Leakage Detection Via Model Based Method

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***Abstract*— This paper presents a model based approach for real time leakage detection in a nonlinear multi tank flow rig. A state space model of the process was derived based on the expansion of the nonlinear function into a Taylor's series about the operating point and the retention of only the linear term. The model based approach performs consistency checks of the process against the state space model of the process. Using fault detection technique, leakage indicating signals were generated. False detection due to uncertainty as a result of using the linearised reference mathematical model and sensor noise can be avoided by referring to real time measurement data recorded when system is operating normally. This method has been implemented in real time and results show promising potentials.**

***Keywords*—multi-tank rig, leakage detection, model based systems, real time.**

I. Introduction

Industrial processes need to satisfy very demanding requirements for the quality of the produced goods, environmental protection and safety regulation. Such unsatisfactory operation conditions are commonly related to a malfunction of a set of actuators, sensors or system parameters, which would directly result in an accident. Prompt diagnosis of faults [1] such as component leakages in industrial processes is essential to minimize production losses and increase the safety of the operators and the equipment [2]. As a direct consequence, more instrumentation and control systems must be introduced into the production process, increasing its complexity and affecting its reliability [3].

Simulation results in fault diagnosis of an industrial gas turbine was carried out by [4] defining a comprehensive methodology for fault diagnosis by using a state estimation approach, in conjunction with residual processing schemes, including a simple threshold detection. It describes how this is achieved in the noise-free case as well as the noisy case using statistical analysis tools, when the data are affected by noise. Discrete-event systems fault diagnosis in sensors and actuators fault based on a generized observer scheme has been carried out by [5] describing a method for detecting and identifying faults that occur in the sensors or in the actuators of dynamical systems with discrete-valued inputs and outputs. The model used in the diagnosis is a stochastic automaton. The generalized observer scheme (GOS) is extended by a fault detection module to cope with plant faults that are different from actuator or sensor faults.

Determining the probability of a fault event given any signature during plant operation can improve the accuracy in correctly identifying and isolating faults [6]. By assigning transition probabilities and marginal probabilities to safe and fault events, it is possible to determine the feasible configurations of alarms (signatures) and their conditional probability given any event.

Plant identification offers a powerful method of projecting some aspects of physical reality into a mathematical description or model. Parametric time-domain models are the form most widely employed for system identification, being particularly suited to digital computing and real-time applications [7]. The identified parameter then can be used to explain the behavior of the target system as well as for prediction and control purposes.

A very useful method for emulating system performance is by means of a mathematical model that will produce the relevant output for a given input. In a model-based fault detection system, the output of the model and that of the plant are compared and the difference is used to generate appropriate fault indicating residuals. Model-based systems have been used in systems situations involving nonlinearity and time-varying parameter successfully [8]. Model based methods has been developed for many model domains, e.g. models from the AI-field which are often logic based [9], or Discrete Event Dynamic Systems for which automata descriptions are common [10]. A third model domain that is commonly considered are models typically found in the field of signals and systems, i.e. models involving continuous variables in continuous or discrete time. Typical model formulations are differential/difference equations, transfer functions, and/or static relations.

A model-based fault detection algorithm which is generic in the sense, that any model correctly describing a functional dependency inside a system can be enclosed easily almost without adjusting any thresholds or other essential parameters [11]. Other description of model based approaches in fault detection in engineering systems are described in [12] for a centrifugal pump; fault detection algorithm is based on an Analytical Redundancy Relation (ARR); hydraulic system in [13-14] - application of a nonlinear model based adaptive robust observer (ARO) to the fault detection and diagnosis of faults and robust filter structures are designed to attenuate the effect of model uncertainties; for induction motor in [15]- using recurrent dynamic neural networks for transient response

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prediction and multi-resolution signal processing for nonstationary signal feature extraction; [16] considered fault estimation using a delayed FIR filter that is designed as a noncausal Wiener filter. The approach was demonstrated using a case study of a process upset in a separation column in a petrochemical plant.

This paper presents a real time model based leakage detection system using fault detection techniques for a nonlinear multi tank flow rig. The system is modelled based on the expansion of its nonlinear function into a Taylor series about the operating point and the retention of only the linear term. Consistency checks of the process against the model of the process are automatically performed online. Leakage in the tanks is detected when output behaviour deviates from expected conditions. Although a linearized model is used, a practical yet effective manner of avoiding false detection is proposed based on measured data from the process concerned.

