Optimal Sensor Rules and Unified Fusion Rules for Multisensor Multi-hypothesis Network Decision Systems with Fading Channels

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***Abstract*—In this paper, we firstly present optimal sensor rules with fading channel for a given fusion rule, in which sensor observations are not necessarily independent of each other. Then as a preliminary result to solve the unified fusion rule problem for multisensor multi-hypothesis network decision systems with fading channels, we propose the unified fusion rule for a specific l sensors parallel binary Bayesian decision system under assumption that the ith sensor is required to transmit a certain number r_i of bits via fading channel while the fusion center can receive its own observation. Since the communication pattern at every node including the fusion center in the multisensor multi-hypothesis decision network are the same as the above parallel binary Bayesian decision system, the above unified fusion rule results can be extended appropriately to more general multisensor multi-hypothesis network decision systems with fading channels, such as the tandem and tree network. More precisely speaking, for these network decision systems, a unified fusion rule is proposed. People only need to optimize sensor rules under the proposed unified fusion rule to achieve global optimal decision performance. More significantly, the unified fusion rule does not depend distributions of sensor observations, decision criterion, and the characteristics of fading channels. Finally, several numerical examples support the above analytic results and show a interesting phenomenon that the two points (0, 0) and (1, 1) may not be the beginning and end points of ROCs when all channels are fading channels, while in ideal channel case they are the start and end.**

***Index Terms*—Distributed decision, optimal sensor rule, global optimization, unified fusion rule, fading channel.**

I. Introduction

With the fast development of the networking, wireless communications, microprocessors, wireless sensor network(**WSN**) have become a significant area, therefore, the studies about fading channels are becoming more and more attractive, as evidenced by recent publication such as [1]-[10]. In [1], Thomopoulos and Zhang applied Neyman-Pearson criterion to design local decision rules in the presence of channel errors. They considered the binary symmetric channels to model the transmission of local decisions to a fusion center. Then, in [2], the person by person optimization was used to determine the likelihood ratio thresholds for both the local sensors and the fusion center. Duman and Salehi extended the result in [2] to multiple sensors model in [3]. After those works, many investigations with regard to fading channels have been made recently. For example, in [6], Chen et al. extended the classical parallel fusion structure by incorporating the fading channel layer that is omnipresent in wireless sensor network, and derived the likelihood ratio based fusion rule given fixed local decision devices. Under the conditional independence assumption, under a given fusion rule, among multiple sensor observations, Chen et al. presented that the optimal local decisions that minimize the error probability at the fusion center amount to a likelihood-ratio test given a particular constrain on the fusion rule [7]. Then, Kashyap et al. significantly improved the Chen's result in [8]. Niu et al. proposed three other sub-optimal fusion rules, which called a two-stage approach using the Chair-Varshney fusion rule, a maximal ratio combiner fusion statistic, and an equal gain combiner fusion statistic, with only requirement of the knowledge of channel statistics [9].

Although WSNs with fading channels have been researched over the last few years, the unified fusion rules with fading channel for multi-hypothesis multisensor network decision systems remain hard tasks. The networks for parallel, tandem and hybrid have been previously addressed in [11] and [13], and have been more investigated in [14] and [15]. After that, the multi-hypothesis multisensor network decision systems attract much research interest. In ideal channel case, Zhu et al. proposed the unified fusion rule in the distributed multi-hypothesis multisensor parallel network decision systems, and then extended them to the tandem network and hybrid network decision systems [17]. Those unified fusion rules have most general form and are independent of the statistical characteristics of observations and decision criteria. However, above results are proposed under the assumption that the transmission channels are ideal. It may not be realistic for many WSNs where the transmitted information has to endure both channel fading and noise/interference.

In this paper, we first present the optimal sensor rule with fading channels for any given fusion rule, in which sensor observations are not necessarily independent of each other, the optimal sensor rules under the assumption of observation independence in [7] is a special example of our results. Then, we aim to propose unified fusion rules with fading channels for the multi-hypothesis multisensor network decision systems. For this goal, as a preliminary result, we first propose a unified fusion rule for a specific l sensors parallel binary Bayesian decision system with fading channels under the assumption that ith sensor is required to transmit r_i bits via fading channel while the fusion center can receive its own observation. The unified fusion rule can achieve globally optimal decision performance in the fading channel case similar to that in the ideal channel case. Then, since the communication pattern at every node including the fusion center in the multisensor decision network are the same as the above parallel binary Bayesian decision system, based on above results, we propose the unified fusion rule for multisensor multi-hypothesis parallel network decision systems with fading channels, and then extend the result to the tandem network and hybrid network decision systems. Finally, several numerical examples support the above analytic results and show an interesting phenomenons that the two points (0, 0) and (1, 1) may not be the beginning and end points of ROCs anymore when all channels are fading channels, while in ideal channel case they are the start and end.

