Theory of Grey Systems and its Application in Electric Load Forecasting

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***Abstract*—In this paper, the basic concept of the grey system theory and the modeling of the grey system are introduced briefly. Based on the analysis of the characteristics of GM(1,1) model, Unbiased GM(1,1) model is proposed here. The comparison of GM(1,1) model and unbiased GM(1,1) model is given, the results of comparison show that Unbiased GM(1,1) model is better than GM(1,1) model. In the final part of the paper, Unbiased GM(1,1) model is used to do electric load forecasting.**

***Keywords*—grey system, GM(1,1) model, Unbiased GM(1,1) model, load forecasting**

I. INTRODUCTION

The grey system theory was initially presented by Julong Deng, a Chinese scholar in 1982[1] [2]. Since then, it has been widely employed in many areas such as agriculture, electric power, IT, transportation, economics, management, etc [3] [4].

The name of grey systems is chosen based on the amount of known information. For example, a "black box" stands for an object such that its internal structure is totally unknown to the investigator. Here, the word "black" represents unknown information, "white" for completely known information, and "grey" for the information which are partially known and partially unknown. Accordingly, systems with completely known information are called as white systems, systems with completely unknown information as black systems, and the systems with partially known and partially unknown information as grey systems, respectively

In practical situations, we often face with incomplete information. There are four possibilities for incomplete information: the information of elements (or parameters) is incomplete; the structural information is incomplete; the boundary information is incomplete; and the behavior information of motion is incomplete.

"Incomplete information" is the fundamental meaning of being "grey". Through over 20 years of studies, the theory of grey systems has been recognized as a powerful tool for both qualitative and quantitative systems analysis [5] [6].

II. GREY SYSTEMS ANALYSIS AND MODELING

A. Grey systems analysis

In the modern history of science, the concepts and theories of differentiation and integration have helped to bring forward many important and magnificent progresses in various scientific fields. To make grey concepts work, one will need to generalize the traditional calculus to discrete data. Let $X^{(0)}$ = ($x^{(0)}(1)$, $x^{(0)}(2)$, … , $x^{(0)}(n)$), according to grey system theory, the integral of the original series $X^{(0)}$ can be defined as

$$x^{(1)}(k)=\sum_{i=0}^{k}x^{(0)}(i)$$

A new time series $X^{(1)}$ = ($x^{(1)}(1)$, $x^{(1)}(2)$, … $x^{(1)}(n)$) can be produced by using the formula.

To avoid confusion with the integral of continuous function, the operator D : $X^{(0)} \rightarrow X^{(1)}$ is called the accumulating operator, therefore $X^{(1)}$ is called the first-order generation by accumulation. The r-th order generation by accumulation of $X^{(0)}$, for r>1, is the sequence,

D^r ($X^{(0)}$)= $X^{(r)}$ = ($x^{(r)}(1), x^{(r)}(2), \ldots x^{(r)}(n)$),

where

$$x^{(r)}(i)=\sum_{k=1}^{i}x^{(r-1)}(k).$$

To generalize the concept of differentiation of calculus to the case of discrete data sequences, for a fixed whole number r, let D^r ($X^{(0)}$) = $X^{(r)}$. The concept of derivative of the sequence

$X^{(r)}$ = ($x^{(r)}(1)$, $x^{(r-1)}(2)$, … $x^{(r)}(n)$) can be defined as follows:

$$\frac{d}{dt}X^{(r)}(k)=\lim_{time\ \ unit\ \rightarrow \min}\frac{X^{(r)}(k)-X^{(r)}(k-1)}{time\ \ unit}$$

$$= \frac{\sum_{i=1}^{k} x^{(r-1)}(i) - \sum_{i=1}^{k-1} x^{(r-1)}(i)}{1}$$

$$= x^{(r-1)}(k)$$

That is, the derivative of $X^{(r)}$ with respect to time is $X^{(r-1)}$ and each ordinary differential equation f ($y^{(n)}, y^{(n-1)}$, … , y', y)=0 can be rewritten for discrete data sequence $X^{(0)}$ as follows:

$$f\ (x^{(0)}(k),\ x^{(1)}(k), \ldots x^{(n-1)}(k), x^{(n)}(k))=0$$

B. Grey systems modeling

In general, Systems modeling goes through the following steps.

