Group Selection by Using Lotka-Volterra Recurrent Neural Networks

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***Abstract*—This paper studies the problem of group selection by using Lotka-Volterra recurrent neural networks. The networks are required to be with self-inhibition and lateral inhibition. The group selection is based on the concepts of permitted and forbidden sets. By restricting the strength of the lateral inhibition and self-inhibition to be in some interval, conditions for group selection are obtained. Under these conditions, groups are related to permitted and forbidden sets, i.e., each group is a permitted set. Thus, groups can be selected by the networks. Simulation results further confirm the theory.**

I. Introduction

The winner-take-all (WTA) competition can be implemented by some variant recurrent neural networks [7][10][13]. Many unsupervised learning neural networks are designed by WTA rules, such as Grossbergs competitive learning, Hamming networks, fuzzy associative memory, and learning vector quantization. In the traditional WTA neural networks, however, the selection of a single winner may easily suffer from noise in inputs because there are only one single neuron which receives the largest input value is active at a steady state [1]. As an extension of WTA, K-winner-take-all (KWTA) selects the k largest values which are uniformly arranged [9]. In the K-winner networks, the solutions could be systematically studied only for homogeneous external inputs, and their dynamic behavior with nonhomogeneous inputs is in general complicated [3]. The winners of KWTA were selected according to the initial conditions of neural networks.

Recently, the Lotka-Volterra (LV) recurrent neural networks have been used for implementing competition problems. The model of LV networks was proposed in [3], and it has been studied by many authors, see for examples, [1][2][11][12]. As initial-condition-independent behavior of neural network, WTA, winner-share-all(WSA) and variant winner-take-all(VWTA) competition solutions with LV recurrent neural networks in steady states were studied in [3]. These solutions are classified according to the number of active neurons that we call winner. Transitions among these three types solutions can been implemented by the ration of the strength of lateral inhibition to that of self-inhibition.

Group selection, performing group winer-take-all and similar to WSA, was studied in [8] by using continuous-time linear threshold recurrent neural networks. A group can be selected based on the concepts of permitted and forbidden sets if the strength of the lateral inhibition is sufficiently strong. It extends the grouping of potential winners in the WTA networks beyond single neuron or uniformly arranged groups of neurons [8]. The competition between arbitrary groups of neurons can be realized by organizing lateral inhibition in the networks.

In this paper, we use LV recurrent neural networks to study the problem of group selection by lateral inhibition and self-inhibition. We will derive conditions on the strength of the lateral inhibition and self-inhibition so that the network can be used for group selection by using the concepts of permitted and forbidden sets [5][8][4]. Simple necessary and sufficient conditions will be established respectively for existence of permitted and forbidden sets, relationships between permitted set and groups.

This paper is organized as follows. The problem formulation is given in Section II. The theory of permitted and forbidden sets in LV recurrent neural networks with lateral inhibition is presented in Section III. Section IV studies the relationships between permitted sets and the groups. Simulations are given in Section V to further illustrate the theory. Conclusions are given in Section VI.

II. The Problem Formulation

The model of Lotka-Volterra recurrent neural networks is described by

$$\dot{x}_i = x_i \left(-x_i + b_i + \sum_{j=1}^{n} w_{ij} x_j \right)$$

for $t \geq 0$, where x_i denotes the activity of neuron i and $x = (x_1, x_2, \cdots, x_n)^T \in R^n$ denotes the state of the network, $W = (w_{ij})_{n\times n}$ is real $n \times n$ matrices, each of their elements denotes the synaptic weight matrix and represents the connection form neuron i to neuron j, $b = (b_1, b_2, \cdots, b_n)^T \in R^n$ denotes the external input. The neural activities x_i of networks will remain nonnegative for all future time and any inputs b when its initialized value is nonnegative. The networks can be also write as vector form

$$\dot{x} = diag\,(x)\,(-x + b + Wx)\,, \tag{1}$$

where $diag(x)$ is a $n \times n$ matrices which the main diagonal elements are x_i $(i = 1 \cdots n)$, the other elements are zero.

Definition 1: Let A be an $n \times n$ matrix, and let $P \subseteq \{1, 2, \cdots, n\}$ be an index set. The matrix A_P is called a submatrix of A if it can be constructed from A simply by removing from A all rows and columns not indexed by P.