II. Optimal Sensor Rule with Fading Channels for Given Fusion Rule

In this section, we consider multisensor multi-hypothesis Bayesian decision system and aim to find the optimal sensor rule for any given fusion rule with fading channels. Firstly, we give some formulations about fading channels, and then we present a necessary condition of the optimal local compression rules for any fixed fusion rule with fading channels, finally, we show that the optimal sensor rules under the assumption of observation independence in [7] is a special example of our results. In this section, some results can be seen in [18].

A. Some formulations in fading channel case

Let $\mathbf{I}_i = (I_{i1}, \dots I_{ir_i})$ denotes the r_i bits compressed by the ith sensor based on its observation y_i, $\mathbf{I}_i^0 = (I_{i1}^0, \dots I_{ir_i}^0)$ denotes the received r_i bit by the fusion center/next sensor from the ith sensor through fading channel, $i = 1, 2, \dots, l$. The fading channels can be described as follows:

Assumption 1. The channels connecting the sensors to the fusion center/senors are not totally reliable. The channel error between the ith sensor and the fusion center/next sensor is defined as follows:

$$P_{ij}^{ce1} = P(I_{ij}^0 = 0 | I_{ij} = 1), \tag{1}$$

$$P_{ij}^{ce0} = P(I_{ij}^0 = 1 | I_{ij} = 0), \; i \leq l, \, j \leq r_i, \tag{2}$$

where P_{ij}^{ce1} describes the probability of a transmission error that the fusion center/next sensor receives 0 while the ith sensor rule is $I_{ij} = 1$, and P_{ij}^{ce0} denotes the probability of another transmission error.

Assumption 2. The link errors are statistically independent of hypotheses. Thus, reliability of the sensor transmission as received by the fusion center can be given by

$$\begin{aligned} P(I_{ij}^0 = 0 | H_k) &= P(I_{ij} = 0 | H_k)(1 - P_{ij}^{ce0}) \\ &\quad + P(I_{ij} = 1 | H_k) P_{ij}^{ce1} \\ P(I_{ij}^0 = 1 | H_j) &= P(I_{ij} = 1 | H_k)(1 - P_i^{ce1}) \\ &\quad + P(I_{ij} = 0 | H_k) P_i^{ce0}, \; k = 0, 1. \end{aligned} \tag{3}$$

Assumption 3. The channels connect the sensors to fusion center/sensors are independent, i.e.,

$$P(\mathbf{I}_1^0, \mathbf{I}_2^0, \dots, \mathbf{I}_l^0 | \mathbf{I}_1, \mathbf{I}_2, \dots, \mathbf{I}_l) = \prod_{j=1}^{l} P(\mathbf{I}_j^0 | \mathbf{I}_j). \tag{4}$$

B. Necessary condition for optimal sensor rules given a fusion rule with fading channels

No matter for what sensor network decision systems (for example, see [17], [18]), an m-ary l-sensor Bayesian cost with fading channels finally can be written as the following form:

$$\begin{aligned} &C(\mathbf{I}_1^0(y_1), \dots, \mathbf{I}_{l-1}^0(y_{l-1}), \mathbf{I}_l^0(y_l); \, F^0) \\ &= \textstyle\sum_{i=0,j=0}^{m-1} C_{ij} P_j P(F^0 = i | H_j), \end{aligned} \tag{5}$$

where each C_{ij} is some suitable cost coefficients, P_j is priori probability for the hypothesis H_i, F^0 is a given fusion rule at the fusion center, and each $P(F^0 = i | H_j)$ denotes the conditional probability of the event that the fusion center's decision $F^0 = i$ when the actual hypothesis is H_j, $i, j = 0, 1, \dots m-1$.

Substituting the conditional probabilities given $H_0, \dots, H_{m-1}$ into Eq. (5) and simplifying, we see that

$$\begin{aligned} C(\mathbf{y}; F^0) = \textstyle\sum_{i=0}^{m-1} C_{ii} P_i + \sum_{i=0}^{m-1} \int_{F^0 = i} \sum_{j=0, j \neq i}^{m-1} \cdot \\ P_j (C_{ij} - C_{jj}) p(\mathbf{y} | H_j) dy_1 \cdots dy_l, \end{aligned} \tag{6}$$

where $\mathbf{y} = (y_1, \dots, y_l)$.

