An Improved Fuzzy Comprehensive Evaluation Method Using Expanded Least Deviations Algorithm

Wei Chen, Xiaohong Hao and Jie Lin
College of Electrical and information Engineering
Lanzhou University of Technology, LZUT
Lanzhou, China
E-mail: chenlin@lut.cn

***Abstract*—Fuzzy Comprehensive evaluation is usually influenced significantly by the matrix of fuzzy relation and weight vector. For a sequential segmentation category, the principle of the lowest cost, the principle of maximum degree of measure and the principle of maximum degree of membership sometimes can get unreasonable conclusion, because they conceal the difference of two degree of membership. First of all, a new expanded least deviations algorithm is presented for combining index weights, then bring out a improved fuzzy Comprehensive evaluation method based on reliability code. The proposed method can overcome the shortages of the traditional fuzzy Comprehensive evaluation. Case results clearly show that the proposed method is attractive and effective.**

***Keywords*—fuzzy Comprehensive evaluation, reliability code, expanded least deviations algorithm**

I. INTRODUCTION

Fuzzy Comprehensive evaluation is one of the most widely used methods in the decision-theoretic [1-8], but it is usually be influenced significantly by the matrix of fuzzy relation and index vector. For a sequential segmentation category, the principle of the lowest cost, the principle of maximum degree of measure and the principle of maximum degree of membership sometimes can get unreasonable conclusion, even sometimes can get error conclusion, because they conceal the difference of two degree of membership [9-12.] In this paper, a new expanded least deviations algorithm is presented for combining index weights, then bring out a improved fuzzy Comprehensive evaluation method based on reliability code. The proposed method can overcome the shortages of the traditional fuzzy Comprehensive evaluation. Case results clearly show that the proposed method is attractive and effective.

II. IMPROVED FUZZY COMPREHENSIVE EVALUATION METHOD

Suppose the index set $X=\{x_1,x_2,\cdots,x_n\}$ and the evaluation set $Y=\{y_1,y_2,\cdots,y_m\}$. R is the fuzzy relation between the indexes and the evaluation results; it is used to express all possibility that the index x_i ($i=1,2,\cdots\cdots n$) belongs to the evaluation results y_j ($j=1,2,\cdots\cdots m$). For instance, r_{ij} is the possibility that x_i belongs to the evaluation Results y_j . $\boldsymbol{\omega}=(\omega_1,\omega_2,\cdots,\omega_n)$ is weight vector, it indicates the importance of the indexes in the evaluation process. The result of the evaluation is fuzzy sets B, here $\boldsymbol{B}=(b_1,b_2,\cdots,b_m)$, b_i ($i=1,2,\cdots\cdots m$) means the degree of membership.

A. Expanded Least Deviations Aalgorithm for Combining Index Weights

In the process of the fuzzy evaluation, determining the proper weight is one of the most important procedures and has direct impact on the results of comprehensive evaluation. The judgment matrix method of the AHP（Analytic Hierarchy Process）is used in this paper. AHP is a method that through the analysis of complex systems and relationship between the factors contained, and then the system will be broken down into different elements, and these elements are incorporated into different levels, so as to form a multi-level analysis model objectively. According to a certain scaling theory, all the elements of each level will be compared so as to get the comparative scales indicating relative importance of the elements and establish the judgment matrix. By calculating the maximum eigenvalue of judgment matrix and the corresponding eigenvector to get the orders of the elements of each level to a certain element from the upper level, and thus the weight vector is determined. For the detailed algorithm about AHP can get in reference[13].

For a number of experts or a group of experts involved in evaluation process, the following expanded least deviation algorithm [14] can be used to combine the multiple expert judgment matrixes to get the final weight vector. The algorithm in one hand can integrate the experts' judgment information to the utmost; in another hand can reduce the dependence on individual expert.

