Improved Damped Least Squares Solution with Joint Limits, Joint Weights and Comfortable Criteria for Controlling Human-like Figures

Meng Na, Bin Yang, Peifa Jia
State Key Lab. of Intelligent Technology and Systems
Department of Computer Science and Technology, Tsinghua University
Beijing 100084, China
nm03@mails.tsinghua.edu.cn

***Abstract*—In order to generate natural posture and motion of virtual human-like figures, damped least squares inverse kinematics method is modified. Physical rules of human being like joint limits, joint weights and comfortable criteria are introduced to the design of damping factors for the improved damped least squares solution. The proposed method performs well on guaranteeing joint limit avoidance and producing natural-looking postures. This new scheme is successfully implemented and tested for real-time control of a seven-degree-of-freedom virtual human skeletal upper limb. Experiment results show that the improved solution is more robust and stable than the original damped least squares method.**

***Keywords*—damped least squares method, joint limits avoidance, joint weights, comfortable criteria, inverse kinematics**

I. Introduction

Inverse kinematics (IK) is originally designed to control the robotic manipulators. A manipulator, always a rigid multibody, consists of a set of links connected by joints. Rather than specifying the joint value of each degree of freedom (DOF), the IK method automatically computes these values in order to satisfy a given task. For a long time extensive study, there are several methods for solving IK problem for robotics application, including cyclic coordinate decent methods [1], Jacobian transpose methods [2, 3], pseudoinverse methods [4], damped least squares (DLS) methods [5, 6], and neural network artificial intelligence methods [7, 8].

Recently, with the development of computer animation, inverse kinematics also plays a key role in the posture or motion control of articulated figures. Because most inverse kinematics algorithms were originally designed to meet the requirements of robotics, they could not straightforward be applied to computer animation. For the application of human-like articulated figures, Jacobian transpose, pseudoinverse and damped least squares methods were analyzed in [9, 10]. The Jacobian transpose method had poor quality and slow convergence for multiple end effectors. The pseudoinverse method was unstable near singularities and performed ill when the target positions were out of reach. The damped least squares method worked substantially better than the above two methods. Through clamping the distance of the target positions, the damped least squares method could solve the unreachable situations very well [10].

However, the postures for a skeletal configuration of a figure are not always acceptable. In order to be realistic, physical rules should be taken into account. Joint limits were once considered in the control of robotic manipulator. Gradient projection method (GPM) [11] has been widely used in the literature for utilizing redundancy to avoid joint limits, where a gradient of a performance criterion, which is defined as a function of the joint angles and their limits, was projected on to the null space to obtain the self-motion. A weighted least-norm solution was proposed in [12]. The scheme automatically chose the appropriate magnitude of the self-motion throughout the workspace and guaranteed the joint limit avoidance. Another redundancy-based solution to avoid the joint limits of a manipulator was proposed in [13]. The control scheme was based on the task function and the method was relied on the interactive computation. All these methods solve the joint limits problem by defining a secondary task as cost functions or using the task priority scheme.

Besides joint limits, other factors need to be considered when simulating the human-like figures rather than robotic manipulators. In this paper, we propose an improved damped least squares method to satisfy the needs of controlling human-like figures, such as the virtual human skeleton model. In order to generate the natural-look postures and motions, joint limits, joint weights and comfortable criteria are all considered when designing the improved damped least squares method for IK. This improvement is achieved only by redefining the damping factors, no new functions increased in the original DLS method. We demonstrate the validation of the approach on a 7-DOF virtual skeletal upper limb through various real experiments.

The remainder of our work is organized as follows. Section 2 first introduces the background knowledge of inverse kinematics and damped least squares method. Then our improved damped least squares method with joint limits, joint weights and comfortable criteria is expatiated in Section 3, followed by the experiment results of controlling a 7-DOF virtual skeleton upper limb in Section 4. Finally, we give some concluding remarks in Section 5.

II. Damped Least Squares for Inverse Kinematics

In this section, the mathematical framework for inverse kinematics problem is stated first. After that, the original

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damped least squares methods [5, 6] and its modified algorithm [10] to solve the unreachable targets are presented.

