

Mode Tracking of Hybrid Systems in FDI Framework

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Abstract—A hybrid system combines continuous and discrete dynamics and runs with a set of modes. In [3],[4], we proposed an efficient health monitoring method for hybrid systems. This method utilizes unified constraint relations, named the Global Analytical Redundancy Relations (GARRs). Using GARRs for hybrid system health monitoring requires knowledge of the system's mode which is provided by a mode tracker. GARRs represent global information (i.e. information relevant to all modes), and the hybrid system properties can be analyzed across system's modes. In this paper, we utilize this unique feature to develop a GARRs based mode tracking approach. The most significant contribution of this development is the Mode-Change Signature Matrix, its derivation from the GARRs and its use for mode tracking.

Keywords—Hybrid systems, model based, fault detection and isolation, mode tracking, mode-change signature.

I. INTRODUCTION

A hybrid system consists of continuous and discrete dynamics and operates in various states represented by a set of modes. In each mode, the system is governed by continuous dynamics and different modes correspond to different continuous models.

Hybrid system monitoring requires measurement or estimation of continuous state-variables and tracing the system discrete dynamics (i.e. the system mode evolution). Discrete events force the system to move from one mode to another; these changes are referred to as mode-changes. Some of the mode-changes are known (e.g. initiated by a supervisory controller) and some are unknown but measurable (e.g. triggered by measured continuous states). The main difficulty when applying model based monitoring techniques to hybrid systems is due to unpredicted mode-changes caused by unknown discrete inputs, and unknown discrete dynamics; these mode-changes can happen at any time and in any order.

Model based health monitoring of dynamical systems is based on residues which measure the distinction between the actual system and its model. The model represents the system's normal condition and as long as the residues' absolute-value is below a certain threshold, we can conclude that the system is normal. When the monitored system is hybrid, and in the case of unpredicted mode-change, the prevailing continuous model (used for system monitoring) is no longer valid for the monitored system. As a result, residues used for model-based health monitoring will exhibit abnormal behaviour which can be interpreted as two different

phenomena, namely component-fault (fault, which is not represented as a mode of the hybrid system), or a mode-change. In health monitoring of hybrid systems, it is essential to distinguish between these two scenarios and to identify the monitored system new mode, in the case of mode-change.

A common approach to model based Fault Detection and Isolation (FDI) of hybrid systems is to develop a monitoring system with two modules. One module is in charge of the continuous monitoring of each mode, and the other module is in charge of identifying, if possible, the current mode of the monitored system. In this paper we study the second module, and propose an efficient method for mode tracking of hybrid systems in FDI framework.

In [8], a hybrid observer is proposed as part of a model based FDI paradigm for hybrid systems, based on a combination of a Kalman filter (for continuous tracking) and a mode change detector. The discrete events are either known controlled mode changes or autonomous mode changes (triggered by the plant's continuous states). Once the mode-change conditions are detected, a Finite State Machine is utilized to determine the new mode. This method does not utilize the mode continuous dynamics for mode identification.

A different approach is taken in [1] and [9], where a bank on N continuous observers (each represents a suspected system's mode) is used simultaneously for mode identification. The disadvantage of these methods is their high demand for computational-resources. If a discrete dynamical model is not given, then the complete set of modes' continuous-dynamics is analyzed on-line simultaneously. These methods may not be applicable to complex systems with large number of modes and states.

In [7] we proposed an energy based approach for mode tracking of hybrid systems. The method is based on a unique representation of each system's mode, with compact power relations, named power-nets (PNs). This method is efficient, and can be utilized for hybrid system's mode identification in general context (not necessarily health monitoring). However, in FDI framework this method does not enjoy the reach and relevant information for mode tracking that is given by the FDI module and its residues.

In previous works [3],[4], we proposed the concept of Global Analytical Redundancy Relations (GARRs), which describe the behavior of the hybrid system quantitatively at all modes. With this concept, we develop a method to analyze the

hybrid system's fault monitoring ability in an efficient manner. Besides the off-line analysis, the GARRs form a set of residues which is used for on-line health monitoring. In this work we study the influence of the mode-change on these residues. These influences are named mode-change signatures and we represent these signatures in a table, named the Mode-Change Signature Matrix (MChSM). We utilize the MChSM to an efficient mode tracking method of hybrid systems in FDI framework.

The paper is organized as follows: section II reviews some of our developments for health monitoring of hybrid systems, and relevant tools are explained. Section III presents the mode tracking method and its function in the health monitoring framework. Section IV concludes the paper.

