An LMI Approach to Robust Controller Designs of Takagi-Sugeno fuzzy Systems with Parametric Uncertainties

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***Abstract*—This paper proposes a new robust stability condition for Takagi-Sugeno (T-S) fuzzy control system with parametric uncertainties. The condition is represented in the form of linear matrix inequalities (LMIs) and is less conservative than some relaxed quadratic stabilization conditions published recently in the literature by collecting the interrelations of fuzzy subsystems into a set of matrices, not just into a single matrix. Based on the LMI-based conditions derived, one can easily synthesize controllers for state feedback robust stabilization for T-S fuzzy control system with parametric uncertainties. Since only a set of LMIs is involved, the controller design is quite simple and numerically tractable. Finally, the validity of the proposed approach is successfully demonstrated in the the control of simulation table.**

***Index Terms*—T-S fuzzy model, parametric uncertainties, robust stability, linear matrix inequalities.**

I. Introduction

There have been several recent studies concerning the stability and the synthesis of controllers and observers for nonlinear systems described by Takagi-Sugeno (T-S) fuzzy models. Based on quadratic Lyapunov function some papers have discussed the feedback control and the state estimation for fuzzy systems [1-6,12-14]. For example in [2], stability conditions for fuzzy control systems are reported to relax the conservatism of the basic conditions by considering the interrelations of every two fuzzy subsystems. To less of conservatism other methods to relaxed quadratic stability conditions are also proposed in [6] and [7], collecting the interrelations of fuzzy subsystems into a single matrix, More recently, in [8] new stability conditions are obtained by relaxing the stability conditions of the previous works, which collect the interrelations into a set of matrices. However, in the above paper, the state feedback controllers with parametric uncertainties are not considered. Consequently the robustness of the closed-loop fuzzy model is not guaranteed. In [3,9-10] an approach to design an robust fuzzy control of uncertain fuzzy models was proposed. Unfortunately, the synthesis conditions are very conservative. In [11] another approach was proposed with less conservativeness, which based on [7].

In this paper news derived stability conditions for robust state feedback stabilization relaxing the conservatism of the conditions obtained by the previous works (such as [11] and references therein) are given. The proposed results are based on the work of [8] developed in the case of state feedback control.

Motivated by the aforementioned works, sufficient conditions are derived for robust asymptotic state feedback robust stabilization using quadratic Lyapunov function. These stability conditions are solved using optimization techniques. A T-S model of simulation table is used to show the effectiveness of the derived conditions.

The following notations are used in this paper. $P > 0$ means P is a real symmetric positive definite matrix. A^T denotes transpose of matrix A. $\Re^{n\times m}$ denotes the set of real matrices of dimensions $n \times m$. The symbol $*$ denotes the transpose elements in the symmetric positions.

II. Preliminaries

In order to consider parametric uncertainties in the T-S fuzzy systems, we propose the continuous-time T-S fuzzy system in which the ith rule is formulated in the following form:

Continuous-time T-S fuzzy model:

Plant Rule $i = 1, 2, \cdots, q$:

IF $x_1(t)$ is Γ_1^i and $\cdots$ and $x_n(t)$ is Γ_n^i THEN

$$\dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \tag{1}$$

where Γ_j^i is a fuzzy set, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input vector, $A_i \in \Re^{n\times n}$ and $B_i \in \Re^{n\times m}$ are system matrix and input matrix, respectively, ΔA_i and ΔB_i are time-varying matrices with appropriate dimensions, which represent parametric uncertainties in the plant model, and q is the number of rules of this T-S fuzzy model.

The defuzzified output of the T-S fuzzy system (1) is represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{q} h_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)) \tag{2}$$

where

$$h_i(x(t)) = \frac{\omega_i(x(t))}{\sum\limits_{i=1}^{q}\omega_i(x(t))}$$
$$\omega_i(x(t)) = \prod_{j=1}^{n}\Gamma_j^i(x_j(t))$$

in which $\Gamma_j^i(x(t))$ is the grade of membership of $x_j(t)$ in Γ_j^i. Some basic propertied of $\omega_i(x(t))$ are

$$\begin{aligned}&\omega_i(x(t)) \geq 0\\ &\sum_{i=1}^{q}\omega_i(x(t)) > 0, i = 1, 2, \cdots, q.\end{aligned} \tag{3}$$

