Solving TSP Using Lotka-Volterra Neural Networks without Self-Excitatory

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***Abstract*— This paper proposes a new approach to solve Traveling Salesman Problems (TSPs) by using a class of Lotka-Volterra neural networks (LVNN) without self-excitatory. Some stability criteria that ensure the convergence of valid solutions are obtained. It is proved that a class of equilibrium states are stable if and only if they correspond to the valid solutions of the TSPs. That is, one can always obtain a valid solution whenever the network convergence to a stable state. A set of analytical conditions for optimal settings of LVNN is derived. The simulation results illustrate the theoretical analysis.**

I. Introduction

The TSP can be defined as: given a set of N cities, find the shortest path linking all the cities such that all cities are visited exactly once. The TSPs are typical problems of combinatorial optimization. The theoretical and practical insight of TSPs can often be useful in solving other combinatorial optimization problems. In fact, much progress in combinatorial optimization can be traced back to research of TSPs. It has been widely studied in mathematics and artificial intelligence communities.

Since the seminal work of Hopfield and Tank [3], there has been an increased interest in applying Hopfield neural network to TSPs and many attempts have been made to try to solve this problem [4], [5], [6], [7], [8]. Many of them study the parameter settings of Hopfield neural network. Most of the study, however, have limitations such as the fine-tuning of the network coefficients, and invalidity of the obtained solutions and so on.

In this paper, we employ a class of LVNN to solve TSPs. The well-known Lotka-Volterra model, which is a differential nonlinear system describing linear growths and quadratic interactions between variables, was first proposed to describe the predator-prey relationship in an ecosystem, and soon became well known and formed the basis of many important models in mathematical biology and population dynamics. It has also been found with successful and interesting applications in physics, chemistry, economics and other fields. The Lotka-Volterra model of recurrent neural networks [11], which are derived from conventional membrane dynamics of competing neurons, providing a mathematical basis for understanding neural selection mechanisms, has found successful applications in winner-take-all problems [11], [12], [13].

The class of Lotka-Volterra neural networks discussed in this paper are without self-excitatory. We address several important properties such as the networks convergence to a set of specific states, which correspond to the permutation matrix. This property presents Winner-Take-All (WTA) within each row and column, which can be successful applied to TSPs. Some simple conditions are derived to guarantee the stability of this class of LVNN. We get a set of analytical conditions guaranteeing that any equilibrium point of the network characterizes a valid tour for the TSPs. A series of experiments are carried out to verify the theoretical analysis. The simulation results show that the proposed network exhibits good performance in terms of solution quality.

The rest of this paper is organized as follows. Section II presents the architecture and dynamics of the proposed network. In Section III, some important properties of the network and the principles for choosing its parameter value are discussed. The simulation results are given in Section IV. Finally, conclusions are drawn in Section V.

II. Some Preliminaries and TSP Mapping

The TSP is an optimization task that arises in many practical situations. Let n be the number of cities and d_{ij} be the distance between the cities i and j, $i, j \in \{1, 2, \cdots, n\}$, then a valid solution of the TSPs can be presented by a $n \times n$ permutation matrix, where each column and each row is associated respectively to a particular city and order in the tour. Let $H = \{X \in [0,1]^{n\times n}\}$ and $H_C = \{X \in \{0,1\}^{n\times n}\}$ be an unit hypercube of R^{n^2} and its corner set, respectively. Given a fixed $X \in H$, define $S_i \equiv \sum_i x_{i\alpha}$ and $S_\alpha \equiv \sum_\alpha x_{i\alpha}$ as the sum of rows and columns. Then the set of valid tour of TSPs is

$$H_T = \{X \in H_C | S_i = 1, S_\alpha = 1, \forall i, \alpha = 1, \cdots, n\},$$

and the invalid tour set $H - H_T$ can be divided into two sets by

$$H_S = \{X \in H | \exists \alpha, \beta \ and \ i, s.t. \ x_{i\alpha} > 0 \ and \ x_{i\beta} > 0\}.$$

and

$$H_R = \{X \in H | \exists i, j \ and \ \alpha, s.t. \ x_{i\alpha} > 0 \ and \ x_{j\alpha} > 0\}.$$

It is clear that H_T represents the set of valid tour for TSPs and $H - H_T$ represents the set of invalid tour for TSPs. Meanwhile, $H_S \cup H_R = H - H_T$ and $H_S \cap H_R$ cannot always be $\varnothing$.