A. Inverse Kinematics

A rigid multibody such as a robotic manipulator or an animated graphics character is composed of links connected at their ends by joints. The joint angles are written as a column vector as $\boldsymbol{\theta} = \left(\theta_1, \theta_2, ..., \theta_n\right)^T$. And the end effector position $\vec{\boldsymbol{s}} = \left(s_1, s_2, ..., s_m\right)^T$ is the function of joint angles. The goal of IK is to compute the vector of joint angles that will cause the end effector to reach the desired target position $\vec{\boldsymbol{t}} = \left(t_1, t_2, ..., t_m\right)^T$, that is

$$\boldsymbol{t} = \boldsymbol{s}\left(\boldsymbol{\theta}\right) \tag{1}$$

Equation (1) can be linearly approximated using the Jacobian matrix. Jacobian is a m by n matrix that relates differential changes of $\boldsymbol{\theta}$ to changes of $\boldsymbol{s}$, which is a defined by $J\left(\boldsymbol{\theta}\right) = \left(\partial \boldsymbol{s} / \partial \boldsymbol{\theta}\right)$. So the change in end effector position is caused by the increment of joint angles.

$$\vec{\boldsymbol{e}} = \vec{\boldsymbol{t}} - \vec{\boldsymbol{s}} = J \Delta \boldsymbol{\theta} \tag{2}$$

Therefore, the current values of $\boldsymbol{\theta}$ can be updated iteratively by

$$\boldsymbol{\theta} = \boldsymbol{\theta} + \Delta \boldsymbol{\theta} \tag{3}$$

In configurations, only when the task Jacobian matrix J is square and full rank, its inverse matrix exists uniquely. So the increment of joint angles can be calculated through the following equation.

$$\Delta \boldsymbol{\theta} = J^{-1} \vec{\boldsymbol{e}} \tag{4}$$

Whereas in most cases, the Jacobian is rank deficient, $rank(J) = r, r < m$. So that the expression (4) is not possible when singular configurations, and the solution becomes ill-conditioned close to singularities.

B. Damped Least Squares Method

To overcome singularities, Nakamura and Hanafusa [5] and Wampler [6] independently proposed to use the damped least-squares technique in the inverse kinematics problem. The solution minimizes the quantity

$$\left\| J \Delta \boldsymbol{\theta} - \vec{\boldsymbol{e}} \right\|^2 + \lambda^2 \left\| \Delta \boldsymbol{\theta} \right\|^2 \tag{5}$$

where $\lambda \in \mathbb{R}$ is the damping factor, which is introduced for compensating the singularity problems, and can be selected dynamically based on the configuration of the articulated multibody [14, 15].

DLS finds the value of $\Delta \boldsymbol{\theta}$ that minimizes both tracking error and joint velocities, which trade off between the accuracy and feasibility of the solution. The method corresponds to solve the equation

$$\left(J^T J + \lambda^2 I\right) \Delta \boldsymbol{\theta} = J^T \vec{\boldsymbol{e}} \tag{6}$$

Thus, the DLS solution gives a numerically stable method of selecting $\Delta \boldsymbol{\theta}$ by

$$\Delta \boldsymbol{\theta} = J^T \left(J J^T + \lambda^2 I\right)^{-1} \vec{\boldsymbol{e}} \tag{7}$$

When using DLS to control the posture of human-like figures, sometimes the target position may be out of reach. In this case, the multibody often oscillates or jitters when attempting unsuccessfully to reach the distant target. To reduce this problem, the target can effectively be “moved” closer to the end effector position by clamping the length of $\vec{\boldsymbol{e}}$ [10].

$$\boldsymbol{e} = ClampMag\left(\boldsymbol{t} - \boldsymbol{s}, D_{\max}\right) \tag{8}$$

where

$$ClampMag\left(\boldsymbol{w}, d\right) = \begin{cases} \boldsymbol{w} & if \ \left\| \boldsymbol{w} \right\| \le d \\ d \dfrac{\boldsymbol{w}}{\left\| \boldsymbol{w} \right\|} & otherwise \end{cases} \tag{9}$$

Here $\left\| \boldsymbol{w} \right\|$ represents the Euclidean norm of $\boldsymbol{w}$. The value $D_{\max}$ is an upper bound on how far an end effector moves in a single update step. Reference [10] recommends setting $D_{\max}$ to be approximately half the length of a typical link.

Damped least squares method performs better than Jacobian transpose and pseudoinverse method when avoiding singularities [9]. Moreover, enforced by clamping the magnitude of $\vec{\boldsymbol{e}}$, this algorithm can reduce oscillation even when the target positions are unreachable. Therefore, DLS method is very applicable to simulate the posture or control the motion of the virtual human-like figures.

III. IMPROVED DAMPED LEAST SQUARES SOLUTION

Unlike robotic manipulators, physical rules need to be considered when controlling the motion of virtual human-like arm. In order to realize the natural-look motion of the virtual human, joint limits, joint weights and comfortable criteria are introduced to DLS method by designing the damping factor.