II. Multi Tank Flow Rig Process Model

The multi tank flow rig is as shown in Fig. 1. It consists of two water tanks (Tanks 1 and 2), two water level sensors (indicating water levels h_1 and h_2), and two actuators that is a water pump (Act_1) and a solenoid valve (Act_2). Water from *Tank 1* can flow to *Tank 2* via an opening of cross sectional area a_1, whilst maximum diameter of valve opening is denoted as a_2. A_1 and A_2 denote base cross sectional area of Tanks 1 and 2 respectively.

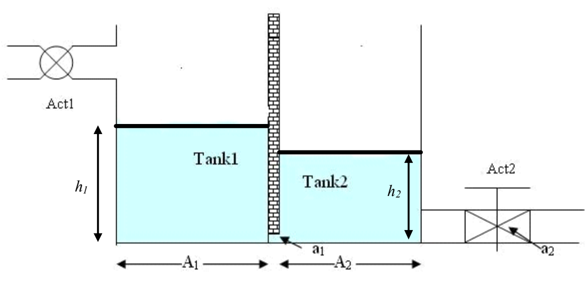


Figure 1. The multi tank flow rig used in case study

A. The nonlinear process model

The process of linearizing nonlinear system makes it possible to apply numerous linear analysis methods that will produce information on the behaviour of nonlinear systems. The linearization procedure presented here is based on the expansion of the function into a Taylor series about the operating point and the retention of only the linear term. Higher order terms will be neglected, since it is small enough; that is, the variables deviate only slightly from the operating condition. Consider a nonlinear system whose output y is a function of two inputs x_1 and x_2, so that,

$$y = f(x_1, x_2) \quad (1)$$

To obtain a linear approximation to this nonlinear system, (1) is expanded into a Taylor's series about the nominal operating point $\bar{x}_1, \bar{x}_2$, where the partial derivatives are evaluated at $x_1 = \bar{x}_1, x_2 = \bar{x}_2$. Near the normal operating point, the higher order terms are neglected. The linear mathematical model of this system in the neighbourhood of the normal operating condition is then given by

$$y - \bar{y} = K_1(x_1 - \bar{x}_1) + K_2(x_2 - \bar{x}_2) \quad (2)$$

where

$$\bar{y} = f(\bar{x}_1, \bar{x}_2)$$

$$K_1 = \left.\frac{\partial f}{\partial x_1}\right|_{x1=\bar{x}1, x2=\bar{x}2}$$

$$K_2 = \left.\frac{\partial f}{\partial x_2}\right|_{x1=\bar{x}1, x2=\bar{x}2}$$

For the flow rig system shown in Fig.1, mass balance for one tank gives us:

$$A\frac{dh}{dt} = Q_{in} - Q_{out} \quad (3)$$

where A is the cross section, h is the water level and Q_{in} and Q_{out} are the flow rate in and out of the tank respectively. The flow out of *Tank 1* can be described by Bernoulli's Law,

$$Q_{out1} = a_1\sqrt{2g(h_1 - h_2)} \quad (4)$$

The flow out of *Tank 2* can be similarly described using Bernoulli's Law as

$$Q_{out2} = a_2\sqrt{2g(h_2)} \quad (5)$$

where a is the cross section of the outlet hole and g is the acceleration of gravity. The input to the process is v (duty cycle of input voltage to the pump [0-100 %]) and the outputs are y_1 and y_2 (water level in Tanks 1 and 2). Assume that the flow generated by the pump can be presented by following equation

$$Q_{in} = k_a(v - v_{min}) \quad (6)$$

Where v_{min} is the minimum duty cycle necessary for the pump starts operating. The model can then be described as

$$\frac{dh_1}{dt} = \frac{k_a}{A_1}(v - v_{\min}) - \frac{a_1}{A_1}\sqrt{2g(h_1 - h_2)} \quad (7)$$

$$\frac{dh_2}{dt} = \frac{a_1}{A_2}\sqrt{2g(h_1 - h_2)} - \frac{a_2}{A_2}\sqrt{2gh_2} \quad (8)$$

And thus,

$$\frac{d(h_1 - h_2)}{dt} = \frac{k_a}{A_1}(v - v_{\min}) - \left(\frac{1}{A_1} + \frac{1}{A_2}\right)\left(a_1\sqrt{2g(h_1 - h_2)}\right) + \frac{a_2}{A_2}\sqrt{2gh_2} \quad (9)$$

where subscript i corresponds to tank i.