(1) Develop a language model of thoughts and concepts.

(2) Examine the factors and the relationship among the factors contained in the language model.

(3) Analyze quantitatively each of the causal relations.

(4) Collect input and output data values for each causal relation to establish a systems model.

(5) Systematically study the dynamic model obtained by Step 4.

In terms of grey systems, a set of unique quantitative method of analysis has been introduced.

Let $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})$ be a sequence of data values with 1, 2, 3, …, n representing time moments. Then, the concept of (grey) derivative of the sequence $D\ (X^{(0)}) = X^{(1)} = (x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)})$ can be defined as

$$d(k) = \lim_{time\ unit \to \min} \frac{X_k^{(1)} - X_{k-1}^{(1)}}{time\ unit} = \frac{\sum_{i=1}^{k} x_i^{(0)} - \sum_{i=1}^{k-1} x_i^{(0)}}{1} = x_k^{(0)}$$

So, the differential equation

$$\frac{dx}{dt} + ax = b$$

of the simplest and the most useful form can be generalized to discrete sequences of historical data as follows:

$$d(k) + ax_k^{(1)} = b \text{ or } x_k^{(0)} + ax_k^{(1)} = b \qquad (1)$$

where a and b are constants.

Now, we know that for a differentiable function $x(t)$,

$$\left| \frac{dx(t)}{dt} - x(t+\Delta t) \right| \approx \left| \frac{dx(t)}{dt} - x(t) \right| \qquad (2)$$

So, equation (1) is modified as follows:

$$x_k^{(0)} + az_k^{(1)} = b \qquad (3)$$

With $z_k^{(1)} = 0.5x_k^{(1)} + 0.5x_{k-1}^{(1)}$

Assume that the sequence $X^{(0)}$ as mentioned earlier is non-negative. If $\hat{a} = [a\ b]^T$ is a sequence of parameters, and

$$Y = \begin{pmatrix} x_2^{(0)} \\ x_3^{(0)} \\ \vdots \\ x_n^{(0)} \end{pmatrix}, \quad B = \begin{pmatrix} -z_2^{(1)} & 1 \\ -z_3^{(1)} & 1 \\ \vdots & \vdots \\ -z_n^{(1)} & 1 \end{pmatrix}$$

then the least-square estimate sequence of the grey differential equation (3) satisfies

$$\hat{a} = [B^T B]^{-1} B^T Y$$

The solution of the grey differential equation

$$x_k^{(0)} + az_k^{(1)} = b$$

is given by

$$\hat{x}_{k+1}^{(1)} = [x_1^{(1)} - \frac{b}{a}]e^{-ak} + \frac{b}{a}, k = 1, 2, \ldots, n$$

The restored values of $x_k^{(1)}s$ are given by

$$\hat{x}_{k+1}^{(0)} = \hat{x}_{k+1}^{(1)} - \hat{x}_k^{(1)}, k = 1, 2, \ldots, n.$$

The above model developed for the sequences of data is called GM(1,1) model, it is the most important model in the theory of grey systems[7]. Many applications are based on it, so it is focused a lot of attention. The more complicated models are GM(m,n) models. For more details, please see relevant references [1-4].

III. UNBIASED GM(1,1) MODEL

A. The characteristics of GM(1,1) model

The first author of the paper has conducted the research to the characteristics of GM(1,1) model, proved GM(1,1) model is a biased exponential model[8].