In this paper, the model of Lotka-Volterra neural networks without self-excitatory can be described by

$$\begin{aligned} \dot{x}_{i\alpha}(t) &= x_{i\alpha}(t)\left[A\left(1-\sum_{\beta=1}^{n}x_{i\beta}(t)\right)\right. \\ &\quad +B\left(1-\sum_{j=1}^{n}x_{j\alpha}(t)\right) \\ &\quad \left.+\sum_{j=1}^{n}w_{ij}x_{j\alpha}(t)+\sum_{\beta=1}^{n}w_{\alpha\beta}x_{i\beta}(t)\right], \quad (1) \end{aligned}$$

where each $x_{i\alpha}$ is the state of neuron $v_{i\alpha}$, w_{ij} is the connection wight between neuron $v_{i\alpha}$ and neuron $v_{j\alpha}$, $w_{\alpha\beta}$ is the connection wight between neuron $v_{i\alpha}$ and neuron $v_{i\beta}$, $w_{ij} = w_{\alpha\beta}$ if $i = \alpha$ and $j = \beta$. Note that $w_{ij} = w_{ji}$.

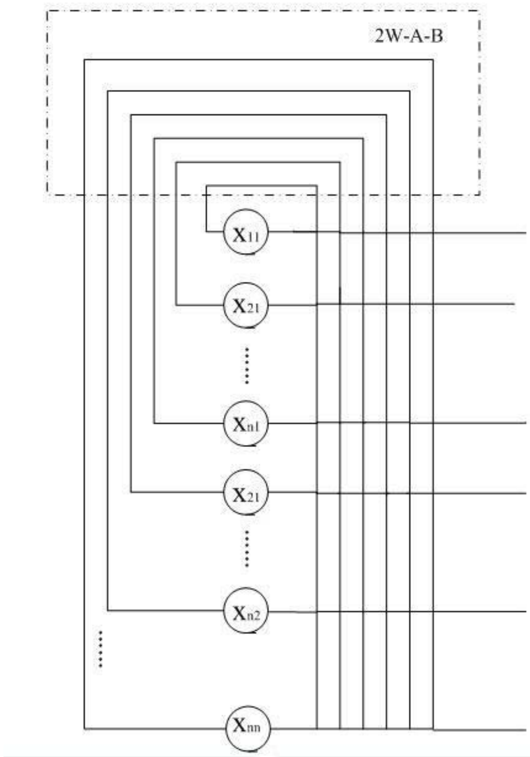

Fig. 1. The connections of the network (1).

Then the network is associated with an energy function in which the lowest energy state corresponds to the shortest tour. This include two aspects. Firstly, the energy function must favor strongly stable states of the form of a permutation matrix. Secondly, all the solutions to the problem correspond to valid tour. We use the method in [2] to obtain the following energy function

$$\begin{aligned} E &= -\sum_{\alpha=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}w_{ij}x_{i\alpha}x_{j\alpha}-\sum_{i=1}^{n}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}w_{\alpha\beta}x_{i\alpha}x_{i\beta} \\ &\quad +\frac{A}{2}\sum_{i=1}^{n}\left(\sum_{\alpha=1}^{n}x_{i\alpha}-1\right)^2+\frac{B}{2}\sum_{\alpha=1}^{n}\left(\sum_{i=1}^{n}x_{i\alpha}-1\right)^2. \end{aligned}$$

It is clear that the third and forth term are constraint conditions, where the first and second term denotes the total cost to minimize.

