Laser Triangulation Displacement Measurement Method Using Prism-Based Optical Structure

ZHANG Haibo,ZHAO Hui,TAO Wei,WANG Zhanbin
Department of Instrument Science & Engineering
Shanghai Jiao Tong University
Shanghai, China
huizhao@sjtu.edu.cn

***Abstract*— This paper presented a novel method using prism-based optical structure to correct the nonlinear problem of Laser Triangulation Displacement Measurement (LTDM). The nonlinearity existed in the common use of LTDM, especially in the wide range measurement. In this paper, a geometric model of the relation between object displacement and image position was built by theoretical derivation. Then two groups of contrast experiments were designed and the results showed the nonlinear problem was significantly improved.**

***Keywords*—displace measurement, trianglation, prism**

I. INTRODUCTION

Laser triangulation method is a common measurement method. It provides a non-contact, nondestructive method and is widely used in measurement of displacement, height, thickness, 3D scanning and so on.[1-4] LTDM is the basic form of laser triangulation method. Fig. 1 shows a typical structure of LTDM. The laser beam is projected to create a reference spot on the surface of an object. The spot is imaged by receiving lens on the optical sensor device. The displacement can be calculated according to the signal of the optical sensor device. Position sensitive detector (PSD), charge coupled device (CCD) are usually used as the optical sensor devices.

The basic equation for the object displacement is [5, 6]

$$Z = \frac{Df}{\Delta p + f\tan(\alpha)} \tag{1}$$

According to imaging formula and Eq. (1), the position change of the spot image with the displacement change, when the object is near from the laser, is much larger than that when the object is far. It means there is a nonlinear problem. As shown in Fig. 1, A is the middle position in the detection range. α is the angle between the laser beam and OA. The bigger α is, the smaller the nonlinearity is. Conversely, the smaller α is, the bigger the nonlinearity is. In the design of displacement measurement device, volume is required to be controlled in a certain range. It means there will be the serious nonlinear problem and enlarge the error of measurement, when α is small, especially in the large range displacement measurement.

To correct the nonlinear problem, this paper presents a novel method using prism-based optical structure, in which the

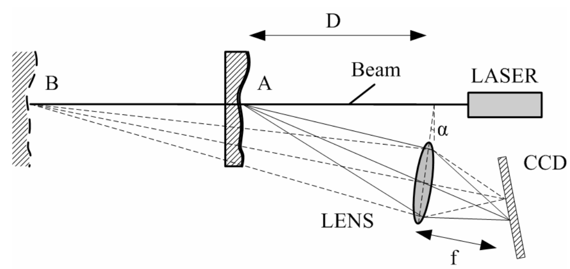

Figure 1. Typical Structure of LTDM

critical change is adding a prism into receiving lens to optimize the image optical path.

This paper is organized as follow. In section I, the nonlinear problem of LTDM and a novel method to solve this problem are introduced. In section II, the method using prism-based optical structure is described and modeled in details. In section III, two contrast experiments using the common method and novel method are carried, and its results are analyzed. In section IV, the conclusion is made.

II. PRISM-BASED OPTICAL STRUCTURE AND GEOMETRIC MODEL

A. *Structure Introduciton*

As shown in Fig. 2, prism-based optical structure has the same principle as the common LTDM. The beam, projected by a diode laser, creates reference spots on the surface of objects like spot A or B. The spots were imaged by prism-based optical structure receiving lens on the sensor device such as linear CCD which is adopted in this paper. The object displacement can be calculated according to the signals of linear CCD.

The critical change of prism-based optical structure is the addition of prism. Using the prism, the imaging rule shown in Fig. 3 will be greatly changed. Adopting proper prism and geometric parameters, the nonlinear problem can be corrected.