Assume an exponential system $x(t) = Ae^{at}$ is written as a discrete manner:

$$x^{(0)}(k) = Ae^{a(k-1)}, k = 1, 2, \ldots, N, \ldots \qquad (4)$$

Take the first N items as raw data sequence, the first-order generation by accumulation is:

$$x^{(1)}(k) = A\frac{1-e^{ak}}{1-e^{a}}, k = 1, 2, \ldots, N \qquad (5)$$

Set up GM(1,1) model ,then

$$B=\begin{bmatrix} -\frac{1}{2}A\frac{2-e^{a}-e^{2a}}{1-e^{a}} & 1 \\ -\frac{1}{2}A\frac{2-e^{2a}-e^{3a}}{1-e^{a}} & 1 \\ \dots & \dots \\ -\frac{1}{2}A\frac{2-e^{(N-1)a}-e^{Na}}{1-e^{a}} & 1 \end{bmatrix},$$

$$Y_N=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(N) \end{bmatrix}$$

Therefore

$$\begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix}=\left(B^T B\right)^{-1} B^T Y_N=\begin{bmatrix} \frac{2\left(1-e^{a}\right)}{1+e^{a}} \\ \frac{2A}{1+e^{a}} \end{bmatrix} \quad (6)$$

Form those results, we can get

$$\hat{x}^{(1)}(k+1)=\frac{-Ae^{a}}{1-e^{a}}e^{-\hat{a}k}+\frac{A}{1-e^{a}},k=1,2,...,N-1 \quad (7)$$

Finally, the fitting results are

$$\hat{x}^{(0)}(1)=A \quad (8)$$

$$\hat{x}^{(0)}(k)=\frac{-Ae^{a}\left(1-e^{\hat{a}}\right)}{1-e^{a}}e^{-\hat{a}(k-1)},k=2,3,...,N \quad (9)$$

Assume $a'=-\hat{a},K=\frac{-e^{a}\left(1-e^{a}\right)}{1-e^{a}}$, then the formula (9) can be written as

$$\hat{x}^{(0)}(k)=KAe^{a'(k-1)},k=2,3,...,N \quad (10)$$

The formula (10) combined with formula (8) is obviously different from the formula (4), so GM(1,1) model is a biased exponential model .

B. Unbiased GM(1,1) model

From the formula (6), we can obtain $a=\ln\frac{2-\hat{a}}{2+\hat{a}},A=\frac{2\hat{u}}{2+\hat{a}}$, that is, using the parameter $\hat{a}$ and $\hat{u}$ of GM(1,1) model ,we can express the parameters a and A for the raw data sequence. Based on it, we propose a unbiased GM(1,1) model. The modeling steps of Unbiased GM(1,1) model are as follows:

1) Give $x^{(1)}$ by applying first-order accumulating generation operator (1-AGO) on $x^{(0)}$

$$x^{(1)}(k)=\sum_{m=1}^{k}x^{(0)}(m),k=1,2,...,N$$

2) Ascertain data matrixes B , Y_N

$$B=\begin{pmatrix} -\frac{1}{2}\left[x^{(1)}(1)+x^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(2)+x^{(1)}(3)\right] & 1 \\ \vdots & \vdots \\ \frac{1}{2}\left[x^{(1)}(N-1)+x^{(1)}(N)\right] & 1 \end{pmatrix},$$

$$Y_N=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(N) \end{bmatrix}$$

3) Solve the parameter column

$$\left[\hat{a} \quad \hat{u}\right]^T=\left(B^T B\right)^{-1}B^T Y_N$$

4) Get the parameters of Unbiased GM(1,1) model

$$a'=\ln\frac{2-\hat{a}}{2+\hat{a}},A'=\frac{2\hat{u}}{2+\hat{a}}$$

5) Establish the model of raw data sequence

$$\hat{x}^{(0)}(1)=x^{(0)}(1)$$

$$\hat{x}^{(0)}(k)=A'e^{a'(k-1)},k=2,3,...$$

C. Comparison of GM(1,1) model and Unbiased GM(1,1) model

Example 1

Let a raw data sequence $x^{(0)}=(e^0,e^1,e^2,e^3,e^4)$ = (1.0000, 2.7183, 7.3891, 20.0855, 54.5981), which comes from a system $x=e^t$. GM(1,1) model established is as follows:

$$\hat{x}^{(0)}(1)=1.0000$$

$$\hat{x}^{(0)}(k)=0.9542e^{0.924234(k-1)},k=2,3,...$$

Unbiased GM(1,1) model established is as follows:

$$\hat{x}^{(0)}(1)=1.0000$$

$$\hat{x}^{(0)}(k)=1.0000e^{1.000000(k-1)},k=2,3,...$$

From above results, we can see there exists error by using GM(1,1) model for a strict exponential sequence, and there exists no error by using Unbiased GM(1,1) model for a strict exponential sequence .