B. Linearization

In water *Tank 1*, assuming that at $t = 0$ the duty cycle controlling the pump is changed from $v = \bar{v}$ to $v = \bar{v} + \Delta v$.

This change causes the water level in *Tank 1* to change from $h_1 = \bar{h}_1$ to $h_1 = \bar{h}_1 + \Delta h_1$, which in turn, causes the water level in *Tank 2* to change from $h_2 = \bar{h}_2$ to $h_2 = \bar{h}_2 + \Delta h_2$. By using the linearization technique shown in (2), (9) can be linearized at steady state: $f_1(\bar{h}_1, \bar{h}_2, \bar{v}) = \frac{d(h_1 - h_2)}{dt} = 0$, the following is derived

$$\frac{d(h_1 - h_2)}{dt} = -\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\frac{a_1\sqrt{g}}{\sqrt{2(\bar{h}_1 - \bar{h}_2)}}[h_1 - h_2 - (\bar{h}_1 - \bar{h}_2)] \ldots + \frac{a_2\sqrt{g}}{A_2\sqrt{2\bar{h}_2}}(h_2 - \bar{h}_2) + \frac{k_a}{A_1}(v - \bar{v}) \tag{10}$$

Similarly for (8), at steady state, $f_2(\bar{h}_1, \bar{h}_2, \bar{v}) = \frac{dh_2}{dt} = 0$,

$$\frac{dh_2}{dt} = \frac{a_1\sqrt{g}}{A_2\sqrt{2(\bar{h}_1 - \bar{h}_2)}}[h_1 - h_2 - (\bar{h}_1 - \bar{h}_2)] - \frac{a_2\sqrt{g}}{A_2\sqrt{2\bar{h}_2}}(h_2 - \bar{h}_2) \tag{11}$$

Finally the state space model of of the linearized process at nominal operating conditions $(\bar{h}_1, \bar{h}_2, \bar{v})$ for $\bar{h}_1 > 0$ is expressed as:

$$\dot{x} = \begin{pmatrix} -\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\left(\frac{(a_1)^2}{a_2}\sqrt{\frac{g}{2\bar{h}_2}}\right) & \frac{a_2}{A_2}\sqrt{\frac{g}{2\bar{h}_2}} \\ \frac{(a_1)^2}{A_2 a_2}\sqrt{\frac{g}{2\bar{h}_2}} & \frac{-a_2}{A_2}\sqrt{\frac{g}{2\bar{h}_2}} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{k_a}{A_1} \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{12}$$

where

- $x_1 = h_1 - h_2 - (\bar{h}_1 - \bar{h}_2)$
- $x_2 = h_2 - \bar{h}_2$
- $u = v - \bar{v}$
- $y_2 = h_2$
- $y_1 = h_1$
- h_1 and h_2 denotes height of water levels in Tanks 1 and 2 respectively
- $\bar{h}_1$ and $\bar{h}_2$ denotes the nominal operating condition for height of water levels in Tanks 1 and 2 respectively
- v denotes the input (duty cycle to controlling pump)
- A_1 and A_2 denotes base cross sectional area of Tanks 1 and 2 respectively
- Cross sectional area of opening in *Tank 1* for water flow to Tank 2 is denoted as a_1
- Maximum cross sectional area of valve opening is denoted as a_2

The numerical values of the physical parameters are as listed in Table 1.

TABLE I. PHYSICAL PARAMETERS

$A_1 = 33cm^2$	$a_1 = 0.053cm^2$
$A_2 = 33cm^2$	$a_2 = 0.03cm^2$
$k_a = 0.57\ cm^3/(duty\ cycle)$	$g = 980cms^{-2}$
$\bar{h}_1 = 10.5$cm	$\bar{h}_2 = 7.8$cm

III. LEAKAGE DETECTION

From linearization of nonlinear mathematical model, state space model of linearization at stationary point has been obtained as in (12). The model based approach performs consistency checks of the process against the state space model of the process. By using data from available sensors, the proposed leakage detection system checks the sensor data and compares it with the expected response derived from system model to generate leakage indicating residuals.