A. Joint Limits

For some types of joints, they can not turn to any 360 degree for their motion range. This situation is very common for virtual animation characters. Let us denote $\theta_{i\max}$ and $\theta_{i\min}$ are the upper and lower limits of the 1-DOF joint value θ_i. So we want to ensure the natural limits are not be violated, $\theta_{i\min} \le \theta_i \le \theta_{i\max}$.

In the original damped least squares method, the damping factor λ is a constant. That means the same effect acts on all joints. Here we introduce the $n \times n$ diagonal matrix $D(\lambda)$ to replace λ, using different λ_i to respectively restrict different joints.

$$D(\lambda) = diag(\lambda) = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} \quad (10)$$

and each λ_i $(1 \le i \le n)$ is defined as follows.

$$\lambda_i = c \cdot \left[\frac{2\theta_i - \theta_{i\max} - \theta_{i\min}}{\theta_{i\max} - \theta_{i\min}} \right]^p \quad (11)$$

where $c, p \in \mathbb{R}^+$ are the user defined constants, and p is an even number.

Therefore, each λ_i makes a restriction to each joint value θ_i. When the joint value is within its motion range, small value of λ_i gives accurate solutions; and when the joint is near or moves away from its limits, large value of λ_i results in a feasible solution. The curve of λ_i is symmetrical, shown in Fig. 1. And the larger the p is, the flatter the bottom is, which implies the approximate low cost to the reasonable joint values.

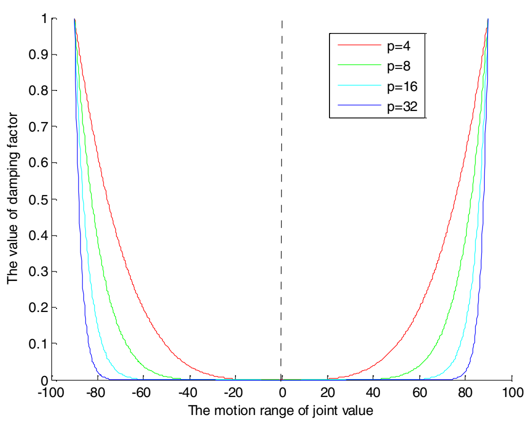


Figure 1. The curve of damping factor λ_i. Here we suppose c=1, $\theta_{i\max} = 90^\circ$, $\theta_{i\min} = -90^\circ$, and p takes the value of 4, 8, 16 and 32 seperately.

B. Comfortable Criteria

From the above improvement we can see that when θ_i is equal to the mean value of $(\theta_{i\max} + \theta_{i\min})/2$, λ_i has the minimum cost of zero. But in fact, the minimum cost of joint value should be at the comfortable joint value θ_{icomf}, has nothing to do with the upper and lower joint limits. For this physical fact, we modify the formula of λ_i as follows.

$$\lambda_i = c \cdot \left[\frac{2(\theta_i - \theta_{icomf})}{\theta_{i\max} - \theta_{i\min}} \right]^p \quad (12)$$

After introducing the comfortable criterion, the curve of λ_i is not always symmetrical, except the comfortable situation is just at the mean value. Therefore, the minimum position falls on the most natural posture, and the cost value of λ_i increases according to the deviation of the comfortable position.

C. Joint Weights

For the articulated multibody, especially the animation characters, the flexibilities of different joints and DOFs are different. The more flexible the joint or DOF is, the higher the weight is. Let's take the human upper limb for example. Shoulder joint is more flexible than wrist joint, and wrist joint is more flexible than elbow joint. So we should add the least cost to shoulder joint, and the less cost to wrist joint than the elbow joint.

A DOF weighting coefficient $w_i \in \mathbb{R}^+$ is introduced to the definition of λ_i.

$$\lambda_i = c \cdot \left[\frac{2(\theta_i - \theta_{icomf})}{\theta_{i\max} - \theta_{i\min}} \right]^p + \frac{1}{w_i} \quad (13)$$

Then the curve of λ_i translates upward, that makes sure all the value of λ_i is greater than zero. When the value of every λ_i is zero, DLS method degenerates to the pseudoinverse method, which is less robust when singularities and less stable when unreachable situations than DLS method. Thus, the introduction of the DOF weighting coefficient ensures good performance of DLS method.