It is clear that

$$\begin{aligned}&h_i(x(t)) \geq 0\\ &\sum_{i=1}^{q}h_i(x(t)) = 1, i = 1, 2, \cdots, q.\end{aligned}$$

Next, a fuzzy model of a state-feedback controller for the continuous-time T-S fuzzy model is formulated as follows:

Controller Rule $i = 1, 2, \cdots, q$:

IF $x_1(t)$ is Γ_j^1 and $\cdots$ and $x_n(t)$ is Γ_j^n THEN

$$u(t) = -K_i x(t), \tag{4}$$

where $K_i \in \Re^{m\times n}$ are constant control gains to be determined.

Since the plant rules (1) have time-varying uncertain matrices, it is not easy to design the controller gain matrices. In order to find these gain matrices K_i , the uncertain matrices should be removed under some reasonable assumptions. Hereforth we assume, as usual, that the uncertain matrices ΔA_i and ΔB_i are admissibly norm-bounded and structured.

Assumption 1: The parameter uncertainties considered here are norm-bounded, in the form

$$\begin{bmatrix}\Delta A_i & \Delta B_i\end{bmatrix} = D_i F_i(t)\begin{bmatrix}E_{1i} & E_{2i}\end{bmatrix}$$

where D_i, E_{Ai} and E_{Bi} are known real constant matrices of appropriate dimensions, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i(t)^T F_i(t) \leq I$, in which I is the identity matrix of appropriate dimension.

III. Robust Stabilization of the T-S fuzzy model

Lemma 1: Given constant matrices D and E and a symmetric constant matrix S of appropriate dimensions, the following inequality holds:

$$S + DFE + E^T F^T D^T < 0$$

where F satisfies $F^T F \leq R$, if and only if for some $\epsilon > 0$.

$$S + \begin{bmatrix}\epsilon^{-1}E^T & \epsilon D\end{bmatrix}\begin{bmatrix}R & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\epsilon^{-1}E\\ \epsilon D^T\end{bmatrix} < 0.$$

Consider a continuous-time T-S fuzzy model (2), the objective is to design a T-S fuzzy-model-based state-feedback controller for robust stabilization of the system (2) in the form

$$u(t) = -\frac{\sum\limits_{i=1}^{q}\omega_i(x(t))K_i(t)x(t)}{\sum\limits_{i=1}^{q}\omega_i(x(t))} = -\sum_{i=1}^{q}h_i(x(t))K_i x(t). \tag{5}$$

The closed-loop system of (2) and (5) is found in (6).

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{q}\sum_{j=1}^{q}h_i(x(t))h_j(z(t))(A_i + \Delta A_i\\ &- (B_i + \Delta B_i)K_j)x(t)\\ &= (\sum_{i=1}^{q}h_i(x(t))\sum_{i=1}^{q}\sum_{j=1}^{q}h_i(x(t))h_j(x(t))\\ &\times (A_i + \Delta A_i - (B_i + \Delta B_i)K_j)x(t)\end{aligned} \tag{6}$$

The main result on the global asymptotic stability of the continuous-time T-S fuzzy model with parametric uncertainties is summarized in the following theorem.

Theorem 1: The continuous-time T-S fuzzy system (2) is asymptotically stabilizable via the T-S fuzzy model-based state-feedback controller (5),if there exist matrices $Q > 0$; N_i, $i = 1, 2, \cdots, q$; Υ_{iii}, $i = 1, 2, \cdots, q$; $\Upsilon_{jii} = \Upsilon_{iij}^T$ and Υ_{iji}, $i = 1, 2, \cdots, q$, $j \neq i$, $j = 1, 2, \cdots, q$ and $\Upsilon_{ijl} = \Upsilon_{lji}^T$, $\Upsilon_{ilj} = \Upsilon_{jli}^T$, $\Upsilon_{jil} = \Upsilon_{lij}^T$, $i = 1, 2, \cdots, q-2$, $j = i+1, \cdots, q-1$, $l = j+1, \cdots, q$ and some scalars $\epsilon_{ijl} > 0, i, j, l = 1, \cdots, q$ such that the following LMIs are satisfied:

$$\begin{bmatrix}\Upsilon_{1i1} & \Upsilon_{1i2} & \cdots & \Upsilon_{1iq}\\ \Upsilon_{2i1} & \Upsilon_{2i2} & \cdots & \Upsilon_{2iq}\\ \vdots & \vdots & \ddots & \vdots\\ \Upsilon_{qi1} & \Upsilon_{qi2} & \cdots & \Upsilon_{qiq}\end{bmatrix} > 0, \tag{7}$$
$$i = 1, 2, \cdots, q$$

$$\begin{bmatrix}\Phi_{iii} & * & *\\ E_{1i}Q - E_{2i}N_i & -\epsilon_{iii}I & *\\ \epsilon_{iii}D_i^T & 0 & -\epsilon_{iii}I\end{bmatrix} < 0 \tag{8}$$
$$i = 1, 2, \cdots, q$$

Because of Scarcity of space, LMI (9) and LMI (10) are listed in the top of next page.

Proof: Consider the Lyapunov function candidate

$$V(x(t)) = x(t)^T P x(t) \tag{11}$$

where P is a time-invariant, symmetric and positive definite matrix. $V(x(t))$ is positive definite and radially unbounded. The time derivative of $V(x(t))$ is

$$\dot{V}(x(t)) = \dot{x}(t)^T P x(t) + x(t)^T P \dot{x}(t) \tag{12}$$

The equilibrium of (2) is quadratically stable if

$$\begin{aligned}\dot{V}(x(t)) &= x(t)^T\{\sum_{i=1}^{q}\sum_{j=1}^{q}h_i h_j(A_i^* - B_i^* K_j)^T P\\ &+P\sum_{i=1}^{q}\sum_{j=1}^{q}h_i h_j(A_i^* - B_i^* K_j)\}x(t) < 0, \forall x(t) \neq 0.\end{aligned}$$

that is

$$\sum_{i=1}^{q}\sum_{j=1}^{q}h_i h_j\{(A_i^* - B_i^* K_j)^T P + P(A_i^* - B_i^* K_j)\} < 0. \tag{13}$$

where $A_i^* = A_i + \Delta A_i, B_i^* = B_i + \Delta B_i$

$$\begin{bmatrix} \Psi_{iij} & * & * & * & * & * \\ E_{1i}Q - E_{2i}N_i & -\epsilon_{iij}I & * & * & * & * \\ E_{1i}Q - E_{2i}N_j & 0 & -\epsilon_{iji}I & * & * & * \\ E_{1j}Q - E_{2j}N_i & 0 & 0 & -\epsilon_{jii}I & * & * \\ (\epsilon_{iij} + \epsilon_{iji})D_i^T & 0 & 0 & 0 & -(\epsilon_{iij} + \epsilon_{iji})I & * \\ \epsilon_{jii}D_j^T & 0 & 0 & 0 & 0 & -\epsilon_{jii}I \end{bmatrix} < 0 \tag{9}$$

$$i = 1, 2, \cdots, q, j = 1, 2, \cdots, q, j \neq i.$$

$$\begin{bmatrix} \mathrm{T}_{ijl} & * & * & * & * & * & * & * & * & * \\ E_{1i}Q - E_{2i}N_j & -\epsilon_{ijl}I & * & * & * & * & * & * & * & * \\ E_{1i}Q - E_{2i}N_l & 0 & -\epsilon_{ilj}I & * & * & * & * & * & * & * \\ E_{1j}Q - E_{2j}N_i & 0 & 0 & -\epsilon_{jil}I & * & * & * & * & * & * \\ E_{1j}Q - E_{2j}N_l & 0 & 0 & 0 & -\epsilon_{jli}I & * & * & * & * & * \\ E_{1l}Q - E_{2l}N_i & 0 & 0 & 0 & 0 & -\epsilon_{lij}I & * & * & * & * \\ E_{1l}Q - E_{2l}N_j & 0 & 0 & 0 & 0 & 0 & -\epsilon_{lji}I & * & * & * \\ (\epsilon_{ijl} + \epsilon_{ilj})D_i^T & 0 & 0 & 0 & 0 & 0 & 0 & -(\epsilon_{ijl} + \epsilon_{ilj})I & * & * \\ (\epsilon_{jil} + \epsilon_{jli})D_j^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\epsilon_{jil} + \epsilon_{jli})I & * \\ (\epsilon_{lij} + \epsilon_{lji})D_l^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\epsilon_{lij} + \epsilon_{lji})I \end{bmatrix} < 0 \tag{10}$$