Fig. 2 is the block diagram showing how the linearized state space model of the process or plant can be used as a reference for the model based leakage detection algorithm to deduce a fault indicating residual generated due to the difference between the output values from state space model and that of the plant.

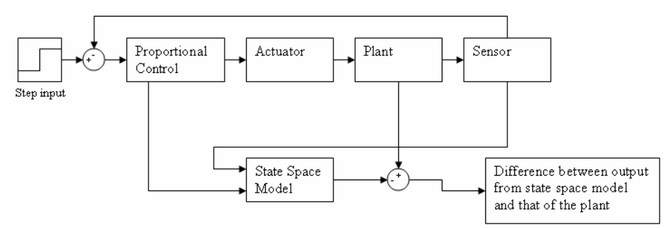


Figure 2. State space model of plant for generating leakage indicating signals

A. Leakage indicating signals

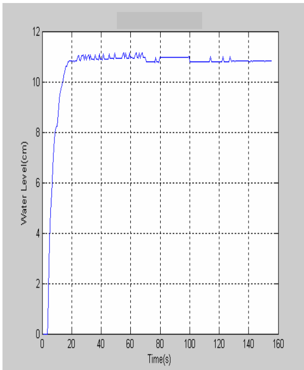
Real time water levels corresponding to time was recorded. When system is operating close to operating point, the proposed fault detection system will compare the real time data obtained from sensors to the online simulation results obtained from state space model (12) to generate a leakage indicating signal, shown as mathematical expressions in (13).

$$D_{t1} = y_1(t + \Delta t) - y_1(t) - \int_t^{t+\Delta t} \dot{x}_1 + \dot{x}_2 dt$$
$$D_{t2} = y_2(t + \Delta t) - y_2(t) - \int_t^{t+\Delta t} \dot{x}_2 dt \tag{13}$$

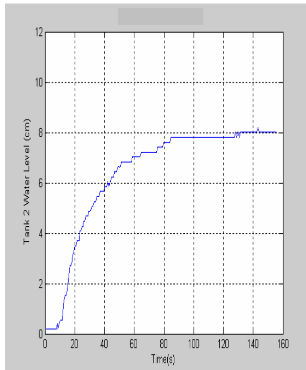
where

- D_{ti} *denotes the fault indicating residual generated*

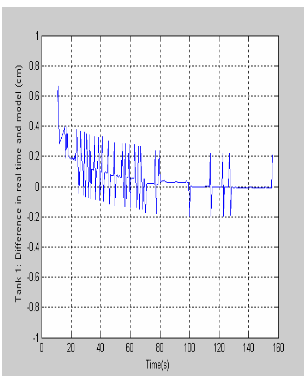
- $y_i(t)$ *denotes actual water level at time* t
- Δt denotes small change in time t



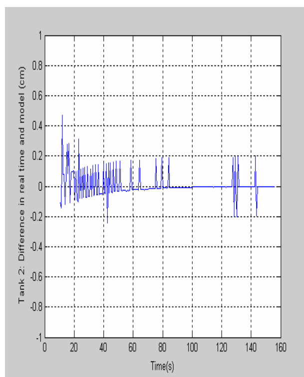
(a)



(b)



(c)



(d)

Figure 3. Results of real time testing under normal operation

(a) Step response of water level in *Tank 1*.

(b) Step response of water level in Tank 2.

(c) Leakage indicating residual generated for Tank 1.

(d) Leakage indicating residual generated for Tank 2

B. Determining nominal operating parameters

State space model of the water tanks system is only an approximation of true system dynamics. Besides, the performance of the system also changes over time and will not be consistent with design. In ideal cases, whereby modelling uncertainty is not present, leakage indicating signal is obviously at zero. However, in reality there always exists difference between mathematical model and actual parameters to a certain limit. As a matter of fact, for the case study presented in this paper, modelling discrepancy always exist due to the linearization process and possibly due to sensor noise.

Nevertheless, the limits mentioned can be derived from real time measurement data- by observing the limits of fault indicating residuals magnitudes during nominal operation. Thus, before the proposed detection system can be implemented, real time measurements to obtain leakage indicating residual magnitudes need to be performed when flow rig is operating under normal conditions. Figs. 3(a) and 3(b) shows the system responses when step input = 0.7 for *Tank 1* and step input = 0.52 for *Tank 2*. (Setpoint of *Tank 1* at 70% of full tank- corresponding to water level, $\bar{h}_1$ = 10.5cm.