The group selection we will study in this paper is based on the concepts of permitted and forbidden sets. A set of neurons is said to be permitted if it can be coactived to a stable equilibrium point by some input. Here, the concept of Lyapunov stability is associated with some equilibrium points. An equilibrium point can be stable or unstable. However, in practice, only the outputs of stable equilibrium points can be observed.

A point $x^* \in R_+^n$ is called an equilibrium point of the network (1), if it satisfies

$$x^* = b + W \cdot x^*.$$

Next, we give the definition of Lyapunov stability for an equilibrium point.

Definition 2: An equilibrium point $x^* \in R_+^n$ is called stable, if given any constant $\epsilon > 0$ there exists a constant $\delta > 0$ such that

$$\|x(0) - x^*\| \leq \delta$$

implies that

$$\|x(t) - x^*\| \leq \epsilon$$

for all $t \geq 0$. An equilibrium point is called unstable if it is not stable.

Next, we given the mathematical definition for permitted and forbidden sets.

Definition 3: A set of neurons with index set P is called permitted if there exists an input b such that the network (1) has a stable equilibrium point x^* with

$$\begin{cases} x_i^* > 0, i \in P \\ x_j^* = 0, j \notin P. \end{cases} \tag{2}$$

A set of neurons is called forbidden if it is not permitted, i.e., given any input b, it is not possible to find a stable equilibrium point satisfies (2).

Lemma 1: [5] If W is symmetric and $I - W$ copositive, a set of neurons with index set P is permitted if and only if each eigenvalue of $(I - W)_P$ is nonnegative.

III. Permitted and Forbidden Sets with Inhibition

A. The model with lateral inhibition

Suppose there are n neurons which are grouped into some groups. We assume that each neuron belongs to at least one group, and that each group contains at least one neuron. One neuron is allowed to belong to more than one group, so that the groups may come into overlapping. Let us first define the lateral inhibitory synaptic matrix.

Definition 4: Let $J = (J_{ij})_{n \times n}$ be an matrix. If both neuron i and j are contained in a group, then $J_{ij} = 0$. If neuron i and j are not contained in any of a same group, then $J_{ij} = 1$. The matrix J is called lateral inhibitory synaptic matrix.

Clearly, the lateral inhibitory synaptic matrix J is symmetric and all of the diagonal elements are zero. The inhibitory matrix J shows that the inhibitory connection between two neurons is established only if the two neurons are not contained in any of a same group. The matrix J can be constructed from a simple learning mechanism [8] described as follows.

Suppose there are m groups of neurons, and the group membership ξ of the ath group is defined by

$$\xi_i^a = \begin{cases} 1, & i \in a \\ 0, & i \notin a \end{cases}$$

for $i = 1, 2, ..., n$. Each group a can be described by the binary vector $\xi^a = (\xi_1^a, \xi_2^a, \cdots, \xi_n^a)^T$. All elements of matrix J are initialized to be unity. The elements of matrix J can be learned simply through the updating rule

$$J_{ij} \leftarrow J_{ij}\left(1 - \xi_i^a \xi_j^a\right),$$

where $a = 1, 2, \cdots, m$ and $i, j = 1, 2, \cdots, n$. This implies that if both neuron i and j belong to pattern a, then the connection between them is removed.

At the beginning of the learning, the initial state of J performs uniform inhibition. During step by step of the learning, the inhibitory connections are removed between neurons that they belong to some same group, then the competition evolves to mediate competition between groups of neurons.

Based on the concept of later inhibition and self-inhibition, the network (1) can be rewritten as follows

$$\dot{x} = diag\,(x)\,(b + (\alpha I - \beta J) \cdot x) \tag{3}$$

for $t \geq 0$. where I is the unit matrix which represents self-inhibition connection weight, α is the strength of self-inhibition, β is the strength of lateral inhibition, J is lateral inhibition synaptic matrix defined by Definition 4, and $diag(x)$ is a $n \times n$ matrices which the main diagonal elements are x_i $(i = 1 \cdots n)$, the other elements are zero. Here, we assume that the networks have the same strength of self-inhibition synaptic and the same strength of lateral inhibitory synaptic. Figure 1 intuitively illustrates the network structure of connection defined by tow group

$$\xi_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \xi_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