D. Final Improved Solution of DLS

Considering all the above factors, joint limits, joint weights and comfortable criteria, the improved DLS method for controlling the motion of virtual figures can be rewritten as follows.

$$\Delta\boldsymbol{\theta} = J^T \left(JJ^T + D(\lambda)^2 \right)^{-1} \vec{\boldsymbol{e}} \quad (14)$$

Therefore, the joint vector $\boldsymbol{\theta}$ can be updated iteratively to getting closer to the target position. At the same time, the improved DLS method makes sure the figure motions satisfy the human natural rules and generates more realistic postures.

IV. EXPERIMENTS

The improved damped least squares solution was implemented in real-time to control a 3D virtual skeletal upper limb from Poser 5. To control the skeleton motion, we simplify the upper limb with 7 DOF of 3 joints. The skeleton model and the joints are illustrated in Fig. 2.

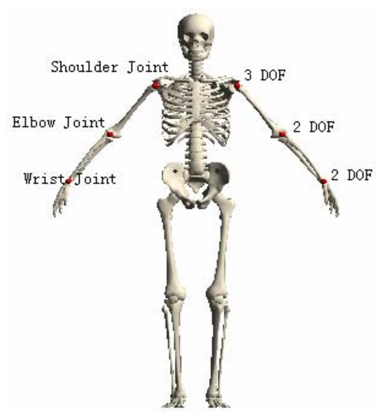


Figure 2. The 3D virtual skeleton model with 3 joints and 7 DOF of the upper limb.

For the real arm of human being, the upper, lower and comfortable angles exist, which limit the motion range of real arm. Table 1. presents the joint parameters of the human upper limb, and from which we can see that the natural position is not always the mean value of the maximal and minimal joint limits. So the minimal cost setting at the comfortable posture is reasonable.

TABLE I. JOINT PARAMETERS OF HUMAN UPPER LIMBP

Joint Name	**Rotational Type**	**Joint Parameters**[a]		
		$\theta_{i\min}$	$\theta_{i\max}$	θ_{icomf}
Shoulder Joint	Flexion	-100°	50°	3°
	Twisting	-90°	90°	0°
	Abduct	-40°	100°	-13°
Elbow Joint	Flexion	-150°	0°	-23°
	Twisting	-90°	90°	0°
Wrist Joint	Flexion	-90°	90°	0°
	Pivot	-60°	60°	0°

a. The unit of the joint value is degree.

A. Experiment 1. Joint Limits

Joint limits are an important component of a joint model, since they restrict the motion space to the realistic range of motion. So that we test the improved DLS method the performance of avoiding the joint limits first.

In this experiment, different utmost target position is given, and traditional DLS method and improved DLS method will calculate the joint angles separately. Without restriction on the joint limits, the arm motion will turn to the easy way to get the target position, computed by the traditional DLS method. But take the motion range into account of the improved DLS method, the trajectory of the end effector and joint values are different, which avoid the excess of joint limits.

Here gives some experiment data on the virtual skeleton model to compare the performance of normal DLS method with the improved DLS method. The red bold numbers represent the violated joint angles calculated by the traditional DLS method, while the black bold values are the normal setting of the joint configuration, computed by the improved DLS method.

TABLE II. EXPERIMENT DATA OF DLS METHOD WITHOUT JOINT LIMITS AND IMPROVED DLS METHOD (IDLS)

Joint Value	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
$\theta_{\min}$	-90	-100	-40	-90	-150	-90	-60
$\theta_{\max}$	90	50	100	90	0	90	60
Initial Value	0	3	-13	90	-2	0	0
DLS	83.3	**101.8**	44.2	33.8	-86.2	8.1	**83.3**
IDLS	85.5	**27.4**	47.0	89.9	-97.0	-87.9	**57.2**
Initial Value	-90	0	0	0	-90	0	0
DLS	**-96.3**	2.2	**-57.3**	-5.4	-122.9	-18.8	2.5
IDLS	**-90.0**	20.9	**-38.7**	-4.1	-90.0	-72.2	2.8
Initial Value	0	0	-13	0	0	90	40
DLS	-21.4	-59.5	-19.5	-10.3	-19.7	**93.9**	44.8
IDLS	-10.4	-67.9	-25.4	-1.8	-6.6	**88.6**	41.1
Initial Value	0	-100	-10	0	0	10	0
DLS	**233.4**	-85.7	20.3	-36.1	-7.2	62.4	-9.1
IDLS	**2.9**	-100.0	-9.5	-11.1	-18.1	66.1	-6.2
Initial Value	0	20	100	0	-30	0	10
DLS	-58.3	**-101.7**	63.1	**-102.9**	-113.1	-51.6	40.6
IDLS	-76.6	**-76.3**	99.9	**-87.2**	-93.6	-60.7	-12.5
Initial Value	90	0	10	0	-20	30	60
DLS	89.7	5.7	7.5	0.8	-22.5	35.3	**68.3**
IDLS	90.0	-6.1	21.2	-37.9	-54.3	44.4	**60.0**