Example 2:

Suppose that there exists a disturbed system $x = e^{0.5t} + 0.01(-1)^t$, from which we can get a raw data sequence $x^{(0)}$ = (1.0100, 1.6387, 2.7283, 4.4717, 7.3991). GM(1,1) model established is as follows:

$$\hat{x}^{(0)}(1) = 1.0100$$

$$\hat{x}^{(0)}(k) = 0.9810e^{0.498072(k-1)}, k = 2,3,...$$

Unbiased GM(1,1) model established is as follows:

$$\hat{x}^{(0)}(1) = 1.0100$$

$$\hat{x}^{(0)}(k) = 0.9882e^{0.501101(k-1)}, k = 2,3,...$$

The comparison of the fitting results of the two models is shown in table 1.

Table 1. Comparison of the fitting results of the two models

Actual values	GM(1,1) model		Unbiased GM(1,1) model	
	Fitting results	Relative error (%)	Fitting results	Relative error (%)
1.0100	1.0100	0	1.0100	0
1.6387	1.6028	2.19	1.6311	0.46
2.7283	2.6185	4.02	2.6921	1.33
4.4717	4.2779	4.33	4.4434	0.63
7.3991	6.9890	5.54	7.3340	0.88

From the table1, we know that the precision of Unbiased GM(1,1) model is superior to GM(1,1) model when the raw data is a exponential sequence with disturbance .

From example 1 and example 2, we can know that Unbiased GM(1,1) model is superior to GM(1,1) model, so the better results can be gotten by using Unbiased GM(1,1) model to substitute GM(1,1) model in practical application where the raw data belong to the style of exponential.

IV. APPLICATION OF UNBIAS GM(1,1) MODEL IN ELECTRIC LOAD FORECASTING

GM(1,1) model has been widely used in many areas[9] [10], load forecasting is an important area in which GM(1,1) model is employed [11-14].

The data of whole society power consumption of Shan Tou city of China from year 2000 to 2005 are listed in table 2.The fitting results of GM(1,1) model and Unbiased GM(1,1) model are also listed in table 2.

Table 2 Raw data and the fitting results of the two models (Unit: 10^4kw • h)

year	2000	2001	2002	2003	2004	2005
Power consumption	439064	469428	543399	676846	783312	875959
GM(1,1)	439064	477728	558377	652640	762817	891593
unbiased GM(1,1)	439064	472929	552944	646495	755874	883759

The comparison of fitting accuracies of GM(1,1) model and Unbiased GM(1,1) model is listed in table 3.

Table 3 Comparison of fitting accuracies of the two models

	GM(1,1) model	unbiased GM(1,1) model
MAPE	2.08%	1.89%

From table 3, we can know the fitting results of Unbiased GM(1,1) model are much better than GM(1,1) model, so Unbiased GM(1,1) model is used to do load forecasting.

The prediction results of whole society power consumption of Shan Tou city from year 2006 to 2010 are listed in table 4.

Table 4 The prediction results of whole society power consumption of Shan Tou city from year 2006 to 2010 (Unit: 10^6 kw • h)

year	2006	2007	2008	2009	2010
Power consumption	10333	12081	14125	16515	19309

V. CONCLUSION

In this paper, we briefly introduce the basic concept of the grey system theory and the modeling of the grey system, and present a modified GM(1,1) model. The modified GM(1,1) model is called Unbiased GM(1,1) mode, it is superior to GM(1,1) model and is suitable for load forecasting .Two examples and a practical case of load forecasting are presented to demonstrate the effectiveness of the modified model.

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