F^0 is actually a disjoint m-ary partition of the set of all local messages $(\mathbf{I}_1^0, \dots, \mathbf{I}_l^0)$. Each of those local messages corresponds uniquely to a polynomial. More precisely speaking, the local message $(I_{11}^0 = d_{11}, \dots, I_{1r_1}^0 = d_{1r_1}; \dots; I_{l1}^0 = d_{l1}, \dots, I_{lr_l}^0 = d_{lr_l}; d_{ij} = 0 \,\text{or}\, 1, j \leq r_i, \, i \leq l)$ corresponds uniquely to a product polynomial $P_{11}(I_{11}^0) P_{12}(I_{12}^0) \cdots P_{lr_l}(I_{lr_l}^0)$, where for any $j \leq r_i$, $i \leq l$

$$P_{ij}(I_{ij}^0) = \begin{cases} 1 - I_{ij}^0, & \text{if} \quad d_{ij} = 0 \\ I_{ij}^0, & \text{if} \quad d_{ij} = 1. \end{cases}$$

Obviously, $P_{H_i}^0(\mathbf{I}_1^0, \dots, \mathbf{I}_l^0)$ is the summation of all $P_{11}(I_{11}^0) P_{12}(I_{12}^0) \cdots P_{lr_l}(I_{lr_l}^0)$ where the local message $(I_{11}^0 = d_{11}, \dots, I_{1r_1}^0 = d_{1r_1}; \dots; I_{l1}^0 = d_{l1}, \dots, I_{lr_l}^0 = d_{lr_l})$ are in the ith subset (H_i decision subset) partitioned by the fusion rule F^0. Thus,

$$\begin{aligned} &\{(\mathbf{I}_1^0, \dots, \mathbf{I}_l^0) : \, F^0(\mathbf{I}_1^0, \dots, \mathbf{I}_l^0) = i\} \\ = \; &\{(\mathbf{I}_1^0, \dots, \mathbf{I}_l^0) : \, P_{H_i}^0(\mathbf{I}_1^0, \dots, \mathbf{I}_l^0) = 1\}, \end{aligned} \tag{7}$$

Based on Assumptions 1-3 and the total probability formula, $P_{H_i}^0(\mathbf{I}_1^0, \dots, \mathbf{I}_l^0)$ can be rewritten as

$$
\begin{aligned}
&P^0_{H_i}(\mathbf{I}^0_1,\ldots,\mathbf{I}^0_l)\\
&=P_{H_i}(\mathbf{I}_1,\mathbf{I}_2,\ldots,\mathbf{I}_l;\mathbf{P}^{ce0},\mathbf{P}^{ce1}),
\end{aligned}
\tag{8}
$$

where $\mathbf{P}^{ce0}=(P^{ce0}_1,\ldots,P^{ce0}_l)$, $\mathbf{P}^{ce1}=(P^{ce1}_1,\ldots,P^{ce1}_l)$, and P_{H_i} is a function of all local sensor rules and transmission error probabilities yielded from $P^0_{H_i}$, which may not be an indicate function of H_i decision region in the observation space.

Thus, using Eqs. (7), (8) and $P_{H_i}(\mathbf{I}_1,\mathbf{I}_2,\ldots,\mathbf{I}_l;\mathbf{P}^{ce0},\mathbf{P}^{ce1})$ to be a linear function of $\mathbf{I}_i$ for any individual i, the integrand of the right hand side of the Eq. (6) can be rewritten as following various versions:

$$
\begin{aligned}
&\textstyle\sum_{i=0}^{m-1}P_{H_i}(\mathbf{I}_1,\mathbf{I}_2,\ldots,\mathbf{I}_l;\mathbf{P}^{ce0},\mathbf{P}^{ce1})\cdot\\
&\textstyle\sum_{j=0,j\neq i}^{m-1}P_j(C_{ij}-C_{jj})p(\mathbf{y}|H_j)\\
=\;&(1-I_{11})P^1_{11}(\tfrac{\mathbf{I}_1}{I_{11}},\mathbf{I}_2,\ldots,\mathbf{I}_l;\mathbf{P}^{ce0},\mathbf{P}^{ce1};\mathbf{y})\\
&+P^1_{12}(\tfrac{\mathbf{I}_1}{I_{11}},\mathbf{I}_2,\ldots,\mathbf{I}_l;\mathbf{P}^{ce0},\mathbf{P}^{ce1};\mathbf{y})\\
&\qquad\cdots\\
=\;&(1-I_{lr_l})P^{r_l}_{l1}(\mathbf{I}_1,\mathbf{I}_2,\ldots,\tfrac{\mathbf{I}_l}{I_{lr_l}};\mathbf{P}^{ce0},\mathbf{P}^{ce1};\mathbf{y})\\
&+P^{r_l}_{l2}(\mathbf{I}_1,\mathbf{I}_2,\ldots,\tfrac{\mathbf{I}_l}{I_{lr_l}};\mathbf{P}^{ce0},\mathbf{P}^{ce1};\mathbf{y}),
\end{aligned}
\tag{9}
$$

where $\mathbf{I}_i/I_{ij}=(I_{i1},\ldots,I_{ij-1},I_{ij+1},\ldots I_{ir_i})$, P^j_{i1} and P^j_{i2} are the functions independent of $I_{ij}, i=1,\ldots,l, j=1,\ldots,r_i$, respectively . Based on above discussion, it is easy to see (for details, cf. [16]) that the following necessary condition of the optimal local compression rules holds.