$$i = 1, 2, \cdots, q-2, j = i+1, \cdots, q-1, l = j+1, \cdots, q.$$

where

$$\begin{aligned} \Phi_{iii} = {} & QA_i^T + A_iQ - N_i^T B_i^T - B_iN_i + \Upsilon_{iii} \\ \Psi_{iij} = {} & 2QA_i^T + QA_j^T + 2A_iQ + A_jQ - (N_i + N_j)^T B_i^T - N_i^T B_j^T - B_i(N_i + N_j) - B_jN_i + \Upsilon_{iij} + \Upsilon_{iji} + \Upsilon_{iij}^T \\ \mathrm{T}_{ijl} = {} & 2Q(A_i + A_j + A_l)^T - (N_j + N_l)^T B_i^T - (N_i + N_l)^T B_j^T - (N_i + N_j)^T B_l^T \\ & +2(A_i + A_j + A_l)Q - B_i(N_j + N_l) - B_j(N_i + N_l) - B_l(N_i + N_j) \\ & +\Upsilon_{ijl} + \Upsilon_{ilj} + \Upsilon_{jil} + \Upsilon_{ijl}^T + \Upsilon_{ilj}^T + \Upsilon_{jil}^T \end{aligned}$$

with the fuzzy local state feedback gains are $K_i = N_iQ^{-1}$, $i = 1, 2 \cdots, q$.

Let $Q = P^{-1}$, $N_i = K_iP^{-1}$. Premultiply and postmultiply (13) by Q, we get

$$\Lambda_c = \sum_{i=1}^{q} h_i^3 \Lambda_1 + \sum_{i=1}^{q} \sum_{\substack{j=1 \\ j \neq i}}^{q} h_i h_j \Lambda_2 + \sum_{i=1}^{q-2} \sum_{j=i+1}^{q-1} \sum_{l=j+1}^{q} h_i h_j h_l \Lambda_3$$

$$\begin{aligned} \Lambda_1 = {} & (QA_i^T + A_iQ - N_i^T B_i^T - B_iN_i) \\ & +(Q\Delta A_i + \Delta A_iQ - N_i^T \Delta B_i^T - \Delta B_iN_i) \end{aligned}$$

$$\begin{aligned} \Lambda_2 = {} & (2QA_i^T + QA_j^T + 2A_iQ + A_jQ - (N_i + N_j)^T B_i^T \\ & -N_i^T B_j^T - B_i(N_i + N_j) - B_jN_i) \\ & +(2Q\Delta A_i^T + Q\Delta A_j^T + 2\Delta A_iQ + \Delta A_jQ \\ & -(N_i + N_j)^T \Delta B_i^T - N_i^T \Delta B_j^T - \Delta B_i(N_i + N_j) \\ & -\Delta B_jN_i) \end{aligned}$$

$$\begin{aligned} \Lambda_3 = {} & (2Q(A_i + A_j + A_l)^T - (N_j + N_l)^T B_i^T \\ & -(N_i + N_l)^T B_j^T - (N_i + N_j)^T B_l^T \\ & +2(A_i + A_j + A_l)Q - B_i(N_j + N_l) \\ & -B_j(N_i + N_l) - B_l(N_i + N_j)) \\ & (2Q(\Delta A_i + \Delta A_j + \Delta A_l)^T - (N_j + N_l)^T \Delta B_i^T \\ & -(N_i + N_l)^T \Delta B_j^T - (N_i + N_j)^T \Delta B_l^T \\ & +2(\Delta A_i + \Delta A_j + \Delta A_l)Q - \Delta B_i(N_j + N_l) \\ & -\Delta B_j(N_i + N_l) - \Delta B_l(N_i + N_j)) \end{aligned}$$