The two parameters α and β in the network (3) play important roles in the group selection. Through out this paper, we assume the two parameters satisfy the following condition:

$$Condition: \begin{cases} \alpha < 0 \\ \|\alpha\| < \beta. \end{cases} \tag{4}$$

B. Permitted Sets and Forbidden Sets

Generally, it is difficult to check whether a given set of neurons is permitted or forbidden by the Definition 3. Although Lemma 1 has already established the sufficient and necessary condition guaranteeing a given set of neurons can be permitted

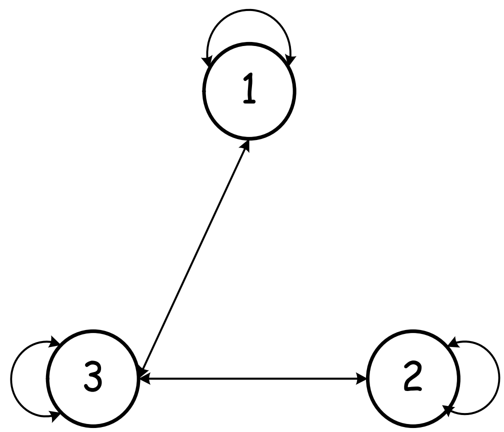

Fig. 1. The connection structure of the network defined by six groups $\xi_1 = \{1, 0, 1\}$ and $\xi_2 = \{0, 1, 1\}$. The connection between neurons represent the self-inhibition and lateral inhibition synaptic.

in networks (1), but based on the concept of later inhibition, we can give another equivalent condition that relate to the matrix J and is more easy to be checked in practice. Next, we relate these concepts with the matrix J and a simple condition will be given.

Theorem 1: A set of neurons with index P is permitted if $J_P = 0$, where 0 is a zero matrix.

Proof: Suppose that $J_P = 0$, we will prove that the set of neurons with index set P is permitted. By networks (1) and networks (3), we can get

$$W = (\alpha + 1)I - \beta J.$$

Clearly, we can get

$$\lambda_{min}\left((I - W)_P\right) = -\alpha > 0.$$

By 4 and Lemma 1 , we can get that the set of neurons with index P is permitted. The proof is completed. ■

The following two theorems are straightforward from the above Theorem 1 and Interlacing Theorem [6].

Theorem 2: A set of neurons with index F is forbidden if $J_F \neq 0$.

Theorem 3: Each subset of a permitted set is permitted, each superset of a forbidden set is forbidden.

IV. Relations between Groups and Permitted and Forbidden Sets

In this section, we investigate the relationships between permitted set and groups. We will address two important problems. First, whether each group is a permitted set? Second, whether a permitted set is contained in some group?

Theorem 4: Each group and its any subgroups are permitted.

Proof: Let P be an index set of a group of neurons. By the definition of lateral inhibitory matrix, it holds that $J_P = 0$. Using Theorem 1, neurons with index set P must be permitted. By Theorem 3, each subset of a permitted set is also permitted, then any subgroup of P is also permitted. The proof is completed. ■

To address the second problem. Let us consider an example, suppose there are group memberships:

$$\xi_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \xi_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \xi_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

Let $P = \{1, 2, 3\}$ be an index set, by Theorem 5, neurons with index set P is permitted. However, such neurons are not contained in any group. This shows that in general there may exist permitted set which are not contained in any group. We call such permitted set as spurious permitted set.

Definition 5: A set of neurons P is called a spurious permitted set, if P is permitted but not contained in any group.

Identifying spurious permitted sets is important for group selection. Next, we derive some results on this problem.

Lemma 2: Let P be a set of neurons. If there are two neurons of P are not contained in any same group, then P must be a forbidden set.

Proof: Let F be the index of the the two neurons of P that are not contained in any same group. Then, by the definition of lateral inhibitory matrix J, we have

$$J_F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \neq 0.$$

By Theorem 2, F must be forbidden. Since P is superset of F, then P is also forbidden. The proof is completed. ■

Theorem 5: Suppose a set of neurons with index set P has at least three elements. Then, P is permitted if and only if any two neurons of P are contained in a same group.

Proof: Suppose that there are two neurons not contained in any same group. By Theorem 2, these two neurons form a forbidden set, then P is also forbidden. This contradiction implies that any two neurons must be contained in some same group.