Suppose $\boldsymbol{P}_l=[b_{ij,l}]_{n\times n}$, $l=1,2,\cdots\cdots m$ is the judgment matrix group, m is the number of the evaluation experts, $\boldsymbol{P}^*=[\omega_i/\omega_j]_{n\times n}$ is the weight matrix to be solved; $\boldsymbol{D}=\left\{\boldsymbol{\omega}\in \boldsymbol{R}^n \left| \sum_{i=1}^{n}\omega_i=1, \omega_i\geq 0, i=1,2,\cdots\cdots n\right.\right\}$ is weights vector

This work was supported in part by the Special Fund of Science and Technology Research of Gansu Province（GS044-A52-001-24）and by the Major International Joint Research Program of Gansu (0708WCGA45).

set, $\boldsymbol{\Omega}=\left\{\lambda\in \boldsymbol{R}^m \middle| \sum_{i=1}^{m}\lambda_l=1,\lambda_l\geq 0, l=1,2,\cdots\cdots m\right\}$ is weighted vector set; $\boldsymbol{e}=(1,1,\cdots\cdots 1)^{\mathrm{T}}\in \boldsymbol{R}^n$.

Suppose the Disturbance Matrix $\boldsymbol{E}_l=[\varepsilon_{ij,l}]_{n\times n}=[b_{ij,l}\frac{\omega_i}{\omega_j}]_{n\times n}$, $l=1,2,\cdots\cdots m$, Constructing the least squares model as follow:

$$\min f(\boldsymbol{\omega})=\frac{1}{2}\sum_{i,j=1}^{n}\sum_{l=1}^{m}\lambda_l(b_{ij,l}\frac{\omega_i}{\omega_j}+b_{ji,l}\frac{\omega_i}{\omega_j}-2)$$

$$\boldsymbol{\omega}\in \boldsymbol{D},\ \boldsymbol{\lambda}\in\boldsymbol{\Omega} \quad (1)$$

This solution to the above optimization problem is given by the Lagrange function:

$$L(\boldsymbol{\omega},k)=h(\boldsymbol{\omega})+k(\sum_{i=1}^{n}\omega_i-1)$$

$$=\frac{1}{2}\sum_{i,j=1}^{n}\sum_{l=1}^{m}\lambda_l(b_{ij,l}\frac{\omega_i}{\omega_j}+b_{ji,l}\frac{\omega_i}{\omega_j}-2)+k(\sum_{i=1}^{n}\omega_i-1) \quad (2)$$

where $k>0$ is the Lagrange multipliers, then making $\frac{\partial L}{\partial\omega_i}=0$, one can obtain

$$\sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_l(b_{ji,l}\frac{\omega_i}{\omega_j}-b_{ij,l}\frac{\omega_i}{\omega_j})+k\omega_i=0 \quad (3)$$

We can obtain $k=0$, then $\boldsymbol{\omega}$ must satisfy:

$$\sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_l(b_{ji,l}\frac{\omega_i}{\omega_j}-b_{ij,l}\frac{\omega_i}{\omega_j})=0 \quad (4)$$

Suppose $\varphi_i(\boldsymbol{\omega}(k))=\sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_l(b_{ji,l}\frac{\omega_i(k)}{\omega_j(k)}-b_{ij,l}\frac{\omega_i(k)}{\omega_j(k)})$

$$i=1,2,\cdots\cdots n \quad (5)$$

The steps of using expanded least deviations algorithm for combining index weights are as follows:

Step1: give failure value $\varepsilon>0$, let $k=0$, give initial value at random $\boldsymbol{\omega}(0)=(\omega_1(0),\omega_2(0),\cdots,\omega_n(0))$.

Step2:compute $\varphi_i(\boldsymbol{\omega}(k))$, $i=1,2,\cdots\cdots n$, if $|\varphi_i(\boldsymbol{\omega}(k))|<0$ for all i, accept the iteration value, it shows $\boldsymbol{\omega}(k)$ is mineral solution of $h(\boldsymbol{\omega})$, end。or wise, turn to step3。

Step3:suppose $|\varphi_s(\boldsymbol{\omega}(k))|=\max|\varphi_i(\boldsymbol{\omega}(k))|$, then compute

$$g(k)=(\sum_{j\neq s}\sum_{l=1}^{m}\lambda_l(b_{sj,l}\frac{\omega_j(k)}{\omega_s(k)})\Big/\sum_{j\neq s}\sum_{l=1}^{m}\lambda_l(b_{js,l}\frac{\omega_s(k)}{\omega_j(k)}))^{\frac{1}{2}} \quad (6)$$

$$\text{making } \phi_i(k)=\begin{cases}g(k)\omega_s(k), & i=s\\ \omega_i(k) & i\neq s\end{cases} \quad (7)$$

$$\omega_i(k+1)=\phi_i(k)\Big/\sum_{j=1}^{n}\phi_j(k),\ i=1,2,\cdots\cdots n \quad (8)$$

Step4: let $k=k+1$, back step2.