If (14)-(16)are feasible, that is

$$\begin{aligned} & QA_i^T + A_iQ - N_i^T B_i^T - B_iN_i + Q\Delta A_i^T \\ & +\Delta A_iQ - N_i^T \Delta B_i^T - \Delta B_iN_i < -\Upsilon_{iii} \end{aligned} \tag{14}$$

$$\begin{aligned} & 2QA_i^T + QA_j^T + 2A_iQ + A_jQ - (N_i + N_j)^T B_i^T \\ & -N_i^T B_j^T - B_i(N_i + N_j) - B_jN_i + 2Q\Delta A_i^T \\ & +Q\Delta A_j^T + 2\Delta A_iQ + \Delta A_jQ - (N_i + N_j)^T \Delta B_i^T \\ & -N_i^T \Delta B_j^T - \Delta B_i(N_i + N_j) - \Delta B_jN_i \\ & < -\Upsilon_{iij} - \Upsilon_{iji} - \Upsilon_{iij}^T \end{aligned} \tag{15}$$

$$\begin{aligned} & 2Q(A_i + A_j + A_l)^T - (N_j + N_l)^T B_i^T \\ & -(N_i + N_l)^T B_j^T - (N_i + N_j)^T B_l^T) \\ & +2(A_i + A_j + A_l)Q - B_i(N_j + N_l) \\ & -B_j(N_i + N_l) - B_l(N_i + N_j) \\ & +2Q(\Delta A_i + \Delta A_j + \Delta A_l)^T - (N_j + N_l)^T \Delta B_i^T \\ & -(N_i + N_l)^T \Delta B_j^T - (N_i + N_j)^T \Delta B_l^T \\ & +2(\Delta A_i + \Delta A_j + \Delta A_l)Q - \Delta B_i(N_j + N_l) \\ & -\Delta B_j(N_i + N_l) - \Delta B_l(N_i + N_j) \\ & < -\Upsilon_{ijl} - \Upsilon_{ilj} - \Upsilon_{jil} - \Upsilon_{ijl}^T - \Upsilon_{ilj}^T - \Upsilon_{jil}^T \end{aligned} \tag{16}$$

so (17) hold.

$$\begin{aligned}\Lambda_c < & -\sum_{i=1}^{q} h_i^3 \Upsilon_{iii} - \sum_{i=1}^{q}\sum_{\substack{j=1\\ j\neq i}}^{q} h_i^2 h_j (\Upsilon_{iij} + \Upsilon_{iji} + \Upsilon_{iij}^T) \\ & -\sum_{i=1}^{q-2}\sum_{j=i+1}^{q-1}\sum_{l=j+1}^{q} h_i h_j h_l (\Upsilon_{ijl} + \Upsilon_{ilj} + \Upsilon_{jil} + \Upsilon_{ijl}^T \\ & +\Upsilon_{ilj}^T + \Upsilon_{jil}^T) \\ = & -Z^T \left(\sum_{i=1}^{r} h_i \begin{bmatrix} \Upsilon_{1i1} & \Upsilon_{1i2} & \cdots & \Upsilon_{1iq} \\ \Upsilon_{2i1} & \Upsilon_{2i2} & \cdots & \Upsilon_{2iq} \\ \vdots & \vdots & \ddots & \vdots \\ \Upsilon_{qi1} & \Upsilon_{qi2} & \cdots & \Upsilon_{qiq} \end{bmatrix} \right) \mathrm{Z}\end{aligned} \tag{17}$$

where $Z = [\ h_1 I \quad h_2 I \quad \cdots \quad h_q I\ ]^T$

Thus if (7) holds, then $\Lambda_c < 0$. That is system (2) is asymptotically stable about its zero equilibrium via the fuzzy controller (5).

Then, using Assumption 1, (14) can be represented as

$$\begin{aligned}&\Phi_{iii} + D_i F_i(t)(E_{1i}Q - E_{2i}N_i) \\ &+ (E_{1i}Q - E_{2i}N_i)^T F_i(t)^T D_i^T < 0\end{aligned} \tag{18}$$

where

$$\Phi_{iii} = QA_i^T + A_iQ - N_i^T B_i^T - B_i N_i + \Upsilon_{iii}$$

Using Lemma 1 repeatedly, the matrix inequality (18) holds for all $F_i(t)$ satisfying