Take the first group experiment data for example, the initial posture of the right upper limb is seen in Fig. 3(a) with the joint values of $\boldsymbol{\theta} = (0, 3, -13, 90, -2, 0, 0)^T$. And the target position, as indicated by the red point, is set at the right back of the left elbow for a special circumstance. Then the inverse kinematics method is used to calculate the joint angles of the right limb to get to the target position. Traditional DLS method without joint limits was adopted first to compute the joint angles. The result (Fig. 3(b)) is unreasonable because two joints exceed its motion range with the overmuch adduction of the shoulder and wrist joints. That is impossible in real case. However, the improved DLS method is more robust and stable to avoid the joint limits, and gives the rational result even though the unique cases (see Fig. 3(c)). Also the illustrations of the whole iterative process of calculating the joint angles with DLS and improved DLS methods are shown in Fig. 4.

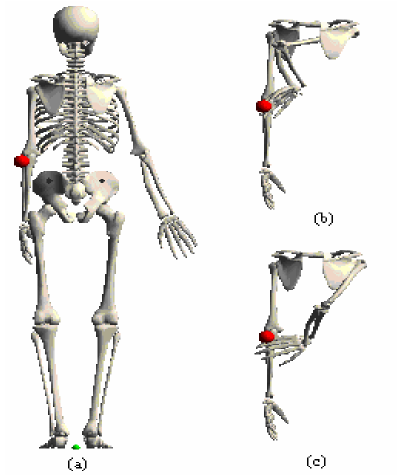
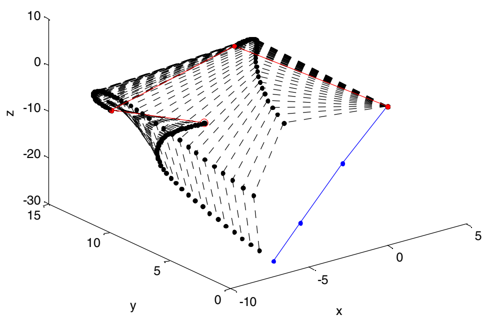
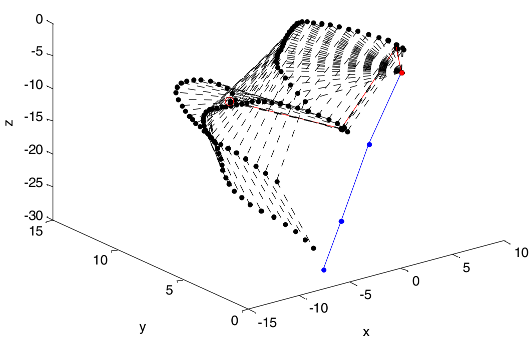


Figure 3. Joint configurations of DLS and IDLS method for reaching the target position. (a) is the initial posture and the red point represents the target position. (b) is the joint configuration of the traditional DLS method. (c) is the joint configuration of the improved DLS method.



(a)



(b)

Figure 4. The iterative process for updating the joint angles of DLS method (a) and improved DLS method (b). The blue line shows the initial configuration of the upper limb posture. The red line shows the final configuration to touch the target position (a red circle). The dots represents the joints and end effector (shoulder, elbow, wrist and middle finger tip) position. And the black dotted lines are intermediate iterative posture of the upper limb.

B. Experiment 2. Comfortable Postures

In our proposed improved DLS method, comfortable joint value replaces the mean value of the motion range for the design of the damping factor λ_i. Because we believe that the more comfortable the joint configuration is, the more realistic the posture is. In this way, the curve of λ_i has minimum when the joint angle is at the comfortable value.

We define the comfortable parameter l_{comf} to measure the performance of the improved DLS method with comfortable criteria.

$$l_{\text{comf}} = \sqrt{\sum_{i=0}^{n} \left(\theta_i - \theta_{i\text{comf}}\right)^2} \tag{15}$$

l_{comf} is the Euclidean distance between the vector $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_{\text{comf}}$. And the relative smaller value of l_{comf} means more natural and comfortable configuration. The algorithm with expression (13) is compared to the one using the mean value to take place the comfortable value.

$$\lambda_i = c \cdot \left[\frac{2\theta_i - \theta_{i\max} - \theta_{i\min}}{\theta_{i\max} - \theta_{i\min}}\right]^p + \frac{1}{w_i} \tag{16}$$

Most results of the improved DLS method with the comfortable criteria is closer to the natural position and more comfortable, see in Table 3. Therefore, the introduction of the comfortable joint position makes the joint configuration more natural.