B. Evaluation Ccriteria

The Reference [12] showed that for a sequential segmentation category, the principle of the lowest cost, the principle of maximum degree of measure and the principle of maximum degree of membership sometimes can get unreasonable conclusion, even sometimes can get error conclusion, because they conceal the difference of two degree of membership. therefore proposed the principle of reliability code.

If evaluation categories $(y_1,y_2,\cdots\cdots,y_k)$ is a sequential segmentation of the attributes space Y, μ_x is the membership, here membership requires to unitary. λ is reliability code, considering the generally range of λ is $0.5<\lambda<1$, here $\lambda=0.6\sim0.7$ .If $y_1,y_2,\cdots\cdots,y_k$ meet $y_1>y_2>\cdots\cdots>y_K$, and

$$k_0=\min\{k:\sum_{l=1}^{k}\mu_{x_i}(y_l)\geqslant\lambda,\ 1\leqslant k\leqslant K\} \quad (9)$$

One can obtain that x_i belongs to category y_{k_0}.

If $y_1,y_2,\cdots\cdots,y_k$ meet $y_1<y_2<\cdots\cdots<y_K$, and

$$k_0=\max\{k:\sum_{l=k}^{K}\mu_{x_i}(y_l)\geqslant\lambda,\ 1\leqslant k\leqslant K\} \quad (10)$$

One can obtain that x_i belongs to category y_{k_0}.

C. Improved Multi-level Fuzzy Comprehensiv Evaluation Method

Step1: Decompose the index set $X=\{x_1,x_2,\cdots,x_n\}$ into s sub-set according some specific attribute, $X_i=\{x_{i1},x_{i2},\cdots,x_{ip_t}\}$, $i=1,2,\cdots s$, satisfy:

$$X=\bigcup_{i=1}^{s}X_i \quad (11)$$

Step2: Use single level fuzzy Comprehensive evaluating to each X_i Suppose the evaluating set is $Y=\{y_1,y_2,\cdots,y_m\}$, $\boldsymbol{\omega}_i=(\omega_{i1},\omega_{i2},\cdots,\omega_{ip_t})$ is the fuzzy weight vector of each element from X_i ,here, requires the ω_{ij} to meet : $\sum_{j=1}^{p_t}\omega_{ij}=1$, $\omega_{ij}\geqslant 0$, $j=1,2,\cdots p_t$. $\boldsymbol{\omega}_i$ can get by using the expanded optimization algorithm.

If the single element evaluating matrix of X_i is $\boldsymbol{R}_i$, then the result of single level fuzzy evaluating is

$$\boldsymbol{B}_i=\boldsymbol{\omega}_i\circ\boldsymbol{R}_i=(b_{i1},b_{i2},\cdots,b_{im}) \quad (12)$$

Step3: Regarding each X_i as a single index, use $\boldsymbol{B}_i$ as the single index evaluating result, we can obtain $\boldsymbol{R}$ which is the matrix of degree of membership

$$\boldsymbol{R}=\begin{bmatrix}\boldsymbol{B}_1\\ \boldsymbol{B}_2\\ \vdots\\ \boldsymbol{B}_s\end{bmatrix}=\begin{bmatrix}b_{11} & b_{12} & \cdots & b_{1m}\\ b_{21} & b_{22} & \cdots & b_{2m}\\ \vdots & & & \vdots\\ b_{s1} & b_{s2} & \cdots & b_{sm}\end{bmatrix} \quad (13)$$

If $\boldsymbol{\omega}=(\omega_1,\omega_2,\cdots,\omega_s)$ is the fuzzy weight vector of element X_i （$i=1,2,\cdots s$） ,then we can get second level fuzzy Comprehensive evaluating vector as follow

$$\boldsymbol{B}=\boldsymbol{\omega}\circ\boldsymbol{R}=(b_1,b_2,\cdots,b_m) \qquad (14)$$

If the element number of X_i （$i=1,2,\cdots s$） get from step(1) is too much, we can continue to decompose X into sub-set like third level or more higher level.