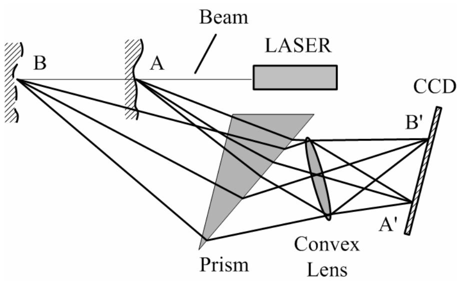

Figure 2. Prism-Based Optical Structure

B. Geometric Model Building

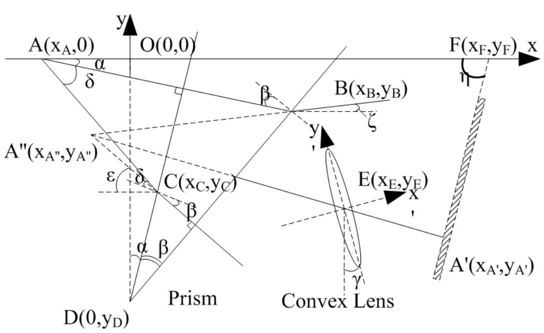

Figure 3. Geometric Relations among Parameters

To get the rule between the object's displacement and the image position, we made a geometric analysis on optical structure. Fig. 3 shows the geometric relations among parameters. We set the projection point, which is projected on laser beam by the prism vertex D (x_D, y_D) as origin of coordinates O $(0,0)$. β is a angle of prism at D. α is the angle between prism's front surface and OD. A (x_A, y_A) is a spot center create by laser beam. A' $(x_{A'}, y_{A'})$ is the image of A on linear CCD. A" $(x_{A''}, y_{A''})$ is the virtual image of A when lights pass though the prism. B (x_B, y_B) is the intersection point between prism's back surface and the ray which is vertical with the front surface. C (x_C, y_C) is the intersection point between prism's front surface and the ray which is vertical with the back surface. E (x_E, y_E) is the center of the convex lens. A linear CCD is place on the imaging plane. F (x_F, y_F) is the intersection between CCD and x axis. L is the distance between F and A".

The relation between B and A is shown as

$$\begin{cases} \dfrac{y_A - y_B}{x_A - x_B} = -\tan\alpha \\ \dfrac{x_B - x_D}{y_B - y_D} = \tan(\alpha+\beta) \end{cases}, \tag{2}$$

where $y_A = 0$, $x_D = 0$.

We get the coordinate of B showed in (3) with (2).

$$\begin{cases} x_B = Ax_A \tan\alpha - B \\ y_B = Ax_A + B\tan\alpha \end{cases} \tag{3}$$

In (3), A and B is defined as

$$A = \frac{\tan\alpha}{1+\tan\alpha\tan(\alpha+\beta)}$$

$$B = \frac{y_D \tan(\alpha+\beta)}{1+\tan\alpha\tan(\alpha+\beta)}$$

The relation between C and A is shown as

$$\begin{cases} \dfrac{y_A - y_C}{x_A - x_C} = -\tan(\alpha+\delta) \\ \dfrac{x_C - x_D}{y_C - y_D} = \tan\alpha \end{cases} \tag{4}$$

where $\sin\delta = n\sin\beta$ and n is refractive index of prism.

Then we also get the

$$\begin{cases} x_C = C\cdot x_A - D \\ y_C = \mathrm{ctan}\,\alpha\cdot C\cdot x_A + \tan(\alpha+\delta)\cdot D \end{cases} \tag{5}$$

In (5), C and D are defined as follows:

$$C = \frac{\tan(\alpha+\delta)}{\tan(\alpha+\delta)+\mathrm{ctan}\,\alpha}$$

$$D = \frac{y_D}{\tan(\alpha+\delta)+\mathrm{ctan}\,\alpha}$$

The relation between A' and A" is shown as

$$\begin{cases} \dfrac{y_{A''} - y_C}{x_{A''} - x_C} = -\tan\varepsilon \\ \dfrac{y_B - y_{A''}}{x_B - x_{A''}} = \tan\zeta \end{cases}, \tag{6}$$

where $\varepsilon = \alpha+\beta$ and $\zeta = \arcsin(n\sin\beta) - \alpha - \beta$.

