A Economic Lot Scheduling Problem for Manufacturing and remanufacturing

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***Abstract*—Recently, remanufacturing has received more and more attention by many manufacturers. It is economical to schedule manufacturing and remanufacturing on the same single product line due to setup cost saving. In this paper, we extend the classical Economic Lot Scheduling Problem to schedule the manufacturing and remanufacturing on the same single product line. We assume that the demand rate and return rate are constant and the product line has limited capacity of manufacturing and remanufacturing. We employ the Common Cycle approach to coordinate manufacturing with remanufacturing such that the total cost of per unit time is minimized. Furthermore, many numerical examples are presented to show the effects of return rate on the performance of this manufacturing/remanufacturing hybrid system.**

***Keywords*—Inventory Control, Remanufacturing, Reverse Logistics**

I. Introduction

Recently, remanufacturing has received more and more attentions for the pressure of the environmental legislations, economic interesting and environmental consciousness. It is economical to accommodate the manufacturing and remanufacturing of the homogenous products on the same single product line. How to schedule the manufacturing and remanufacturing on the same product line is aroused. Our aim is to find a scheduling to minimize the average total cost.

The management of this material flow opposite to the conventional supply chain flow is addressed in the rapidly expanding field of *reverse logistics* [1]. Fleischmann et al. [1] divided this new field into three main areas, namely, distribution planning, inventory control, and production planning. There were many research papers concerning on the inventory control of manufacturing/remanufacturing hybrid system. Sherbrooke [2] presented a METRIC model to control the repairable products. The METRIC model is based on the assumption that return and demand are perfectly correlated. Heyman [3] and Muckstadt and Issac [4] have investigated the situation of independent demand and return. E. van der Lann et al. [5][6][7][8] presented so called PUSH and PULL strategies to control the inventory of the manufacturing/remanufacturing hybrid system. More details can be referred to the extended review of Fleischmann et al. [1].

All the above researches assumed that manufacturing and remanufacturing are processed at different facilities. As we know, there are no researches concerning the problem of accommodating manufacturing and remanufacturing on the same single product line.

The classic Economic Lot Scheduling Problem (denoted by ELSP) accommodates the production of a number of items on a single product line. Most of the published literatures concern the situations of constant demand, non-backorder and finite production rate. There are several approaches to solve this problem. One of the approaches is Common Cycle approach [9]. For the Common Cycle approach, the optimal common cycle can be easily obtained, but this approach is not optimal. Then an alternative approach named Basic Period was presented by Bomberger [10]. Bomberger [10] employed dynamic programming to solve the optimal Basic Period model. The Common Cycle and Bomberger's approaches are simple and will always provide a feasible solution (if a feasible solution exists). Besides the above analytical approaches, some heuristic approaches have be widely used to solve the ELSP [11][12][13]. The computation of the heuristic approaches is very easy. The main disadvantage of these heuristic approaches is that there are no guarantee for even a feasible solution.

In this paper, we implement the Common Cycle approach to accommodate the manufacturing and remanufacturing of the homogenous products on the same single product line. Our problem shares the same feature with the classic Economical Lot Scheduling Problem since remanufacturing represents a alternative production process. But our problem has distinct features that the number of returned products is limited and the returns are imperative. The products return process make the scheduling of manufacturing and remanufacturing much more complicated than the classic ESLP. The rest of this paper is organized as follow: the mathematic model is derived in the next section. In section 3, we present some numerical examples to investigate the system behaviors. Section 4 is our conclusion, and the last section is our acknowledgement.

II. Mathematic model

In this section, we will derive the mathematic model to accommodate the manufacturing and remanufacturing of the homogenous products on the same single product line. There are two inventory in the considered system, namely, the return inventory and serviceable inventory. The return inventory is used to hold the returned products and the serviceable inventory is used to stock the manufacturing or remanufacturing finished products to satisfy the demands. Before we give the mathematic model derivation, we make the following assumptions:

- Manufacturing or remanufacturing can not occupy the product line at the same time;
- The products return rate and demand rate are deterministic constant, furthermore, the demand rate is greater than return rate;
- The manufacturing rate and remanufacturing rate are deterministic and constant;
- No disposal of returned item are permitted;
- No backorder are permitted;
- There are setup costs and setup time associated with manufacturing and remanufacturing respectively.

Based on the above specifications, we introduce the following notations:

h_r: holding cost of a returned products of per unit time;
h_n: holding cost of a serviceable products of per unit time;
A_r: setup cost for remanufacturing;
A_m: setup cost for manufacturing;
r: constant return rate;
d: constant demand rate;
p_r: remanufacture rate;
p_m: manufacture rate;
s_r: setup time for remanufacturing;
s_m: setup time for manufacturing;
T: common cycle for manufacturing and remanufacturing.