Step4: Get evaluation results by using the reliability code proposed in section 2.

D. Logical Operator

The factor "$\circ$" used in the fuzzy Comprehensive evaluating model $\boldsymbol{B}=\boldsymbol{\omega}\circ\boldsymbol{R}$ is weighted average factor

$$b_j=\sum_{k=1}^{s}\omega_k b_{kj}\ , \qquad j=1,2,\cdots,m \qquad (15)$$

It is relatively better than the other factors, because it can save the process information uttermost.

III. APPLICATION IN THE COMPREHENSIVE EVALUATING OF POWER QUALITY

The information of power quality is not only a significant part of technology support system of electricity market, but also is one of the constraint conditions of setting electricity tariff [15]. At the same time, to improve the quality of electricity is a key part of ancillary service in electricity market. Therefore, it is a trend and request to monitor and evaluate the power quality under the circumstance of electricity market.

The elements impacting the quality of power commodities are in large and complex, namely, power quality needs of a number of indicators to measure. How will the sub-indicators reasonable description and organization to reflect the quality of power together is actually a very complex multi-attribute Comprehensive evaluation and decision-making.

China has promulgated with the power quality national standards are: supply voltage allowed deviations; power system frequency allowed deviation; utility grid harmonics; voltage fluctuation and flicker; three-phase voltage allowed imbalance limitation; temporary over-voltage and transient over-voltage. In addition, some power quality problems are more and more attention by the people, such as the voltage sags and interruption, voltage transient movement, voltage swell, short-term over-voltage and under-voltage, these indicators are mostly transient power quality problems so far no uniform standard or index system. In the electricity market, another power quality indicator - the service indicators are attracting increasing attention. Based on above steady and transient factors, the hierarchical model for power quality comprehensive evaluation is established as shown in Figure 1. There are four first-level indicators of power quality; each first-level indicator has some second-level component indicators.

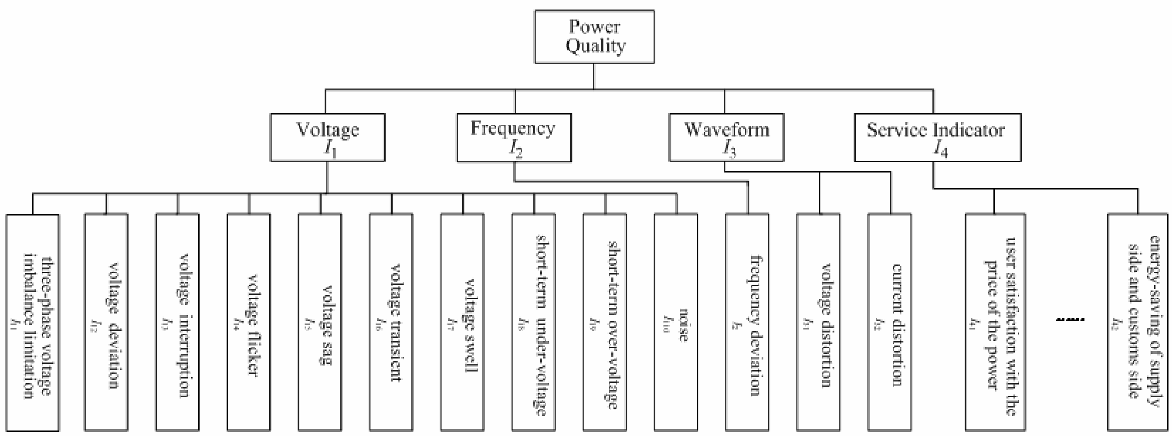

Figure 1 The Hierarchy model for power quality evaluation

Conducting Comprehensive evaluation of static indicators figured up with the data from the 95% probability value of the monitoring period and used SARFI (System Average RMS Variation Frequency Index) [16] for transient indicators.