According to (3), (5) and (6), we get the coordinate of spot image A" showed in Eq. (7).

$$\begin{cases} x_{A''} = Ex_A + F \\ y_{A''} = Gx_A + H \end{cases} \tag{7}$$

where

$$E=\frac{A\tan\alpha\tan\zeta+C\operatorname{ctan}\alpha-A+C\tan\varepsilon}{\tan\zeta+\tan\varepsilon}$$

$$F=\frac{-B(\tan\zeta+\tan\alpha)-D\tan\varepsilon+D\tan(\alpha+\delta)}{\tan\zeta+\tan\varepsilon}$$

$$G=\frac{\operatorname{ctan}\alpha C\tan\zeta+(C-A\tan\alpha)A\tan\varepsilon\tan\zeta\tan\varepsilon}{\tan\zeta+\tan\varepsilon}$$

$$H=\frac{B\tan\alpha\tan\varepsilon+D\tan(\alpha+\delta)\tan\zeta}{\tan\zeta+\tan\varepsilon}-\frac{(D+B)\tan\zeta\tan\varepsilon}{\tan\zeta+\tan\varepsilon}$$

The relation between A' and A" is shown in (8).

$$\begin{cases}\dfrac{y_F-y_{A'}}{x_F-x_{A'}}=\tan\eta\\[2ex]\dfrac{y_E-y_{A'}}{x_E-x_{A'}}=\dfrac{y_E-y_{A''}}{x_E-x_{A''}}\end{cases}\tag{8}$$

According to (7) and (8), we get the coordinate of spot image A" showed in Eq. (7).

$$\begin{cases}x_{A'}=\dfrac{(x_EG-x_FE\tan\eta)x_A-x_F\tan\eta F+x_EH+x_Fx_E\tan\eta}{(G-E\tan\eta)x_A-F\tan\eta+x_E\tan\eta+H-y_E}\\[2ex]y_{A'}=-x_F\tan\eta+\dfrac{(x_EG-x_F\tan\eta E)x_A-x_F\tan\eta F+x_EH+x_Fx_E\tan\eta}{(G-E\tan\eta)x_A-F\tan\eta+x_E\tan\eta+H-y_E}\tan\eta\end{cases}\tag{9}$$

At last, we get the relations between the object's displacement and the image's displacement.

$$L=\frac{(x_{A'}-x_F)}{\cos\eta}=\frac{(x_EG-x_FE\tan\eta)x_A-x_F\tan\eta F+x_EH+x_Fx_E\tan\eta}{(G-E\tan\eta)\cos\eta x_A-F\tan\eta\cos\eta+x_E\tan\eta\cos\eta+H\cos\eta-y_E\cos\eta}-\frac{x_F}{\cos\eta}\tag{10}$$

III. EXPERIMENT

A. Experiment Devices

Two groups of contrast experiments were designed to verify our method. Experiment devices structure was shown in Fig 4. A data acquistion card in a PC controlled a stepper motor and the intensity of a diode laser with 650 nm-wavelength. The object's displacement was changed under the control of motor which was drove by PC. A 5340 element linear CCD was used to detect the image of spot formed by the receiving lens. The PC acquired the signals of the linear CCD by a CCD driver. The parameters of the system were shown is Fig. 3. Fig. 7 is a photo of the practical experiment devices.

In experiment A, we used a single convex lens (f=50 *mm*) as the receiving lens. The lens were placed 30 mm far from the laser beam. In experiment B, prism-based optical structure was used as the receiving lens. In the two experiments, the parameters were the same:: $\alpha=25°$, $\beta=30°$, $\eta=20°$, $y_D=-52mm$, $x_F=250mm$, $x_E=36mm$, $y_E=-37mm$. All of the parameters above were determined by the software simulation and the field needs.