III. Performance Analysis for Network (1)

Assume the initial condition as $x_{i\alpha}(0) = \phi_{i\alpha}(0) > 0, (i, \alpha = 1, \cdots, n)$. Form [2] we can easily know that each solution $x(t)$ of (1) with the initial condition $x_{i\alpha}(0) = \phi_{i\alpha}(0) > 0, (i, \alpha = 1, \cdots, n)$ satisfies $x_{i\alpha}(t) > 0, (i, \alpha = 1, \cdots, n)$ for all $t \geq 0$.

Definition 1: The network (1) is called completely stable, if each trajectory of (1) converges to an equilibrium point.

Lemma 1: Denote $\omega = \max\{\omega_{ij}\}$, if it holds that $\rho = A + B - 2n\omega > 0, (i, \alpha = 1, \cdots, n)$, the network (1) is bounded, and the network (1) is completely stable.

Proof. See [2]. ■

A. Non-convergence of invalid solution

Theorem 1: Suppose that $w_{ii} = 0$ for $\forall i = 1, 2, \cdots, n$ and $w_{ij} < -\max(A, B), (i \neq j)$. The equilibrium point is unstable if the equilibrium point not correspond to a valid tour of the TSP, namely that the equilibrium point is not a $n \times n$ permutation matrix.

Proof. Three steps will be used for the prove.

Case $x = 0$. It is clearly that the linearization system of (1) is given by

$$\dot{x}_{i\alpha} = (A + B)x_{i\alpha}, (i, \alpha = 1, 2, \cdots, n).$$

Clearly its eigenvalues of the matrix are positive. Then,

$$\lim_{t\to+\infty} x_{i\alpha}(t) = \lim_{t\to+\infty} x_{i\alpha}(0)e^{(A+B)t} = +\infty.$$

So that $x^* = 0$ is unstable.

Case $x^* \in H_S$. Suppose that x^* is an equilibrium point. we prove that if there are more than one elements larger than zero in a column, the equilibrium x^* is unstable.

Suppose that there exist constants α and β such that $x^*_{i\alpha} > 0, x^*_{i\beta} > 0$, then it must have

$$A\left(1-\sum_{\gamma=1}^{n}x^*_{i\gamma}\right)+B\left(1-\sum_{j=1}^{n}x^*_{j\alpha}\right)+\sum_{j=1}^{n}w_{ij}x^*_{j\alpha} +\sum_{\gamma=1}^{n}w_{\alpha\gamma}x^*_{i\gamma}=0,$$

$$A\left(1-\sum_{\gamma=1}^{n}x^*_{i\gamma}\right)+B\left(1-\sum_{j=1}^{n}x^*_{j\beta}\right)+\sum_{j=1}^{n}w_{ij}x^*_{j\beta} +\sum_{\gamma=1}^{n}w_{\beta\gamma}x^*_{j\gamma}=0.$$

It is sufficient to prove the following system

$$\begin{cases} \dot{x}_{i\alpha} = x_{i\alpha}\Bigg[A\Bigg(1 - x_{i\alpha} - x_{i\beta} - \sum_{\gamma\neq\alpha,\beta}^{n} x_{i\gamma}^*\Bigg) \\ \qquad +B\Bigg(1 - x_{i\alpha} - \sum_{j\neq i}^{n} x_{j\alpha}^*\Bigg) + w_{ii}x_{i\alpha} \\ \qquad + \sum_{j\neq i}^{n} w_{ij}x_{j\alpha}^* + w_{\alpha\alpha}x_{i\alpha} \\ \qquad +w_{\alpha\beta}x_{i\beta} + \sum_{\gamma\neq\alpha,\beta}^{n} w_{\alpha\gamma}x_{i\gamma}^*\Bigg] \\ \dot{x}_{i\beta} = x_{i\beta}\Bigg[A\Bigg(1 - x_{i\alpha} - x_{i\beta} - \sum_{\gamma\neq\alpha,\beta}^{n} x_{i\gamma}^*\Bigg) \\ \qquad +B\Bigg(1 - x_{i\beta} - \sum_{j\neq i}^{n} x_{j\beta}^*\Bigg) + w_{ii}x_{i\beta} \\ \qquad + \sum_{j\neq i}^{n} w_{ij}x_{j\beta}^* + w_{\beta\beta}x_{i\beta} \\ \qquad +w_{\beta\alpha}x_{i\alpha} + \sum_{\gamma\neq\alpha,\beta}^{n} w_{\beta\gamma}x_{j\gamma}^*\Bigg] \end{cases}$$