Firstly, With the data provided by Electrotek Concepts which recorded by monitoring of a PCC(Point of common coupling) in a distribution power network to illustrate the evaluation methods. Detailed data are shown in Appendix Table 1.

The weights of 10 second-level indicators of voltage indicators is calculated by AHP Methods, the scores given by four experts used and the expanded optimization algorithm expounded in section 2.2, the weight vector is ω_1 = （0.1461，0.0931，0.0679，0.0998，0.2564，0.0441，0.1994，0.0392，0.0392，0.0146）. By using the same way we can get the weight vector of the first-level indicator as $\boldsymbol{\omega}$ =（0.5481，0.1301，0.2785，0.0433）.

The determination of the membership degree function of the indicators is divided into three categories, one is the steady-state power quality indicators, such as voltage deviation, frequency deviation, voltage fluctuation and flicker, and so on, most of them are landed within a certain interval and closing to zero, charged zero in on both sides of the probability of a random. The farther away from the zero value, the probability is smaller. So they have in line with the characteristics of normal distribution and adopted the characteristics of a Gaussian normal distribution as a function of its membership degree; another is the transient power quality (of the incident) indicators, such as the voltage sags and interruption, etc. Although, its appearance times in their examination period is random, but the frequency is very low, for the convenience, we have a triangular function as its membership degree; The third is service indicators, we can send out questionnaires to the customs in the area (regional) of the monitoring lines, and then use the statistical results of as the value of the membership degree function of the services indicators.

In accordance with the above improved fuzzy Comprehensive evaluation method, 12 group data of power quality is used ,the Comprehensive evaluation results shows in Table Ⅰ, it is consistent with the conclusions provided in references 17.

TABLE I. COMPREHENSIVE EVALUATION RESULTS OF POWER QUALITY

sample	1	2	3	4	5	6	7	8	9	10
Results	m	g	g	m	m	e	g	g	q	q

Here: q means qualified, m means medium, g means good, e means excellent, u means unqualified.

For Appendix Table Ⅱ, if using conventional fuzzy Comprehensive evaluation method, about more than 40% (5 group) data will obtain unreasonable even wrong evaluation results. For example, to evaluate sample 8th by proposed method in this paper, one can obtain $k_0 = 2$, and therefore the power quality of this time period belong to C_2 categories, promptly, the Comprehensive evaluation results of the sample is good. According to the conventional method of fuzzy Comprehensive evaluation of the evaluation results should be C_1, in fact we can see that sample belong to C_1 or C_2 are the properties of more or less equal measure, and samples belong to C_1 and C_2 together equivalent to 0.67, accounting for the entire attribute more than half, so that the sample belongs to C_1 attribute category is unreasonable.

IV. CONCLUSIONS

An improved fuzzy Comprehensive evaluation method is proposed in this paper, the method effectively overcome the shortages that the evaluation results seriously affected by the fuzzy relationship matrix and weight vector in using traditional fuzzy Comprehensive evaluation method. The application in Comprehensive evaluation of the quality of the power commodities illustrates the effectiveness and the feasibility of the proposed method.

REFERENCES

[1] L.C. Briand, E.l Emam and B.G Freimut, "A comprehensive evaluation of capture-recapture models for estimating software defect content", IEEE Transactions on Software Engineering,vol.26, no.6, pp.518 – 540,2000.

[2] J. Ma and D. Zhou, "Fuzzy set approach to the assessment of student-centered learning", IEEE Transactions on Education, vol.43, no.2, pp.237-241, 2000.

[3] Y.H. Li and Y.Q. Hu, "A Model of Multilevel Fuzzy Comprehensive Evaluation for Investment Risk of High and New Technology Project", Proceeding of 2006 International Conference on Machine Learning and Cybernetics, Dalian, pp.1942 – 1947,Aug. 2006,

[4] J.S. Gao, X.D. Shi and L.W. Wang, "Study on Machine Equipment Selection in Dynamic Alliance Based on Fuzzy Comprehensive Evaluation," Proceeding of the Sixth World Congress on Intelligent Control and Automation, vol.2, Dalian, pp.6787 – 6791, June 2006.