Furthermore, we assume that $r < d < p_m$ and $r, d \leq p_r$.

In this paper, we employe the Common Cycle approach to schedule the production inventory system with return-flow. The Common Cycle approach behavior as: In each time cycle T, there are a remanufacturing batch and a manufacturing batch. If we assume that remanufacturing starts at the beginning of a production cycle, the remanufacturing process is ended when there are no returned items left. In consequence, the manufacturing starts when the serviceable items are exhausted. The manufacturing batch equals to the extra demands which can not satisfied by the returned items in each production cycle. Then a remanufacturing batch starts again when the serviceable inventory is empty and a new production cycle begins. The schemes of the return inventory and the serviceable inventory are illustrated in Figure 1. We should note that the manufacturing and remanufacturing should be prepared in advance since there is setup time.

During each cycle T, the number of remanufactured products is rT, and remanufacturing time (including setup time) is $s_r + rT/p_r$. The number of manufactured products is $(d-r)T$, and the manufacturing time (including setup time) is $s_m + (d-r)T/p_m$. We definite the busy rate of the product line as:

$$\rho = \frac{1}{T}\left[s_r + \frac{rT}{p_r} + s_m + \frac{(d-r)T}{p_m}\right] = \frac{s_r + s_m}{T} + \frac{r}{p_r} + \frac{d-r}{p_m} \tag{1}$$

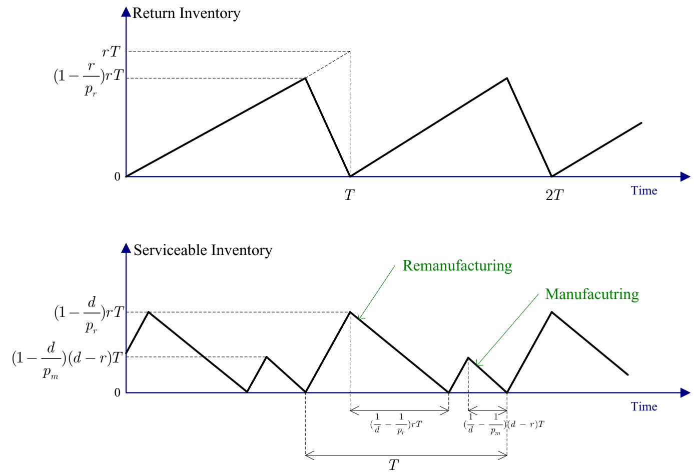

Fig. 1. The scheme of the return inventory and serviceable inventory

From the scheme shown in Figure 1, we can easily calculate that the holding cost of returned products in each period T is:

$$C_1 = \frac{1}{2}h_r\left(1 - \frac{r}{p_r}\right)rT^2 \tag{2}$$

The holding cost of remanufacturing finished in each period T is:

$$C_2 = \frac{1}{2}h_n\left(\frac{1}{d} - \frac{1}{p_r}\right)^2 dr^2T^2 \tag{3}$$

The holding cost of manufacturing finished in each period T is:

$$C_3 = \frac{1}{2}h_n\left(\frac{1}{d} - \frac{1}{p_m}\right)(d-r)^2T^2 \tag{4}$$

The total cost in each period T is:

$$\begin{aligned} C_T =& C_1 + C_2 + C_3 + (A_r + A_m) \\ =& \frac{1}{2}h_r\left(1 - \frac{r}{p_r}\right)rT^2 + \frac{1}{2}h_n\left(\frac{1}{d} - \frac{1}{p_r}\right)^2 dr^2T^2 \\ &+ \frac{1}{2}h_n\left(\frac{1}{d} - \frac{1}{p_m}\right)(d-r)^2T^2 + (A_r + A_m) \end{aligned} \tag{5}$$

The average total cost per time unit is:

$$\begin{aligned} C =& \frac{1}{2}h_r\left(1 - \frac{r}{p_r}\right)rT + \frac{1}{2}h_n\left(\frac{1}{d} - \frac{1}{p_r}\right)^2 dr^2T \\ &+ \frac{1}{2}h_n\left(\frac{1}{d} - \frac{1}{p_m}\right)(d-r)^2T + \frac{A_r + A_m}{T} \end{aligned} \tag{6}$$

which subject to the capacity constraint:

$$\left(\frac{1}{d} - \frac{1}{p_r}\right)rT \geq s_m \tag{7}$$

$$\left(\frac{1}{d} - \frac{1}{p_m}\right)(d-r)T \geq s_r \tag{8}$$

$$s_r + \frac{r}{p_r}T + s_m + \frac{d-r}{p_m}T \leq T \tag{9}$$

The constraints (7) and (8) ensure that the facility have enough time for manufacturing setup and remanufacturing setup respectively. The constraint (9) guarantee that a feasible solution exists. Let:

$$\begin{aligned} A &= A_r + A_m \\ B &= \frac{1}{2}h_r\left(1-\frac{r}{p_r}\right)r + \frac{1}{2}h_n\left(\frac{1}{d}-\frac{1}{p_r}\right)^2 dr^2 \\ &\quad + \frac{1}{2}h_n\left(\frac{1}{d}-\frac{1}{p_m}\right)(d-r)^2 \end{aligned}$$