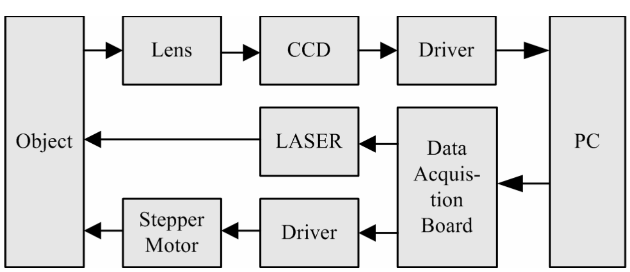

Figure 4. Experiment devices structure

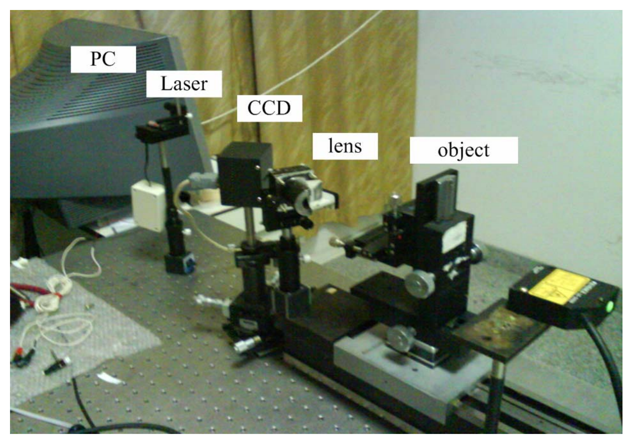

Figure 5. A photo of experiment devices

B. Singnal Processing

It was very important to adopt a proper algorithm to determine the image position, for the accuracy of algorithm is one of key factors which limited the accuracy of displacement measurement. The common methods were centroid, fitting, interpolation and so on. [7] Because of the prism's addition, the image signals' waveform became irregular, and was very difficult to described using a simple function like gauss function. Taken this into account, we chose centroid method for its applications to waveforms of any shape.

$$y = \sum_{i=1}^{N} i \cdot x_i \Big/ \sum_{i=1}^{N} x_i = y_c + \sum_{i=1}^{N} (i - y_c) \cdot x_i \Big/ \sum_{i=1}^{N} x_i \tag{11}$$

$$y = \sum_{i=1}^{N} i \cdot x_i^2 \Big/ \sum_{i=1}^{N} x_i^2 = y_c + \sum_{i=1}^{N} (i - y_c) \cdot x_i^2 \Big/ \sum_{i=1}^{N} x_i^2 \tag{12}$$

Equation (11) and (12) are two kinds of centroid methods. Equation (11) is the basic centroid method, and (12) is called square weighting centroid method. In (11) and (12), the signals of a N element linear CCD are $x_1, x_2, \cdots, x_N$, i is the element position number, y_c is the centroid position. Obviously, the part after y_c in (11) is equal to zero. That means the datum far from y_c has much more influence than the datum near y_c in the basic centroid method. There is a very similar situation in square weighting centroid method. But in our application, data at centroid parts have much bigger amplitudes and less relative errors than the data at edge parts. By square weighting, the effect of data at centroid parts is greatly enlarged. So we chose square weighting centroid method.

C. Experiment Results

As shown in Fig. 3, we measured the range where $x_A \in [-300, -100]$. We set the displacement where $x_A = -100mm$ as 0 *mm* and $x_A = -300mm$ as 200 *mm*. According to (10) and experiment parameters, we got the theoretical results shown in Fig. 6. The results of the experiment A and B are shown in Fig. 7 and Fig. 8. The results showed the relation between the object's displacement and the image position, which were the sensors' sensitivities.

As shown in Fig. 7, the sensitivity was about 58 *pixels/mm* when the object was at 0 *mm*, and only about 6 pixels *pixels/mm* when the object was at 200 mm. The nonlinearity was about 34 percents. In Fig. 8, the sensitivity was about 20 *pixels/mm* when the object was at 0 *mm*, only about 11 *pixels/mm* when the object was at 200 *mm*. The nonlinearity was only about 8 percents. Obviously, the sensitivity rule was changed for the prism's addition. The nonlinear problem, which was serious in the single lens method, was greatly improved in the prism-based method.