Suppose any two neurons of P are contained in some same group. Then, it follows that $J_P = 0$. By Theorem 1, P is permitted. The proof is completed. ■

Theorem 6: A set of neurons P is a spurious permitted set if and only if P is not contained in any group and any two neurons of P are contained in some group.

Proof: By the definition of spurious permitted set and theorem 5, the result follows and the proof is completed. ■

V. Simulation Results

In this section, some simulations will be carried out to illustrate the theories. Consider a ring Lotka-Volterra recurrent neural network as showing by Figure 2.

There are 12 groups to can be stored in the ring networks as shown by Figure 2. We can define twelve group memberships

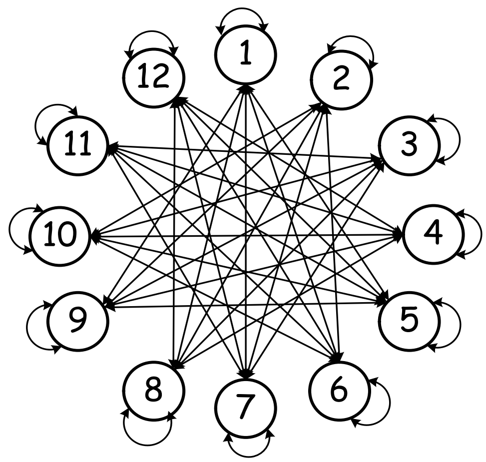

Fig. 2. The structure of ring LV network consist of 12 neurons. The connection on each neuron represent the self-inhibition synaptic of neuron. The connection among neurons represent the lateral inhibition synaptic which they are not belong to set of 4 contiguous neurons.

by

$$\xi_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \xi_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, ..., \xi_{11} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 1 \end{pmatrix}, \xi_{12} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}.$$

By the learning mechanism of lateral inhibitory synaptic matrix in Section II, we can get the 12×12 inhibitory synaptic matrix J as

$$J = \begin{pmatrix} 0&0&0&0&1&1&1&1&1&0&0&0 \\ 0&0&0&0&0&1&1&1&1&1&0&0 \\ 0&0&0&0&0&0&1&1&1&1&1&0 \\ 0&0&0&0&0&0&0&1&1&1&1&1 \\ 1&0&0&0&0&0&0&0&1&1&1&1 \\ 1&1&0&0&0&0&0&0&0&1&1&1 \\ 1&1&1&0&0&0&0&0&0&0&1&1 \\ 1&1&1&1&0&0&0&0&0&0&0&1 \\ 1&1&1&1&1&0&0&0&0&0&0&0 \\ 0&1&1&1&1&1&0&0&0&0&0&0 \\ 0&0&1&1&1&1&1&0&0&0&0&0 \\ 0&0&0&1&1&1&1&1&0&0&0&0 \end{pmatrix}.$$

Clearly, each 4×4 submatrix of J is zero. Thus, the dynamics of the network shown in Figure 2 can be described by

$$\dot{x}(t) = diag\left(x(t)\right)\left(b + (-0.5I - 0.85J) \cdot x(t)\right),$$

where I is the 12×12 unit matrix. By Theorem 1, it can be checked that neurons with the following index set

$$\begin{aligned} P_1 &= \{1, 2, 3, 4\},\ P_2 = \{2, 3, 4, 5\}, ..., \\ P_{11} &= \{11, 12, 1, 2\},\ P_{12} = \{12, 1, 2, 3\} \end{aligned}$$

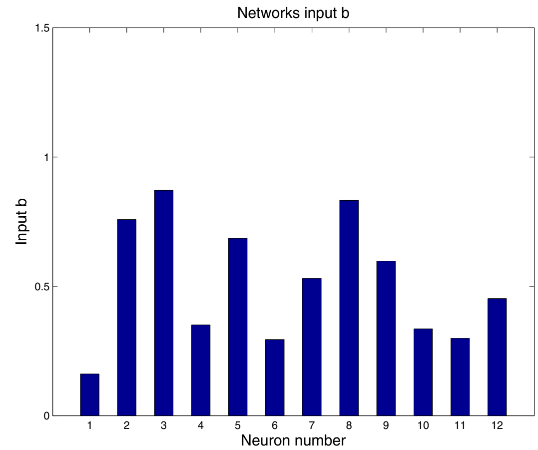

Fig. 3. Networks input b = {0.1611, 0.7581, 0.8711, 0.3508, 0.6855, 0.2941, 0.5306,0.8324, 0.5975, 0.3353, 0.2992, 0.4526}