TABLE III. EXPERIMENT DATA OF IMPROVED DLS METHOD WITH OR WITHOUT COMFORTABLE CRITERIA.

Joint Value	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	l_{comf}
θ_{comf}	0	3	-13	0	-23	0	0	0
Comf	49.7	-0.4	12.5	-8.2	-3.4	13.7	49.0	**78.6**
Mean	49.4	-0.1	14.0	-15.0	-8.3	14.7	49.7	79.4
Comf	40.6	41.2	-20.3	-13.7	-14.6	17.4	49.3	**78.5**
Mean	41.5	38.8	-19.0	-29.7	-17.1	18.5	50.8	83.1
Comf	32.6	25.0	31.6	-53.7	-68.4	-39.5	5.8	**100.5**
Mean	41.9	5.0	41.9	-14.1	-34.3	-65.5	-38.7	104.4
Comf	15.1	45.5	0.3	24.9	-93.4	-79.2	45.4	**127.0**
Mean	24.2	41.3	30.5	58.6	-117	-64.6	5.0	143.0
Comf	1.0	10.8	-9.0	-12.1	-52.0	-0.6	0.6	**32.6**
Mean	1.0	10.8	-9.1	-14.0	-52.0	-0.7	0.7	33.4
Comf	2.8	5.2	-15.8	55.8	-43.2	-2.5	-3.7	**59.7**
Mean	3.4	5.1	-15.2	59.3	-43.2	-3.6	-4.4	63.1

C. Experiment 3. Joint Weights

The employment of joint weights into the improved DLS method, on one hand can prevent the algorithm from degenerating to the pseudoinverse method. On the other hand it takes the flexibility of joint and DOF into consideration. We design the experiments as follows. In one try, the weights of all the DOF are the same. While in the other try, the weights are different according to the flexibility of the joints and DOFs. The comfortable parameter l_{comf} is still used to compare the performance of two experiments, and the results are shown in the table below.

TABLE IV. EXPERIMENT DATA OF IMPROVED DLS METHOD WITH DIFFERENT WEIGHTS (DIFF W) AND SAME WEIGHTS (SAME W).

Joint Value	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	l_{comf}
θ_{comf}	0	3	-13	0	-23	0	0	0
Diff W	5.9	55.6	9.8	-1.8	-47.6	-12.6	2.1	**64.0**
Same W	8.8	55.9	10.2	-11.7	-49.9	-8.6	5.9	66.2
Diff W	-19.0	3.3	16.4	-5.2	-95.2	-84.9	1.7	**117.0**
Same W	-10.8	0.9	10.8	-22.1	-97.6	-84.7	-10.1	118.4
Diff W	3.1	-13.0	-17.7	1.0	-59.0	-26.3	-1.2	**47.8**
Same W	2.9	-11.0	-16.6	6.1	-67.3	-11.8	-2.4	48.6
Diff W	7.7	-1.0	32.6	-2.5	-108	96.9	2.0	**137.2**
Same W	1.5	0.4	36.3	-23.8	-107	94.5	-42.2	144.7
Diff W	5.7	0.3	-24.1	1.2	-28.3	20.6	-1.1	**24.9**
Same W	4.9	-1.1	-24.4	7.7	-27.1	23.5	-4.2	28.7
Diff W	42.6	-36.9	-35.0	38.6	-70.4	-42.2	16.6	**98.5**
Same W	36.4	-37.1	-29.7	64.7	-82.9	-32.3	3.8	110.1

The experiment results show that the contrasts of the different and same weights solution are subtle. But different weights set by the flexibility of the joints and DOFs could produce more natural and comfortable joint configurations.

V. CONCLUSION

In this paper, the traditional DLS method has been improved for controlling the virtual human-like figures. With the introduction of joint limits, joint weights and comfortable criteria, our improved DLS method performs better than the original one on avoiding joint limits and generating natural-look postures of virtual human.

The same as the traditional DLS method, parameter selection is very important to the final result. So in the future research, we will work on the dynamic parameter adjustment of damping factor for the improved DLS method.

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