is unstable at $(x_{i\alpha}^*, x_{i\beta}^*)$.

Consider the Jacobe matrix of the above system at $(x_{i\alpha}^*, x_{i\beta}^*)$

$$J_1 = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix},$$

where $J_{11} = -(A+B-w_{ii}-w_{\alpha\alpha})x_{i\alpha}^*$, $J_{12} = -(A-w_{\alpha\beta})x_{i\alpha}^*$, $J_{21} = -(A - w_{\beta\alpha})x_{i\beta}^*$, $J_{22} = -(A + B - w_{ii} - w_{\beta\beta})x_{i\beta}^*$.

Let λ_1, λ_2 be the eigenvalues of matrix J_1, it is clear that

$$\begin{aligned} & \lambda_1\lambda_2 \\ = \; & (A + B - w_{ii} - w_{\alpha\alpha})x_{i\alpha}^*(A + B - w_{ii} - w_{\beta\beta})x_{i\beta}^* \\ & -(A - w_{\alpha\beta})(A - w_{\beta\alpha})x_{i\alpha}^*x_{i\beta}^* \\ = \; & x_{i\alpha}^*x_{i\beta}^*\left[(A + B)^2 - (A - w_{\alpha\beta})^2\right] \\ < \; & 0. \end{aligned}$$

It implies that there is at least one of $\lambda_i(i = 1, 2)$ must be positive, thus above system is unstable at $(x_{i\alpha}^*, x_{i\beta}^*)$.

Case $x^* \in H_R$. we discuss the case that there are two elements in the same row larger than zero. Suppose that there exist constants i and j such that $x_{i\alpha}^* > 0, x_{j\alpha}^* > 0$, then it must have

$$A\left(1 - \sum_{\gamma=1}^{n} x_{i\gamma}^*\right) + B\left(1 - \sum_{k=1}^{n} x_{k\alpha}^*\right) + \sum_{k=1}^{n} w_{ik}x_{k\alpha}^* + \sum_{\gamma=1}^{n} w_{\alpha\gamma}x_{i\gamma}^* = 0,$$

$$A\left(1 - \sum_{\gamma=1}^{n} x_{j\gamma}^*\right) + B\left(1 - \sum_{k=1}^{n} x_{k\alpha}^*\right) + \sum_{k=1}^{n} w_{jk}x_{k\alpha}^* + \sum_{\gamma=1}^{n} w_{\alpha\gamma}x_{j\gamma}^* = 0.$$

