}$ means that sensor $S_{i}$ compresses its own data $y_{i}$ together with the received $s_{i-1}$ bits to $s_{i}$ bits, and transmits these $s_{i}$ bits to the next sensor $S_{i+1}$ through fading channel.

Combining the above two basic structures, an arbitrary hybrid network can be constructed.

V. UNIFIED FUSION RULES FOR MULTISensor MULTI-HYPOTHESIS NETWORK DECISION SYSTEMS WITH FADING CHANNELS

We have the following two observations:

Observation 1. An $m$-ary decision can be expressed by $n$ binary decisions, where $m - 1 < 2^{n} \leq m$ (for details, see [17] and [18]).

Observation 2. Obviously, at each intermediate node in the tandem and hybrid networks, when we view the node as a local fusion center, its information structure is actually a $m$-ary parallel network, where $m = 2^{n}$, $n$ is the number of bits transmitted to the next node by this node. Therefore, if a unified fusion rule can be derived for the parallel network, then, a unified fusion rule can be yielded for any tandem and hybrid networks from the fusion center to the bottom node iteratively.

VI. NUMERICAL RESULTS

In this section, we consider parallel Bayesian decision system with fading channels including 3-sensor, 2-ary and 3-ary detection system for Gaussian signals in additive Gaussian noise. All the examples presented have symmetric channel errors and $c_{ij} = 1$ as $i \neq j$, $c_{ii} = 0$, $P_{01} = 1/2$, $P_{02} = P_{21} = 1/4$ for 3-ary detection system. In this case, the Bayesian cost function is actually decision error probability $P_{e}$; In all figures and tables below, the “PN”, “TN”, denote parallel network, tandem network respectively, “IC”, “FC” denote ideal channel and fading channel respectively, “UFR” denotes unified fusion rule.

A. Parallel Bayesian binary decision system with fading channels

In the following example, we consider binary decision system with two sensors, the observations consist of a signal $s$ plus noises $v_{i}, i = 1, 2$ or noises only. Hence, the observations can be modeled as follows.

$$H_{1} : y_{1} = s + v_{1}, \quad y_{2} = s + v_{2},$$

$$H_{0} : y_{1} = v_{1}, \quad y_{2} = v_{2},$$

wheres, $v_{1}$ and $v_{2}$ are all mutually independent ,and

$$s \sim N(2,1), \quad v_{1} \sim N(0,2), \quad v_{2} \sim N(0,1)$$

Example 6.1.1. In this example, we consider two sensor parallel binary Bayesian decision system under the assumption that every sensor is required to transmit a bit through fading channel with $P_{1e}^{01} = P_{1e}^{11} = P_{2e}^{02} = P_{2e}^{12} = p$. The Receiver Operating Characteristic curves (ROC) for AND, OR, and XOR rules with $p = 0.15, 0.3$ are provided respectively in Fig. 3 below.

![Fig. 3. The ROCs for different fusion rules with fading channels](image-url)
An interesting phenomenon is found in Fig. 3 that the ROCs for different fusion rules may not reach the two points (0, 0) and (1, 1) when all the channels are fading channels.

**B. Three sensors decision system**

The three sensor 3-ary decision model is

\[ H_0 : y_1 = \nu_1, \quad y_2 = \nu_2, \quad y_3 = \nu_3; \]
\[ H_1 : y_1 = s_1 + \nu_1, \quad y_2 = s_1 + \nu_2, \quad y_3 = s_1 + \nu_3; \]
\[ H_2 : y_1 = s_2 + \nu_1, \quad y_2 = s_2 + \nu_2, \quad y_3 = s_2 + \nu_3; \]

where the signals \( s_1 \) and \( s_2 \) and the noise \( \nu_1, \nu_2 \) and \( \nu_3 \) are all mutually independent, and

\[
\begin{align*}
    s_1 & \sim N(2, 3), \quad s_2 \sim N(-2, 3), \quad \nu_1 \sim N(0, 3), \\
    \nu_2 & \sim N(0, 2), \quad \nu_3 \sim N(0, 1)
\end{align*}
\]

**Example 6.2.1(a):** The parallel and tandem binary decision information structures are

\[
\left( y_1^{(1)} \left\| y_2^{(1)} \right\| \right) \Rightarrow \left( T_1^{(0)} \left\| T_2^{(1)} \right\| \right) \Rightarrow \left( y_3 \left\| F \right\| \Rightarrow \{2\} \right)
\]

\[
\begin{align*}
    (y_1^{(1)} \rightarrow T_1^{(1)} \rightarrow y_2^{(1)} \rightarrow T_2^{(1)} \rightarrow y_3 \left\| \left\| F \right\| \Rightarrow \{2\})
\end{align*}
\]

The ROCs for unified fusion rule of the above information structure are provide in Fig. 4 below, which shows that the performance decreases with the increase of the channel error and the performance of the unified fusion rule for ideal channels is always better than that of the unified fusion rule for fading channels.

**Example 6.2.1(b):** The parallel and tandem 3-ary information structures are

\[
\left( y_1^{(1)} \left\| y_2^{(1)} \right\| \right) \Rightarrow \left( T_1^{(0)} \left\| T_2^{(1)} \right\| \right) \Rightarrow \left( y_3 \left\| F \right\| \Rightarrow \{3\} \right)
\]

\[
\begin{align*}
    (y_1^{(1)} \rightarrow T_1^{(1)} \rightarrow y_2^{(1)} \rightarrow T_2^{(1)} \rightarrow y_3 \left\| \left\| F \right\| \Rightarrow \{3\})
\end{align*}
\]

Based on the result of the unified fusion rules for 3-ary parallel and tandem network decision system with ideal channels in [17], those unified fusion rules with fading channels can be easily written. In Table 1 below, their probabilities of decision error \( P_e \)'s are given. We can see that they for both the parallel network and tandem network increase with increase of the channel error, and the performance of the unified fusion rule for parallel network is better than that of tandem network.

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