Theorem 2.1. For distributed m-ary multisensor decision system employing the fusion rule F^0. The set of optimal sensor rules $(I_{11},\ldots,I_{1r_1};\ldots;I_{l1},\ldots,I_{lr_l})$ which minimizes the cost functional of Eq. (6) must satisfy following integral equations

$$
\begin{aligned}
I_{11}(y_1)&=I[\textstyle\int P^1_{11}(\tfrac{\mathbf{I}_1}{I_{11}},\ldots,\mathbf{I}_l;\mathbf{P}^{ce0},\mathbf{P}^{ce1};\mathbf{y})dy_2\cdots dy_l]\\
I_{l1}(y_l)&=I[\textstyle\int P^1_{l1}(\mathbf{I}_1,\ldots,\tfrac{\mathbf{I}_l}{I_{l1}};\mathbf{P}^{ce0},\mathbf{P}^{ce1};\mathbf{y})dy_1\cdots dy_{l-1}]\\
&\cdots\\
I_{lr_l}(y_l)&=I[\textstyle\int P^{r_l}_{l1}(\mathbf{I}_1,\ldots,\tfrac{\mathbf{I}_l}{I_{lr_l}};\mathbf{P}^{ce0},\mathbf{P}^{ce1};\mathbf{y})dy_1\cdots dy_{l-1}],
\end{aligned}
\tag{10}
$$

where $I[\cdot]$ is defined by

$$
I[x]=\begin{cases}1, & \text{if}\quad x>0\\ 0, & \text{if}\quad x\leq 0.\end{cases}
\tag{11}
$$

Remark 2.2. In the ideal channel case, the final decision rule is a deterministic decision rule for the observation space, P_{H_0} is actually the indicate function of the final H_0 decision region in the observation space, therefore, its values are either of 0 and 1 other than that in [18]. In this paper, the final decision rule for the observation space is a randomized fusion rule, i.e., the observation space cannot be partitioned into disjoint decision regions, therefore, $P^0_{H_0}$ is not the indicate function of the final H_0 decision region anymore, and it can be some value in the interval $[0,1]$.

Now, since, using Theorem 2.1, the corresponding arguments on the algorithm convergence in the ideal channel case (see [16], [18]) can be used for the fading channel case without essential difficulty.

C. A special case

In [7], Chen et al. proposed optimal sensor rules under the assumption of conditional independence between sensor observations, based on the results of above subsection, we find out that optimal sensor rules in [7] is a particular instance of our results, furthermore, we present a more general optimal sensor rules, in which we remove the inequality assumption therein.

For the parallel binary Bayesian decision system with fading channels, under the assumption of conditionally independent observations, Eq. (9) can be rewritten as follows:

$$
\begin{aligned}
&P_{i1}L(y_1,y_2,\ldots,y_l)\\
=\;&P_{i1}(a\textstyle\prod_{n=1}^{l}p(y_n|H_1)-b\prod_{n=1}^{l}p(y_n|H_0))\\
=\;&ap(y_i|H_1)P_{i1}\textstyle\prod_{n\neq i}p(y_n|H_1)\\
&-bp(y_i|H_0)P_{i1}\textstyle\prod_{n\neq i}p(y_n|H_1)
\end{aligned}
\tag{12}
$$

Therefore, the Eq. (10) can be rewritten as follows:

$$
\begin{aligned}
I_i(y_i)\;&=\;I[A_ip(y_i|H_1)-B_ip(y_i|H_0)]\\
&\qquad i=1,2,\ldots,l,
\end{aligned}
\tag{13}
$$

where A_i and B_i defined as follows respectively:

$$
A_i=a\int P_{i1}\prod_{n\neq i}p(y_n|H_1)dy_1\cdots dy_{i-1}dy_{i+1}\cdots dy_l \tag{14}
$$

$$
B_i=b\int P_{i1}\prod_{n\neq i}p(y_n|H_0)dy_1\cdots dy_{i-1}dy_{i+1}\cdots dy_l. \tag{15}
$$

Using Theorem 2.1, one can easily derive the similar result described in Theorem 1 [7] by Eq. (13), that is to say, under a given fusion rule, optimal sensor rules for the conventional LRT and the reverse LRT can be written as follows:

$$
P(U_k=1|x_k)=\begin{cases}1, & \frac{P(x_k|H_1)}{P(x_k|H_0)}\geq\eta_k\\ 0, & \text{otherwise}\end{cases},
$$

or

$$
P(U_k=1|x_k)=\begin{cases}0, & \frac{P(x_k|H_1)}{P(x_k|H_0)}\geq\eta_k\\ 1, & \text{otherwise}\end{cases}, \tag{16}
$$

where the optimal threshold η_k is a proper number one needs to search for in interval $[0,\infty)$.