To complete the proof, it is sufficient to prove the following system

$$\begin{cases} \dot{x}_{i\alpha} = x_{i\alpha}\Bigg[A\Bigg(1 - x_{i\alpha} - \sum_{\gamma\neq\alpha}^{n} x_{i\gamma}^*\Bigg) \\ \qquad +B\Bigg(1 - x_{i\alpha} - x_{j\alpha} - \sum_{k\neq i,j}^{n} x_{k\alpha}^*\Bigg) \\ \qquad +w_{ii}x_{i\alpha} + w_{ij}x_{j\alpha} + \sum_{k\neq i,j}^{n} w_{ik}x_{k\alpha}^* \\ \qquad +w_{\alpha\alpha}x_{i\alpha} + \sum_{\gamma\neq\alpha}^{n} w_{\alpha\gamma}x_{i\gamma}^*\Bigg] \\ \dot{x}_{j\alpha} = x_{j\alpha}\Bigg[A\Bigg(1 - x_{j\alpha} - \sum_{\gamma\neq\alpha}^{n} x_{j\gamma}^*\Bigg) \\ \qquad +B\Bigg(1 - x_{i\alpha} - x_{j\alpha} - \sum_{k\neq i,j}^{n} x_{k\alpha}^*\Bigg) \\ \qquad +w_{ji}x_{i\alpha} + w_{jj}x_{j\alpha} + \sum_{k\neq i,j}^{n} w_{jk}x_{k\alpha}^* \\ \qquad +w_{\alpha\alpha}x_{i\alpha} + \sum_{\gamma\neq\alpha}^{n} w_{\alpha\gamma}x_{j\gamma}^*\Bigg] \end{cases}$$

is unstable at $(x_{i\alpha}^*, x_{j\alpha}^*)$.

Consider the Jacobe matrix of the above system at $(x_{i\alpha}^*, x_{j\alpha}^*)$

$$J_2 = \begin{pmatrix} J'_{11} & J'_{12} \\ J'_{21} & J'_{22} \end{pmatrix},$$

where $J'_{11} = -(A+B-w_{ii}-w_{\alpha\alpha})x_{i\alpha}^*$, $J'_{12} = -(B-w_{ij})x_{i\alpha}^*$, $J'_{21} = -(B - w_{ji})x_{j\alpha}^*$, $J'_{22} = -(A + B - w_{jj} - w_{\alpha\alpha})x_{j\alpha}^*$

Let λ_1, λ_2 be the eigenvalues of matrix J_2, it is clear that

$$\begin{aligned} & \lambda_1\lambda_2 \\ = \; & (A + B - w_{ii} - w_{\alpha\alpha})x_{i\alpha}^*(A + B - w_{jj} - w_{\alpha\alpha})x_{j\alpha}^* \\ & -(B - w_{ij})x_{i\alpha}^*(B - w_{ji})x_{j\alpha}^* \\ = \; & x_{i\alpha}^*x_{j\alpha}^*\left[(A + B)^2 - (B - w_{ij})^2\right] \\ < \; & 0. \end{aligned}$$

It implies that there is at least one of $\lambda_i(i = 1, 2)$ must be positive, thus above system is unstable at $(x_{i\alpha}^*, x_{j\alpha}^*)$. ■

B. Convergence of valid solution

Theorem 2: A stable equilibrium point corresponds to a valid tour. That is, if x^* is a stable equilibrium point, then $x^* \in H_T$.

Proof. Suppose the statement is not true, namely that there exist a stable equilibrium point corresponding to an invalid tour. Then there must be at least two elements within a column or row larger than 0. From Theorem 3 we know that this equilibrium point is not stable, this poses a contradiction to the assumption. Hence a stable equilibrium point is corresponding to a valid tour of TSPs. ■

IV. Simulation Result

The convergence conditions of network (1) have been discussed above. To verify the theoretical result and the effectiveness of the network applied to TSPs, the following experiments have been carried out.

Set initial states $x_{ij}(0) \in [0,1], (i,j = 1,2,\cdots,8)$,$n = 8, A = 2, B = 1$ and the connection inhibition matrix $W = (w_{ij}), (i,j = 1,2,\cdots,8)$ as follows