Then the objective function (6) can be rewritten as:

$$C = \frac{A}{T} + BT \tag{10}$$

Note that $A, B > 0$, the objective is obviously convex in T. The optimal cycle time when disregarding the capacity constraints (7)-(9) is:

$$\widehat{T} = \sqrt{\frac{A}{B}} \tag{11}$$

Note that the constraint (7), (8) and (9) can also be expressed as:

$$T \geq \frac{dp_r s_m}{(p_r-d)r} = T_1 \tag{12}$$

$$T \geq \frac{dp_m s_r}{(p_m-d)(d-r)} = T_2 \tag{13}$$

$$T \geq \frac{(s_m+s_r)p_r p_m}{(p_r-r)p_m-(d-r)p_r} = T_3 \tag{14}$$

Since the objective function (10) is convex in T, the optimal solution, T^*, is easily obtained as:

$$T^* = \max(\widehat{T}, T_1, T_2, T_3) \tag{15}$$

III. NUMERICAL EXAMPLES

In this section, some numerical examples are presented to illustrate the system behaviors when the Common Cycle approach is implemented. Since return is the distinct feature in this paper, we give the numerical examples under different return rates.

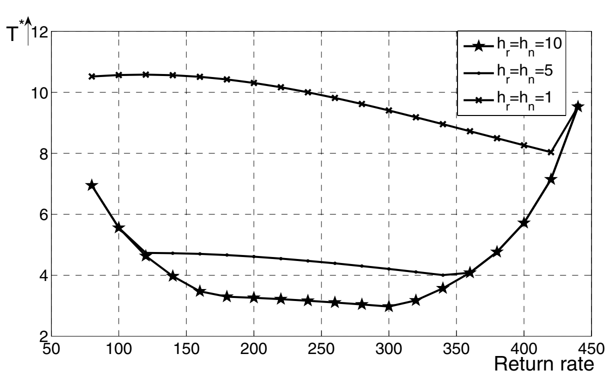

Fig. 2. The optimal production cycle under different holding cost, in which $A_r = 10000, A_m = 12000, d = 500, p_r = 5000, p_m = 4000, s_r = s_m = 1$

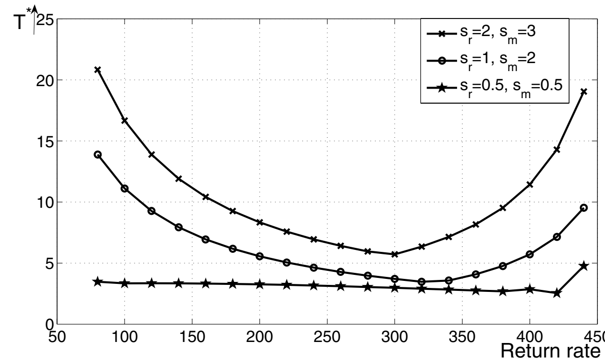

Fig. 3. The optimal production cycle under different setup cost, in which $A_r = 10000, A_m = 12000, d = 500, p_r = 5000, p_m = 4000, h_r = h_n = 10$

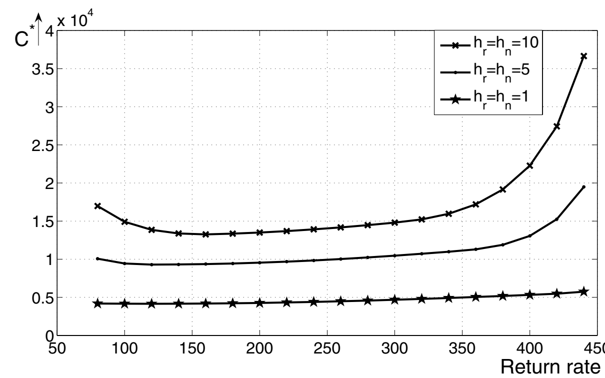

Fig. 4. The optimal total cost under different holding cost, in which $A_r = 10000, A_m = 12000, d = 500, p_r = 5000, p_m = 4000, s_r = s_m = 1$