Since one does not know A_k and B_k before finding optimal sensor rules, the above two kinds of likelihood ratio tests are both required to search for the optimal thresholds in practical applications. Eq. (16) has the same formulation as the result in [7], expect using “reverse” likelihood ration test and removing the inequality assumption therein.

III. The Unified Fusion Rule in Parallel Binary Bayesian Decision System with Fading Channels

In this section, we propose the unified fusion rule for a specific l-sensor parallel binary Bayesian decision system under the assumption that the ith sensor is required to transmit a certain number r_i of bits via fading channel while the fusion center can receive its own observation y_0. It can be seen that each node in sensor networks has this kind of information compression patterns since we suppose the node can also observe data.

Theorem 3.1. The unified fusion rule for the above information structure is $F^0(I_{11}^0, \ldots, I_{1r_1}^0; \ldots; I_{l1}^0, \ldots, I_{lr_l}^0; y_0)$, this unified fusion rule can be equally written as following fusion rule:

$$\begin{array}{l} \{I_{11}^0, \ldots, I_{1r_1}^0; \ldots; I_{l1}^0, \ldots, I_{lr_l}^0; y_0 : F^0 = 0\} \\ = \left\{ \begin{array}{cccc} I_{11}^0 = 0, & \ldots, & I_{lr_l}^0 = 0; & I_{01}(y_0) = 0 \\ I_{11}^0 = 1, & \ldots, & I_{lr_l}^0 = 0; & I_{02}(y_0) = 0 \\ & & \ldots, & \\ I_{11}^0 = 1, & \ldots, & I_{lr_l}^0 = 1; & I_{02^N}(y_0) = 0 \end{array} \right\}, \end{array} \tag{17}$$

where $N = \sum_{i=1}^{l} r_i$.

Proof. Consider a set of sensor rules $I_{11}^0, \ldots, I_{1r_1}^0; \ldots; I_{l1}^0, \ldots, I_{lr_l}^0$, observation y_0, and a general fusion rule :

$$\begin{array}{l} \{I_{11}^0, \ldots, I_{1r_1}^0; \ldots; I_{l1}^0, \ldots, I_{lr_l}^0; y_0 : F^0 = 0\} \\ = \left\{ \begin{array}{c} \{I_{11}^0, \ldots, I_{1r_1}^0; \ldots; I_{l1}^0, \ldots, I_{lr_l}^0; y_0\} : \\ F^0(I_{11}^0 = d_{11}^1, \ldots, I_{lr_l}^0 = d_{lr_l}^1; y_0) = 0 \\ F^0(I_{11}^0 = d_{11}^2, \ldots, I_{lr_l}^0 = d_{lr_l}^2; y_0) = 0 \\ \ldots, \\ F^0(I_{11}^0 = d_{11}^L, \ldots, I_l^{0r_l} = d_{lr_l}^L; y_0) = 0 \end{array} \right\}, \end{array} \tag{18}$$

where d_{ij}^k, $j \leq r_i$, $i \leq l$, $k = 1, \ldots, L$ equal 0 or 1. Therefore, there are at most 2^N different groups in Eq. (18). If $L = 2^N$, we define a new set of binary compression rules as follows:

$$I_{0i}(y_0) = F^0(d_{11}^i, \ldots, d_{1r_i}^i; \ldots; d_{l1}^i, \ldots, d_{lr_l}; y_0). \tag{19}$$

If $L < N$, for example, $I_{11}^0 = 0, \ldots, I_{1r_1}^0 = 0; \ldots; I_{l1}^0 = 0, \ldots, I_{lr_l}^0 = 0; I_{02^N}(y_0) = 0$ does not appear in the rewritten Eq. (18) via Eq.(19), we can add $F(I_{11}^0 = 0, \ldots, I_{1r_1}^0 = 0; \ldots; I_{l1}^0 = 0, \ldots, I_{lr_l}^0 = 0; y_0) = 0$ into Eq. (18), but let sensor rule $I_{02^N}(y_0) \equiv 1$, in fact, $I_{11}^0 = 0, \ldots, I_{1r_1}^0 = 0; \ldots; I_{l1}^0 = 0, \ldots, I_{lr_l}^0 = 0; I_{02^N}(y_0) = 0$ never happen. Obviously, this modification does not change the rewritten Eq. (18) at all. Thus, our fusion rule (17) with a proper set of sensor rules allow us to represent any rule of the form (18). Moreover, using $I_{0i}(y_0)$ with other 2^N bits I_{ij}^0, $j \leq r_i$, $i \leq l$ received by the fusion center and Eq. (17) insures that the overall scheme produces the same output as the original scheme using the rule from Eq. (18). Q.E.D.