$$W = \begin{pmatrix} 0 & -1.3361 & -1.3141 & -1.3601 & -1.5111 & -1.5176 & -1.2982 & -1.4564 \\ -1.3361 & 0 & -1.1107 & -1.6149 & -1.8407 & -1.8083 & -1.5815 & -1.6418 \\ -1.3141 & -1.1107 & 0 & -1.5349 & -1.7919 & -1.8207 & -1.5941 & -1.6908 \\ -1.3601 & -1.6149 & -1.5349 & 0 & -1.3397 & -1.6528 & -1.5171 & -1.7375 \\ -1.5111 & -1.8407 & -1.7919 & -1.3397 & 0 & -1.4579 & -1.4529 & -1.6686 \\ -1.5176 & -1.8083 & -1.8207 & -1.6528 & -1.4579 & 0 & -1.2274 & -1.2937 \\ -1.2982 & -1.5815 & -1.5941 & -1.5171 & -1.4529 & -1.2274 & 0 & -1.2277 \\ -1.4564 & -1.6418 & -1.6908 & -1.7375 & -1.6686 & -1.2937 & -1.2277 & 0 \end{pmatrix}$$

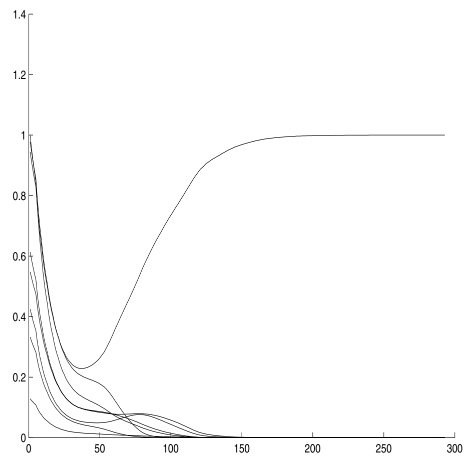

Fig. 2. Trajectories of the network in a random column.

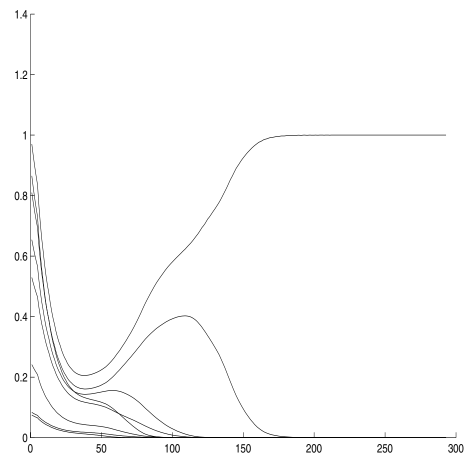

Fig. 3. Trajectories of the network in a random row.

Fig. 2 and Fig. 3 show the trajectories of the variables in a random row and column, respectively. Fig. 4 show the trajectories of all variables. Clearly it follows the rule WTA within each row or column, however, it shows a winner-share-all phenomenon in the global view. Those properties illustrated in the figures are not the same as the properties in [11]. Fig. 2, 3, 4 correspond to a permutation matrix which column and row are associated respectively to a particular city and order in the tour of TSPs.

TABLE I
10-CITY PROBELM

i	1	2	3	4	5	6	7	8	9	10
X	1	2	3	4	5	1	2	3	4	5
Y	1	2	1	2	1	3	4	3	4	3

The second experiment is on a 10-city problem which was presented in [8]. Its city coordinates are shown in Table I. Its minimal tour length is 15.3137. The initial states were randomly generated by $x_{i\alpha} \in (0,1)$. The item "good" indicated the number of tours with distance within 150% of the optimum distance.

TABLE II
PERFORMANCE OF LVNN WITH DIFFERENT PARAMETER SETTING

	Good(%)	Min Length	Ave Length
$A = B = 0.1$	30%	15.3137	23.9717
$A = B = 0.5$	27%	15.3137	23.0063

Simulation results are shown in Table II, where data of the two rows show the performance of the proposed network. The network gets the shortest tour with different parameter setting shown in the table. Fig. a and b are two optimum solutions get by LVNN with varied parameters, respectively.