II. HYBRID BOND GRAPH AND GLOBAL ANALYTICAL REDUNDANCY RELATIONS (GARRS)

An efficient health monitoring approach to complex dynamical systems is model-based. This approach uses a system dynamical model as reference to the system normal condition. A set of constraint relations, known as Analytical Redundancy Relations (ARRs), is derived from the system dynamical model. ARRs represent the monitored system normal dynamics, and form a set of residues. When the monitored system is normal, all of its residues have a value near zero. After a fault occurrence, some or all of these residues will exhibit absolute value greater than zero and trigger a fault detection and isolation process. Any component-fault (i.e. a change of component's physical parameter with respect to its normal value) has a specific influence on these residues; this influence is named a signature and presented in the Fault Signature Matrix (FSM). The FSM is utilized for on-line fault isolation, and for off-line monitoring ability analysis. If a component-fault has a nonzero signature, then we say that the component-fault is detectable, if the component-fault has a unique signature, we say that the component-fault is isolable.

To derive an ARR, we need to eliminate all unknown variables (such as unmeasured state variables) from the model constraint relations; this is not trivial, especially for large complex systems.

Bond graph model [5] and its unique causality representation is a powerful tool for ARR derivation. Cause-effect relations between physical variables are clearly presented in the graph and lay a foundation to systematic and effective ARRs derivation techniques.

In bond graph, the system structure is modeled by junctions, which are based on the energy conservation principle. Junctions' constitutive relations are a useful way for ARRs derivation. The notion of the junction's output-variable is important to our developments and therefore is explained. We consider the 1-type junction in Figure 2 as an example. For a 1-type junction, the flow variable is identical at all bonds connected to the junction, and the effort is constrained by the energy conservation principle, as describe in (1).

$$\begin{aligned} f_1 &= f_2 = f_3 \\ f_1 e_1 &= f_2 e_2 - f_3 e_3 \Rightarrow e_1 = e_2 - e_3 \end{aligned} \quad (1)$$

In a 1-type junction, only one bond determines the junction common flow, this is the bond that its causal stroke is away of the junction. Only one effort variable is determined by the junction's constitutive relation, we name this variable the *junction's output-variable*. For the 1-type junction in Figure 1, the junction's output-variable is the effort e_1 .

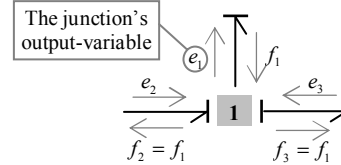


Figure 1. Bond graph junction

Model-based monitoring of hybrid systems requires hybrid-dynamics modeling approach, and we utilize the Hybrid Bond Graph (HBG) [6]. The HBG uses the concept of controlled-junctions to represent the system discrete dynamics. A single controlled-junction has two discrete states, ON and OFF. When the junction is ON it functions as a standard bond graph junction. When the junction is OFF it enforces its common power variable to zero. A 1-type controlled-junction enforces zero flow, and a 0-type controlled-junction enforces zero effort. The operating principle of a 1-type controlled-junction is demonstrated in Figure 2 (where X represents a bond graph component).

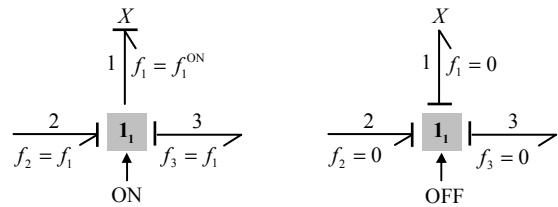


Figure 2. The 1-type controlled-junction operation

One important property of controlled-junction is causality inversion. We consider the 1-type controlled-junction in Figure 2 as an example. When the junction is ON, it functions as a standard 1-type junction. One bond determines the junction common flow and its causal stroke is directed away of the junction (bond-1 in the figure); this bond also carries the junction's output-variable. When the junction is OFF the junction enforces zero flow in all bonds adjacently connected to the junction and all causal strokes of adjacent bonds are directed towards the junction (i.e. all flows are determined by the junction). Consequently, the junction's output-variable bond reverses its causality, none of the effort variables is constrained by the junction, and the junction constitutive relation, of the ON state, is no longer valid.

In HBG, system's modes are defined by the controlled-junctions' states and mode-change is modeled by the controlled-junctions' change-of-state. A simple ARRs derivation method for hybrid systems is to consider each system's mode individually, e.g. for any possible combination

of controlled-junctions' state, we derive the ARR, using standard derivation methods of continuous systems. However this method is tedious, inefficient and does not enjoy the unique global representation of the HBG.

In [3] and [4], we extended the concept of ARR-based health monitoring, to hybrid systems, and developed the Global Analytical Redundancy Relations (GARRs). GARRs are unified constraint relations which describe the behavior of the system quantitatively at all modes. Using GARRs for health monitoring requires knowledge of the system' mode; this information is provided by a mode tracker.