By Eq. (17), we only consider the polynomial $P_{H_0}^0$ of the local sensor rules. The polynomial $P_{H_0}^0$ of Eq. (17) can be written as

$$\begin{array}{rl} & P_{H_0}^0(I_{11}^0, \ldots, I_{lr_l}^0; I_{01}(y_0), \ldots, I_{02^N}(y_0)) \\ = & \sum_{k=1}^{2^N} (\sum_{i=1}^{l} \prod_{j=1}^{r_i} f_{ij}(1 - I_{0k}(y_0))), \end{array} \tag{20}$$

where f_{ij} is I_{ij}^0 or $1 - I_{ij}^0$. Thus, the remaining problem is to search for optimal sensor rules given a fusion rule which has solved before in section 2.

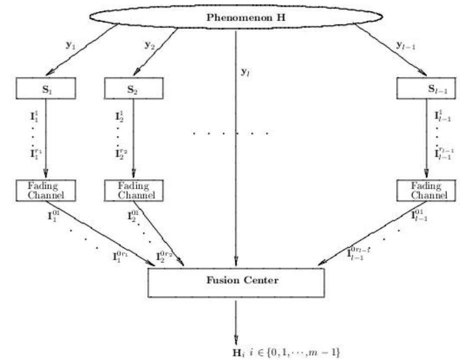

Fig. 1. The parallel Multisensor multi-hypothesis network decision system

IV. Multisensor Multi-hypothesis Network Structures with Fading Channels

In this section, we consider more general multisensor multi-hypothesis decision systems with fading channels: parallel and tandem network with m hypotheses, $H_0, H_1, \ldots, H_{m-1}$, and l sensors, $S_1, \ldots, S_l$ with multiple observation data $y_1, \ldots, y_l$ in space $\Re^{n_1} \times \ldots \times \Re^{n_l}$. in addition, we assume that m known conditional pdf $p(y_1, \ldots, y_l|H_0), \ldots, (y_1, \ldots, y_l|H_{m-1})$ are of arbitrary forms.

A. Parallel network with fading channels

In this subsection, we consider modified parallel network, which the fusion center can also observe data, i.e. the ith sensors transmit r_i bits to a fusion center $\mathcal{F}$ through fading channels. Then, based on the received $(\mathbf{I}_1^0, \ldots, \mathbf{I}_{l-1}^0)$ and observation data y_l, fusion center $\mathcal{F}$ makes a final decision under a given fusion rule (see Fig. 1).

Thus, the above information structure can be expressed by

$$\begin{array}{c} (\{y_1^{(r_1)} \| y_2^{(r_2)} \| \ldots \| y_{l-1}^{(r_{l-1})}\} \Rightarrow \\ \{\mathbf{I}_1^0 \| \mathbf{I}_2^0 \| \ldots \| \mathbf{I}_{l-1}^0\} \Rightarrow (y_l \cdot \mathcal{F}) \rightarrow \{m\}), \end{array}$$

where $\{\cdot \| \cdot \| \ldots \| \cdot\}$ means that all sensors inside "$\{\}$" are in parallel position without communications between sensors. Besides, "$y_i^{(r_i)}$" expresses the sensor S_i observes its own data y_i and then compresses them to r_i information bits, $i = 1 \ldots, l-1$, and then the r_i information bits are sent over a fading channel, characterized by "$\mathbf{I}_i^0$". Moreover, "$\{y_1^{(r_1)} \| y_2^{(r_2)} \| \ldots \| y_{l-1}^{(r_{l-1})}\} \Rightarrow \{\mathbf{I}_1^0 \| \mathbf{I}_2^0 \| \ldots \| \mathbf{I}_{l-1}^0\} \Rightarrow (y_l \cdot \mathcal{F}) \rightarrow \{m\}$" implies that all sensor in "$\{\}$" transmit their compressed data-information bits to the fusion center $\mathcal{F}$ through fading channel, then the fusion center makes m-ary decision together with observation data y_l.

The fusion rule F_{para} of fusion center for a parallel network is given by an m-valued function

$$F_{para}^0(\mathbf{I}_1^0(y_1),\ldots,\mathbf{I}_{l-1}^0(y_{l-1});y_l): \tag{21}$$
$$\{0,1\}^N \longmapsto \{0,1,\ldots,m-1\}, \tag{22}$$

where $N=\sum_{i=1}^{l-1} r_i$.