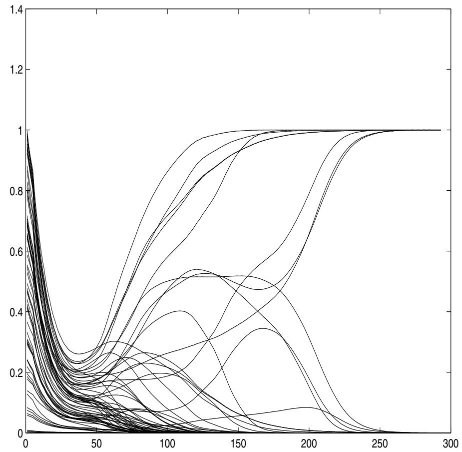

Fig. 4. Trajectories of all the variables.

V. Conclusions

In this paper, a class of LVNN is used to solve the TSPs. The dynamics of the networks are analyzed in terms of stability of specific equilibrium points, including the conditions derived for the convergence. The WTA within each row and column enable the networks can solve TSPs. The theoretical results have been illustrated with a series of experiments.

These analytical conditions depend upon all the lateral inhibitory of the network, including the parameters of the network. For simplicity we consider the parameters A equal to B in the simulations in this paper. If the coefficient A and B are different, the model would be more better, more research on this issue is required.

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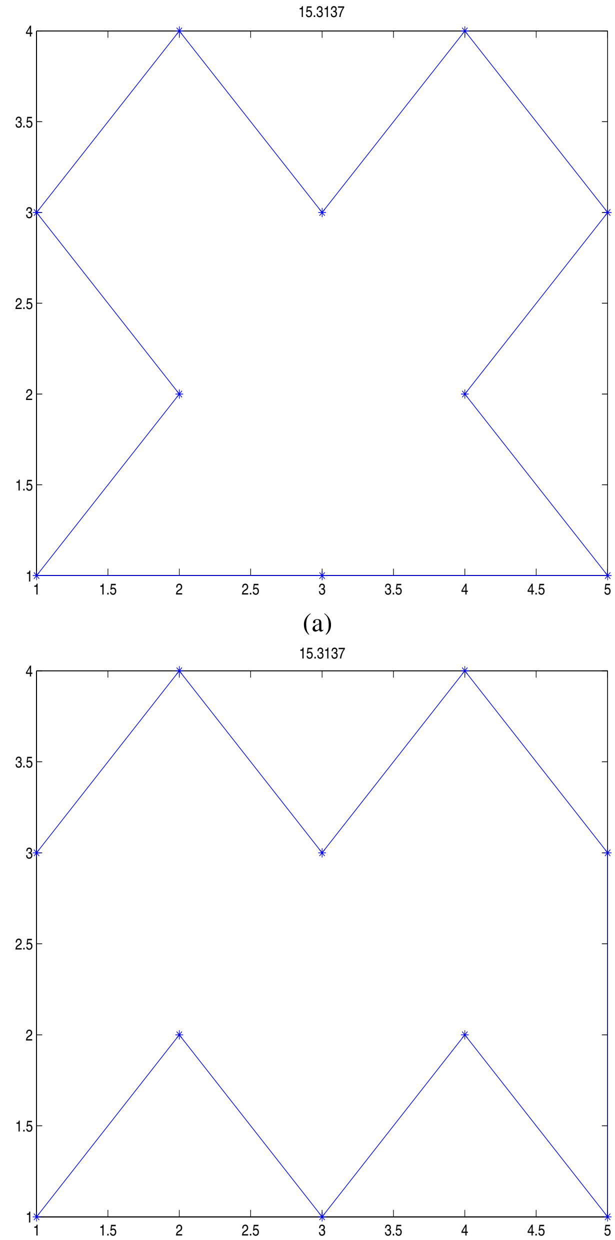

(a)

(b)

Fig. 5. The two Figs. presents the two optimal tours get by the proposed network. the parameters setting of (a) are $A = 0.5, B = 0.5, w_{ij} = -d_{ij} - 2$; the parameters setting of (b) are $A = 0.1, B = 0.1, w_{ij} = -d_{ij} - 0.2$;

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