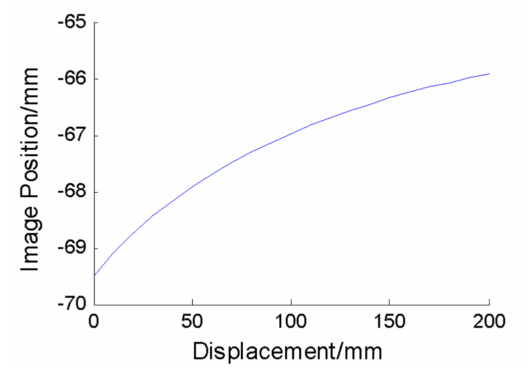

Figure 6. Theoretical results

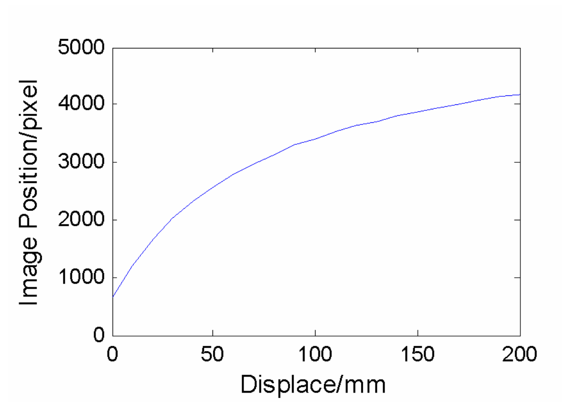

Figure 7. Single lens method's results

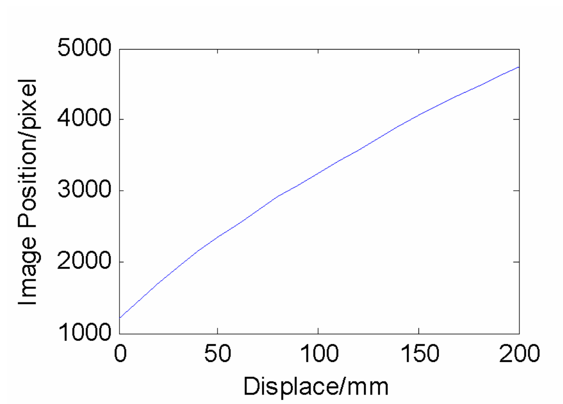

Figure 8. Prism-based method's results

Fig. 9 and Fig. 10 showed the maximum deviation of the single lens and prism-based method. The maximum deviation of single lens method reduced rapidly with the increase of displacement and the maximum deviation of prism method was kept in a stable range. That meant the precision of the prism-based method was more uniform than that of the single lens method in the whole measurement range.

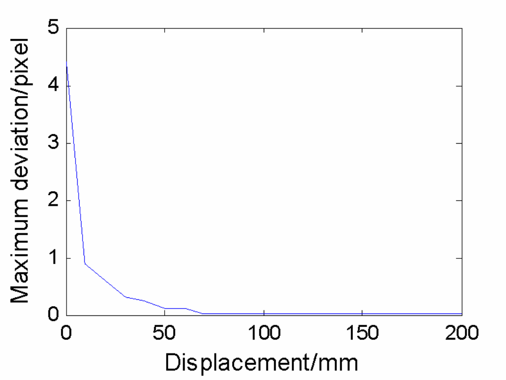

Figure 9. Maximum deviation of single lens method

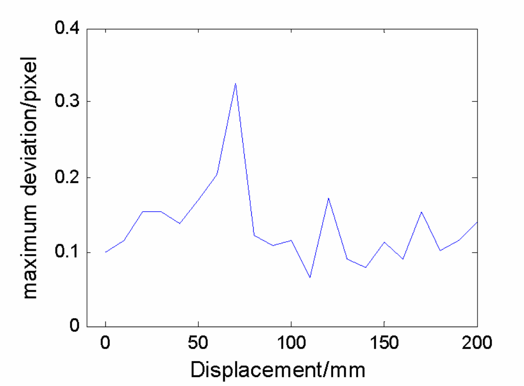

Figure 10. Maximum deviation of prism method

IV. Conclusion

The nonlinear problem exists in common LTDM. The problem may be serious when the measurement range is large. To correct the problem, a prism-based optical structure was presented. In this paper, we got a geometric model after the mathematics analysis. And then, two contrast experiments were carried to prove the novel method. According to the results of experiments, following conclusions could be achieved.

- The novel method could lower the nonlinear problem significantly. The resolution was improved in the whole measurement range.
- The nonlinear problem was not eliminated, but be weaken under the acceptable level in the novel method.
- For the addition of prism, the signals of CCD were changed and not as regular as before. So the signal processing method should also be changed.
- At the same time, the image quality was not as good as single lens method with the prism's addition. It may limit the resolution improvement of system. To get higher quality image, more precise Prism-Based Optical Structure should be designed.

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