[5] S.A. Farghal, M,S, Kandil and A. Elmitwally, "Quantifying Electric Power Quality via Fuzzy Modeling and Analytic Hierarchy Processing", IEE Proceedings-Generation, Transmission and Distribution ,vol.149,no.1,pp.44-49,2002.

[6] Qinquan Jia, Jiahua Song，Hua Lan et al, "Quality of Electricity Commodity and Its Fuzzy Evaluation", Power System Technology,vol.24,no.6,pp.46-49, 2000.

[7] Huizhi Tang, Jianchun Peng, "Research on Comprehensive and Quantificated Appraisal Index of Power Quality Based on Fuzzy Theory", Power System Technology, vol.27,no.12,pp.85-88,2003.

[8] Xia Zhao,Chenyong Zhao and Xiufang Jia et al, "Fuzzy Evaluation of Power Quality Based on Changeable Weight", Power System Technology, vol.29,no.6,pp.11-16,2005.

[9] Yantai Chen, Guohong Chen, Meijuan Li. "Classification & Research Advancement of Comprehensive Evaluation Methods", Journal of Management Sciences in China, vol.7,no.2,pp.69-79, 2004.

[10] D. Ramot; R. Milo and M. Friedman, "Complex fuzzy sets", IEEE Transactions on Fuzzy Systems, vol.10, no.2, pp.171 – 186,2002.

[11] L.X. Wang, "Analysis and design of hierarchical fuzzy systems", IEEE Transactions on Fuzzy Systems, vol.7, no.5,pp. 617 – 624,1999.

[12] Qiansheng Cheng, "Attribute Recognition Theoretical Model With Application", Acta Scicentiarum Naturalun Uuniversitis Pekinesis, vol.33, no.1, pp.12-20, 1997.

[13] T.L. Sstty. The Analytic Hierarchy Process. McGraw- Hill Inc, New York,1988.

[14] Shoukang Qin, Comprehensive Evaluation Methods and Application, Electronic Press, Beijin, 2003.

[15] R. Sasdelli,G.D. Gobbo and G. Iuculano, "Quality Management for Electricity as a Processed Material" , IEEE Transactions on Instrumentation and Measurement,vol.49,no.2,pp.460-467, 2000.

[16] M.H.J. Bollen, D.D. Sabin, R.S. Thallam, "Voltage-sag Indices - Recent Developments in IEEE PI564 Task Force", Proceeding of CIGRE/IEEE PES International Symposium on Quality and Security of Electric Power Delivery Systems,vol.1, pp.34 – 41,2003.

[17] Wei Chen, Xiaohong Hao and Jie Lin, "Comprehensive Evaluation System and Method about Electrical Energy Quality Based on Attribute Recognition Theory and AHP", Electrical Engineering, vol.7,no.5,pp.26-30,2006.

Appendix:

TABLE II. MEASURED DATA OF PCC OF DISTRIBUTION NETWORK

Sample	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	I_{16}	I_{17}	I_{18}	I_{19}	I_{110}	I_2	I_3
1	5	2	1.67	0.89	0	4	3	0	0	1.15	0.116	2.47
2	11	2	1.40	0.78	1	1	1	0	0	1.55	0.06	2.39
3	13.7	2	1.11	0.78	0	1	0	0	0	1.20	0.65	2.40

4	7.3	1.1	1.56	0.85	0	2	2	0	0	1.15	0.092	3.50
5	6	2	1.53	0.75	1	3	0	0	0	0.90	0.07	3.19
6	3	1	0.96	0.71	0	1	0	0	0	0.94	0.06	1.61
7	5	2	1.24	0.77	0	2	1	0	0	1.15	0.066	2.74
8	5	2	1.17	0.73	1	1	0	0	0	1.05	0.06	2.59
9	4	2	2.08	0.96	5	8	0	0	0	1.87	0.12	4.28
10	6	2	2.17	0.97	9	10	4	0	0	2.01	0.132	4.34