B. Tandem network with fading channels

In the tandem system, the sensor network is a team of sensors in tandem, i.e., the first sensor S_1 compresses its observation data y_l to r_1 information bits $\mathbf{I}_1=(I_1^1(y_1),\ldots,I_1^{r_1}(y_1))$ and transmits them to the second sensor S_2 through fading channel, then, due to communication bandwidth limits, the second sensor S_2 has to compress its observation y_2 and the received message $\mathbf{T}_1^0=(I_1^{01}(y_1),\ldots,I_1^{0r_1}(y_1))$ to s_2 bits $(T_2^1(\mathbf{T}_1^0;y_2),\ldots,T_2^{s_2}(\mathbf{T}_1^0;y_2))$ and transmits them to the next sensor S_3 through fading channel, and so on. This procedure is repeated until the $(l-1)$th sensor S_{l-1}. The last sensor S_l also uses its observation y_l together with the receive message $(T_{l-1}^{01},\ldots,T_{l-1}^{0s_{l-1}})$ to make a final m-ary decision under a given final fusion rule F_{tan}, therefore, in this model, the sensor S_l and decision center are built in the same station, (see Fig. 2).

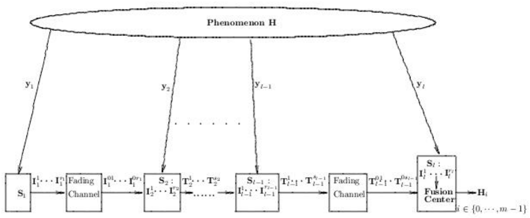

Fig. 2. The tandem multisensor multi-hypothesis network decision system

Therefore, similarly to the notation of the information structure in parallel networks, denote the information structure for the tandem network simply by

$$(y_1^{(r_1)} \overset{(\mathrm{s}_1)}{\longmapsto} \mathbf{T}_1^0 \overset{(\mathrm{s}_1)}{\longmapsto} y_2^{(r_2)} \overset{(\mathrm{s}_2)}{\longmapsto} \ldots \overset{(\mathrm{s}_{l-2})}{\longmapsto} y_{l-1}^{r_{l-1}}$$
$$\overset{(\mathrm{s}_{l-1})}{\longmapsto} \mathbf{T}_{l-1}^0 \overset{(\mathrm{s}_{l-1})}{\longmapsto} (y_l\cdot\mathcal{F}) \longmapsto \{m\}),$$

where $r_1=s_1$, $\mathbf{T}_i^0=(T_i^{01},\ldots,T_i^{0s_i})$ and "$\overset{(\mathrm{s}_{i-1})}{\longmapsto} y_i^{r_i} \overset{(\mathrm{s}_i)}{\longmapsto} \mathbf{T}_i^0 \overset{(\mathrm{s}_i)}{\longmapsto}$" means that sensor S_i compresses its own data y_i together with the received s_{i-1} bits to s_i bits, and transmits these s_i bits to the next sensor S_{i+1} through fading channel.

Combing the above two basic structures, an arbitrary hybrid network can be constructed.

V. Unified Fusion Rules for Multisensor Multi-hypothesis Network Decision Systems with Fading Channels

We have the following two observations:

Observation 1. An m-ary decision can be expressed by n binary decisions, where $m-1<2^n\le m$ (for details, see [17] and [18].

Observation 2. Obviously, at each intermediate node in the tandem and hybrid networks, when we view the node as a local fusion center, its information structure is actually a m-ary parallel network, where $m=2^n$, n is the number of bits transmitted to the next node by this node. Therefore, if a unified fusion rule can be derived for the parallel network, then, a unified fusion rule can be yielded for any tandem and hybrid networks from the fusion center to the bottom node iteratively.

VI. Numerical Results

In this section, we consider parallel Bayesian decision system with fading channels including 3-sensor, 2-ary and 3-ary detection system for Gaussian signals in additive Gaussian noises. All the examples presented have symmetric channel errors and $c_{ij}=1$ as $i\ne j$, $c_{ii}=0$, $P_0=1/2$, $P_1=P_2=1/4$ for 3-ary detection system. In this case, the Bayesian cost function is actually decision error probability P_e; In all figures and tables below, the "PN", "TN", denote parallel network, tandem network respectively, "IC", "FC" denote ideal channel and fading channel respectively, "UFR" denotes unified fusion rule.

A. Parallel Bayesian binary decision system with fading channels

In the following example, we consider binary decision system with two sensors, the observations consist of a signal s plus noises $v_i, i=1,2$ or noises only. Hence, the observations can be modeled as follows.

$$H_1: y_1=s+v_1, \quad y_2=s+v_2,$$
$$H_0: y_1=v_1, \quad y_2=v_2,$$

wheres,v_1 and v_2 are all mutually independent ,and

$$s\sim N(2,1), \quad v_1\sim N(0,2), \quad v_2\sim N(0,1)$$

Example 6.1.1. In this example, we consider two sensor parallel binary Bayesian decision system under the assumption that every sensor is required to transmit a bit through fading channel with $P_1^{ce0}=P_1^{ce1}=P_2^{ce0}=P_2^{ce1}=p$. The Receiver Operating Characteristic curves (ROC) for AND, OR, and XOR rules with $p=0.15$, 0.3 are provided respectively in Fig. 3 below.