$$F_i(t)^T F_i(t) \leq I$$

if and only if there exists a constant $\epsilon_{iii}^{1/2} > 0$ such that

$$\begin{aligned}& \Phi_{iii} + \begin{bmatrix} \epsilon_{iii}^{-1/2}(E_{1i}Q - E_{2i}N_i)^T & \varepsilon_{iii}^{1/2} D_i \end{bmatrix} \\ & \times \begin{bmatrix} \varepsilon_{iii}^{-1/2}(E_{1i}Q - E_{2i}N_i) \\ \varepsilon_{iii}^{1/2} D_i^T \end{bmatrix} \\ = & \ \Phi_{iii} + \begin{bmatrix} (E_{1i}Q - E_{2i}N_i)^T & D_i \end{bmatrix} \\ & \times \begin{bmatrix} \varepsilon_{iii}^{-1} I & 0 \\ 0 & \varepsilon_{iii} I \end{bmatrix} \times \begin{bmatrix} (E_{1i}Q - E_{2i}N_i) \\ D_i^T \end{bmatrix} \\ < & \ 0\end{aligned} \tag{19}$$

Applying Schur complement to (19), we get

$$\begin{bmatrix} \Phi_{iii} & * & * \\ E_{1i}Q - E_{2i}N_i & -\varepsilon_{iii} I & * \\ D_i^T & 0 & -\varepsilon_{iii}^{-1} I \end{bmatrix} < 0$$

which is equivalent with inequality (8).

In the similar procedures, we can obtain The LMIs (9) and (10) from (15) and (16), respectively. This completes the proof of Theorem 1.

IV. Example of Simulation Table

Consider a T-S fuzzy system of simulation table defined by the following rules:

Plant Rule 1: If $x_3(t)$ is Positive,

Then $\dot{x}(t) = (A_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u(t)$.

Plant Rule 2: If $x_3(t)$ is Zero,

Then $\dot{x}(t) = (A_2 + \Delta A_2)x(t) + (B_2 + \Delta B_2)u(t)$.

Plant Rule 3: If $x_3(t)$ is Negative,

Then $\dot{x}(t) = (A_3 + \Delta A_3)x(t) + (B_3 + \Delta B_3)u(t)$,

where $x_3(t)$ represent the angular velocity of rotor, which adopt triangular degree of membership.

According to the Brush-less Dc Motor selected,we get the parameters as follows:

$$A_1 = \begin{bmatrix} -130 & 83 & 362.74 & 0 \\ -83 & -130 & 0 & 0 \\ 1.09 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -130 & 0 & -362.74 & 0 \\ 0 & -130 & 0 & 0 \\ 1.09 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -130 & 83 & 362.74 & 0 \\ 83 & -130 & 0 & 0 \\ 1.09 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_1 = B_2 = B_3 = \begin{bmatrix} 5.44 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The norms of parameter uncertainties are chosen as follows:

$$D_1 = D_2 = D_3 = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}$$

$$E_{A1} = E_{A2} = E_{A3} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$$E_{B1} = E_{B2} = E_{B3} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By Theorem 1, we solve LMIs (7)-(10) and give the main solution matrices as follows:

$$Q = \begin{bmatrix} 3.3680 & 0.0000 & -0.1792 & 0.0101 \\ 0.0000 & 3.8249 & -0.0000 & -0.0000 \\ -0.1792 & -0.0000 & 0.1499 & -0.2562 \\ 0.0101 & -0.0000 & -0.2562 & 0.4995 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} -50.9291 & 6.9394 & -5.1049 & 16.7879 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} -51.0065 & -0.0000 & -5.0988 & 16.7910 \end{bmatrix}$$

$$N_3 = \begin{bmatrix} -50.9291 & -6.9394 & -5.1049 & 16.7879 \end{bmatrix}$$

So the feedback gains are

$$K_1 = \begin{bmatrix} -10.5487 & 1.8142 & 90.4730 & 80.2410 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -10.5521 & -0.0000 & 90.8520 & 80.4416 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -10.5487 & -1.8142 & 90.4730 & 80.2410 \end{bmatrix}$$

We can then determine the fuzzy controller (5) according to these feedback gains and h_1, h_2, h_3.

V. CONCLUSION

This paper presents new stability conditions for robust asymptotic stabilization of Nonlinear Systems with Parametric Uncertainties. The derived results are based on the use of the quadratic Lyapunov function and solved using LMI optimization techniques. The proposed stability conditions are less conservative than previous results and include previous ones as special cases. The approach used in this paper can be also applied to solve the fuzzy robust static output feedback satbilization problems.

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