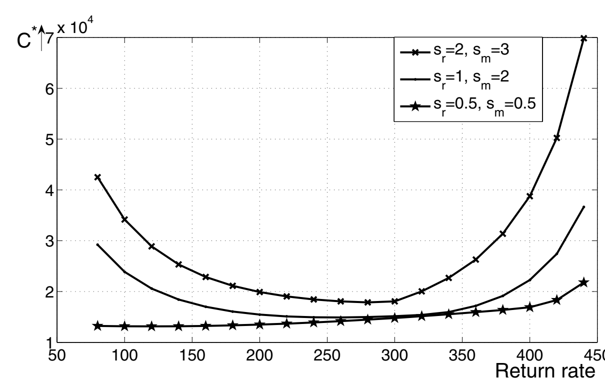

Fig. 5. The optimal total cost under different setup cost, in which $A_r = 10000, A_m = 12000, d = 500, p_r = 5000, p_m = 4000, h_r = h_n = 10$

First, we consider the performance of the optimal production cycle T^* under different return rates. The numerical results are illustrated in Figure 2 and Figure 3.

Figure 2 shows the comparison of the optimal production cycles under different holding cost h_r and h_n. We find that higher holding cost usually results in shorter production cycle, because that short production cycle can guard against higher holding cost. When the return rate is high (namely, $r = 360 \sim 440$), the optimal production cycles of $h_r = h_n = 5$ and $h_r = h_n = 10$ are the same and increase sharply, since that the optimal production cycle is determined by T_2 (see constraint (13)) when the return return rate r is close to demand rate d. The sharp decrease of the optimal production cycle in relatively low return return can be explained by the constraint (12).

In Figure 3, we can find that optimal product cycle decrease at first and then increase over the return rate, which is similar with the curve in Figure 2. These phenomena can be explained as: the optimal production cycle are mainly determined by constraint (12) when return rates are relative low, and the optimal production cycles are mainly determined by constraint (13) when return rates are relative high. These phenomena and explanations are accordant with the above discussion. We should note that the production cycles are nearly unchange over the return rate when $s_r = s_m = 0.5$, since that the optimal production cycle are mainly determined by the equation (11) when the setup time is relative low. We also find that the high setup time setting results higher production cycle, this result also can be explained by the constraint (12) and (13).

The performance of the optimal total cost of per unit time are presented in Figure 4 and Figure 5.

The optimal total cost of per unit time under different holding cost are illustrated in Figure 4. We can find the optimal total cost increase sharply when the return rate is close to the demand rate. This can be explained as: the optimal production cycle increases sharply when the return rate is close to demand rate (see Figure 2), and hence the total holding cost increases greatly. The fluctuation of the optimal total cost is rather flat when the holding cost is quite low (namely, $h_r = h_n = 1$).

From Figure 5 we can find that longer setup time is correlated with higher optimal total cost. To our surprise, the curve of Figure 5 resemble the curve of Figure 3. Since that the higher production cycle usually results in higher average inventory and then induces higher optimal total cost. The optimal production cycle with low setup time (namely, $s_r = s_m = 0.5$) is nearly unchange over return rate, as correspondence, the optimal total cost seldom change over return rate under same setup time setting.

IV. Conclusion

For the pressure of environmental legislations and economic interesting, many manufacturers have to take their products back after use and handle the returned products in environment-friendly manners. Remanufacturing is one of the most environment-friendly ways to deal with the returned products. How to integrate the remanufacturing into manufacturing process is a difficult problem. In this paper we have investigated the inventory control of a manufacturing/remanufacturing hybrid system, in which manufacturing and remanufacturing are carried on the same single product line. In comparison with the manner that remanufacturing is processed at a special product line, the single product line scheduling has the following advantage: (a) Saving the vast investment on a special product line for remanufacturing; (b) Improving the busy rate of the existing product line. In contrast with classical Economic Lot Scheduling Problem, the manufacturing/remanufacturing hybrid system have a distinct feature: the returned products are limited and can not be rejected. This return-flow make this scheduling problem much more complicated than the classic ESLP. In this paper, we extended the classical ESLP and used a Common Cycle approach to schedule the manufacturing and remanufacturing on the same product line. Our aim is to determine the optimal production cycle to balance the holding cost and setup cost such that the total cost is minimized.

In the numerical examples, we found that the optimal production cycles are mainly determined by the constraints in (12) and (13) when the difference between demand and return is large. Through comparing the optimal production cycle and optimal total cost, we found that the relative larger optimal production cycle is usually correlated with higher optimal total cost, especially when the holding cost is relative high.

As we know, this paper is the first research to investigate the scheduling problem that manufacturing and remanufacturing are accommodated on the same single product line. Unfortunately, the optimal policy structure is not know yet. Further research can focus on the optimal policy structure of this problem. By many numerical examples, we find that the busy rate of the product line, i.e. ρ, is not exceed 0.5 in most situations. It sounds economical to coordination the production of another items into the same production line. The problem of scheduling multi-item's manufacturing and remanufacturing on the same production line is much more difficult and can be an alternative focus in the future research.

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