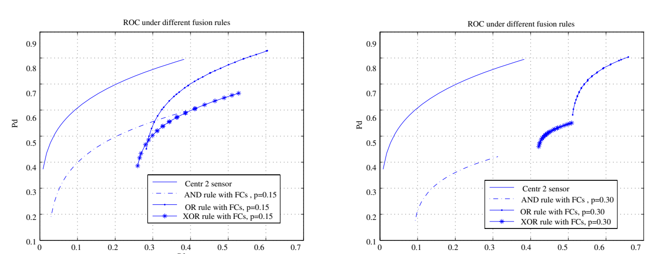

Fig. 3. The ROCs for different fusion rules with fading channels

An interesting phenomenon is found in Fig. 3 that the ROCs for different fusion rules may not reach the two points (0, 0) and (1, 1) when all the channels are fading channels.

B. Three sensors decision system

The three sensor 3-ary decision model is

$$\begin{aligned} H_0 &: y_1 = \nu_1, \quad y_2 = \nu_2, \quad y_3 = \nu_3; \\ H_1 &: y_1 = s_1 + \nu_1,\ y_2 = s_1 + \nu_2,\ y_3 = s_1 + \nu_3; \\ H_2 &: y_1 = s_2 + \nu_1,\ y_2 = s_2 + \nu_2,\ y_3 = s_2 + \nu_3; \end{aligned}$$

where the signals s_1 and s_2 and the noise ν_1, ν_2 and ν_3 are all mutually independent, and

$$s_1 \sim N(2,3), s_2 \sim N(-2,3), \nu_1 \sim N(0,3), \\ \nu_2 \sim N(0,2), \nu_3 \sim N(0,1)$$

Example 6.2.1(a): The parallel and tandem binary decision information structures are

$$\left(\{y_1^{(1)} \| y_2^{(1)}\} \Rightarrow \{\mathbf{I}_1^{0(1)} \| \mathbf{I}_2^{0(1)}\} \Rightarrow (y_3^{(4)} \cdot \mathcal{F}) \to \{2\}\right)$$

$$(y_1^{(1)} \xmapsto{(1)} \mathbf{T}_1^0 \xmapsto{(1)} y_2^{(2)} \xmapsto{(1)} \mathbf{T}_2^0 \xmapsto{(1)} (y_3^{(2)} \cdot \mathcal{F}) \longmapsto \{2\})$$

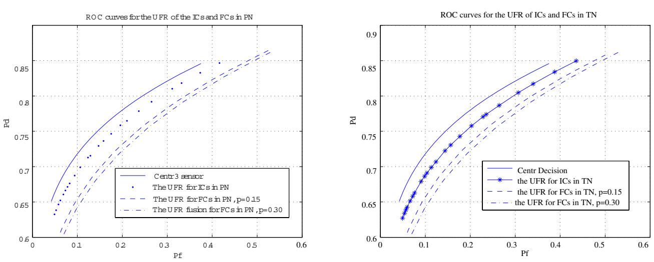

Fig. 4. The ROCs of the unified fusion rules for the parallel network and tandem network binary decision system with ideal channels or fading channels

The ROCs for unified fusion rule of the above information structure are provide in Fig. 4 below, which shows that the performance decreases with the increase of the channel error and the performance of the unified fusion rule for ideal channels is always better than that of the unified fusion rule for fading channels.

Example 6.2.1(b): The parallel and tandem 3-ary information structures are

$$\left(\{y_1^{(1)} \| y_2^{(1)}\} \Rightarrow \{\mathbf{I}_1^{0} \| \mathbf{I}_2^{0}\} \Rightarrow (y_3 \cdot \mathcal{F}) \to \{3\}\right)$$

$$(y_1^{(1)} \xmapsto{(1)} \mathbf{T}_1^{0(1)} \xmapsto{(1)} y_2^{2} \xmapsto{(1)} \mathbf{T}_2^0 \xmapsto{(1)} (y_3 \cdot \mathcal{F}) \longmapsto \{3\})$$

Based on the result of the unified fusion rules for 3-ary parallel and tandem network decision system with ideal channels in [17], those unified fusion rules with fading channels can be easily written. In Table 1 below, their probabilities of decision error P_es are given. We can see that they for both the parallel network and tandem network increase with increase of the channel error, and the performance of the unified fusion rule for parallel network is better than that of tandem network.

TABLE I

DECISION COSTS FOR UNIFIED FUSION RULES WITH DIFFERENT CHANNEL ERRORS

UFR	Channel error			
	P=0	P=0.1	P=0.3	P=0.5
P_e for PN (three sensors)	0.2392	0.2429	0.2575	0.2620
P_e for TN (three sensors)	0.2487	0.2578	0.2647	0.2665

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