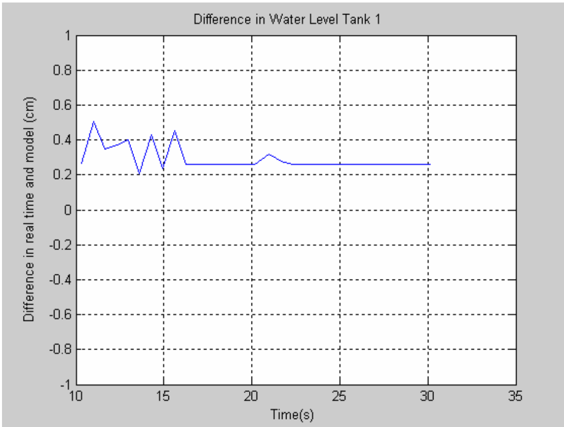
Setpoint of *Tank 2* at 52% of full tanks- corresponding to water level, $\bar{h}_2$ = 7.8cm). Figs. 3(c) and 3(d) shows the fault indicating residuals generated. All devices and parameters of the process are checked to ensure normal operating conditions without the possibility of leakage occurring.

Measurements of the leakage indicating residuals are recorded in Table II. From Table II, it is observed that the average magnitude of the leakage indicating residuals, $\boldsymbol{D_{t1}}$ and $\boldsymbol{D_{t2}}$ approaches zero when water levels in both tanks approaches their respective operating points. Note that the results shown in Table II are averaged between the respective time periods. A practical conclusion from the results shown in Table 1 is that any magnitude of leakage indicating residuals lying within the the range $|D_{t1}| < 0.2$ and $|D_{t2}| < 0.02$ indicates normal system operation. It is reiterated that this observation is important in order to avoid false detection due to the use of a linearized mathematical model as reference and due to uncertainty such as sensor noise in the system.

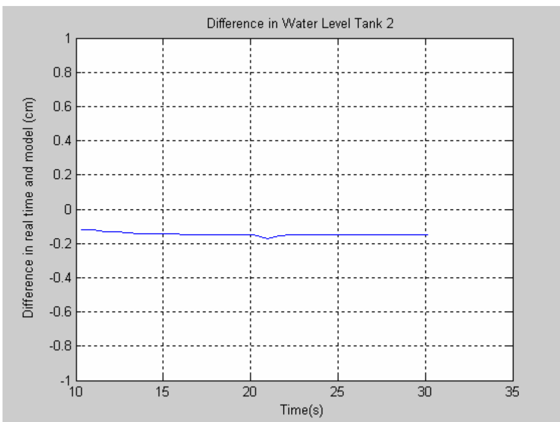
TABLE II. AVERAGE MAGNITUDE OF LEAKAGE INDICATING SIGNALS

Time (s)	**D_{t1}(cm)**	**D_{t2}(cm)**
0-30	*0.20*	*0.02*
30-60	*0.10*	*0.05*
60-90	*0.04*	*-0.03*
90-120	*0.02*	*-0.01*
120-150	*0.00*	*0*

A leak was introduced into the system after the process has reached steady state. Fig. 4 shows the leakage indicating residuals generated for both tanks. The average value for the leakage indicating residuals generated are, $\boldsymbol{D_{t1}}$= */*0.26*/ cm* which is larger than *0.1cm* and $\boldsymbol{D_{t2}}$ = */-0.146/ cm* which is larger than *0.05cm* indicating that a leak had occurred in the system.



(a)



(b)

Figure 4. Real time results of leakage indicating signals

(a) Fault indicating residual generated for *Tank 1*

(b) Fault indicating residual generated for *Tank 2*

IV. CONCLUSION

This paper has proposed a model based leakage detection method for a nonlinear multi tank flow rig. By comparing real time results to results obtained from state space model, a leakage indicating residual is able to indicate the possibility of leaks in the system. The proposed algorithm presented in this paper was tested in real time and have taken into consideration the uncertain condition that can occur in real time for avoiding false detection- by ensuring that the magnitudes of fault indicating residuals although not ideally at zero when system is operating normally lie between a certain range of values.

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