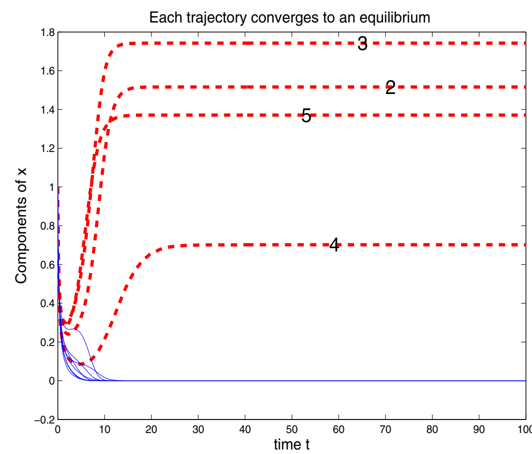

Fig. 4. Each trajectory of ring network with 12 neurons where $\alpha = -0.5$ and $\beta = 0.85$ converges to an equilibrium. The output of networks with index set $P = \{2, 3, 4, 5\}$ is positive value, P is a permitted set.

are permitted. The network can select a group by using random input b. Figure 4 illustration that neurons with index set $P = \{2, 3, 4, 5\}$ is permitted when each trajectory of the networks converges to an stable equilibrium point by input as shown in Figure 3. Figure 5 shows that the group represented by neurons with index set $P = \{2, 3, 4, 5\}$ can be selected by input as shown in Figure 3.

Figure 6 shows that one group formed by 100 contiguous neurons can be selected by a randomly input b in the ring network consists of 300 neurons.

VI. Conclusion

In this paper, we have developed a class LV recurrent neural networks to implement group selection. Some necessary and interesting conditions are established for existence of permitted and forbidden sets, as well as the relations between groups and permitted sets. It shows that competition between arbitrary groups of neurons can be realized by LV recurrent neural networks with lateral inhibition and self-inhibition.

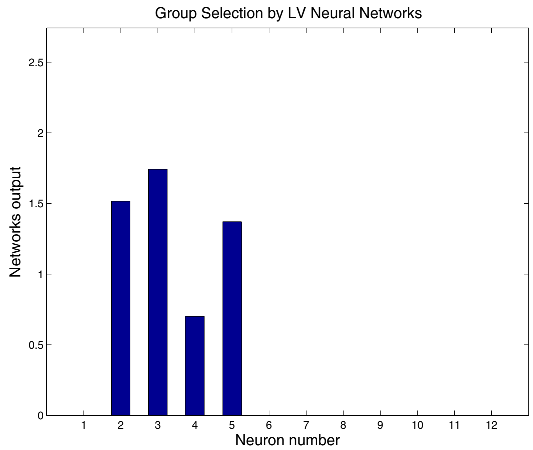

Fig. 5. Group selection by ring network with 12 neurons where $\alpha = -0.5$ and $\beta = 0.85$. The neurons index set $P = \{2, 3, 4, 5\}$ holding positive output is a permitted set and indicates a group.

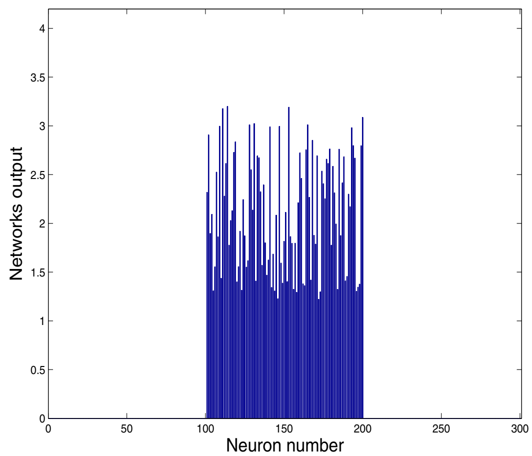

Fig. 6. Group selection by ring network with 300 neurons. The output of networks with index set P is positive value, $P = \{101, 102, ...199, 200\}$ consists of 100 neurons is a permitted set and indicates a group.

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