GARRs derivation is inspired by the principles of classical ARRs derivation methods for continuous systems; these methods utilize the concept of causal-paths. It is clear that an efficient derivation of global constraint relations in hybrid systems requires consistent causal-paths at all modes. Such causality description is achieved in a DHBG [3]. We use the following definitions:

Definition 1: An inactive bond-graph component can be one of the following: an inactive controlled-junction (i.e. OFF state), a null source (of effort or flow), or any other bond graph components such that any input-variable of the component is enforced to zero by a controlled-junction. Any component that is not inactive is considered active.

Definition 2: A Diagnostic Hybrid Bond Graph (DHBG) is a hybrid bond graph that is assigned with suitable set of causalities, such that the causality of every active bond graph component is valid and consistent at all operating modes.

In Figure 2 we can see that the controlled-junction ON and OFF descriptions are different. For the GARRs derivation, a controlled-junction unified description is necessary (i.e. a single description of the junction at its two states). We use the unified description as presented in Figure 3.

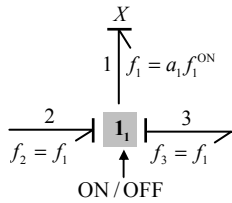


Figure 3. A unified description of controlled-junction

The binary state variable $a_1 \in \{0,1\}$ represents the junction state (and is named the controlled-junction's state-variable), when the junction is ON its state variable has a value $a_1=1$ and when the junction is OFF the variable is zero. At the ON state, component X determines the junction common flow; this flow is notated by f_1^{ON} . When the junction is OFF, $a_1=0$ and from the figure it is clear that all flows of adjacent bonds are zero. In addition the causality of bond-1 (the controlled-junction's output-variable bond) is inverted; this causality change can be problematic in a DHBG. Property 1 states a necessary and sufficient condition, to achieve a DHBG.

Property 1: A DHBG is achieved, if and only if any controlled-junction's output-variable is an input-variable to a component that is inactive when the junction is OFF.

The DHBG is aimed to prevent the causality conflicts that may occur after a mode change. The DHBG limits the causal change only to the controlled-junction's output-variable bond. The causal change implication is not extended to other bonds and causality reassignment is not required. The causal-paths (that are utilized for the GARRs derivation) do not change their structure but only some of the sub-paths are eliminated, due to the OFF state of the controlled-junctions.

The GARRs derivation from the DHBG and their use in health monitoring is demonstrated by an example. We consider the electric circuit in Figure 4; this circuit includes two electrical switches.

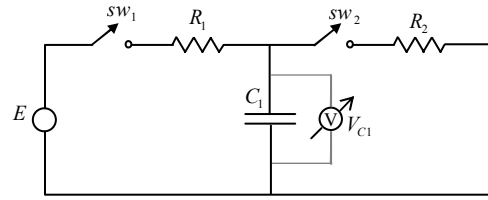


Figure 4. An electric circuit hybrid system - case 1

A DHBG is given in Figure 5, where the two controlled-junctions 1_1 and 1_2 model the two switches sw_1 and sw_2 respectively, the storage component C_1 is assigned with a derivative causality and the sensor's causality is inverted.

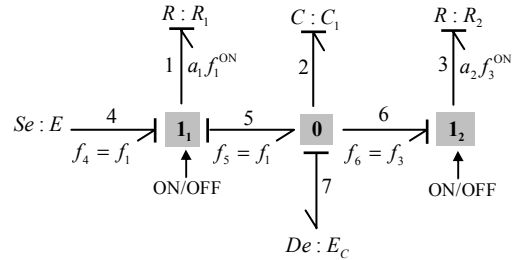


Figure 5. The electric circuit DHBG - case 1

To derive a GARR we use the concept of the unified controlled-junction description, together with the ARRs derivation methods of [2] (hence we only consider junctions with attached sensor). The 0-junction's constitutive-relation is:

$$\begin{aligned} e_7 = e_5 = e_2 = e_6 = E_{C1} \\ f_7 = f_5 - f_2 - f_6 = 0 \end{aligned} \quad (2)$$

The unknowns are f_5 , f_2 and f_6 (the sensor flow is assumed zero). Using the bond graph causal-paths, we have:

$$\begin{aligned} f_5 = a_1 f_1^{ON} = a_1 \frac{e_1}{R_1} = a_1 \frac{e_4 - e_5}{R_1} = a_1 \frac{E - E_{C1}}{R_1} \\ f_2 = C_1 \dot{e}_2 = C_1 \dot{E}_{C1} \\ f_6 = a_2 f_3^{ON} = a_2 \frac{e_3}{R_2} = a_2 \frac{e_6}{R_2} = a_2 \frac{E_{C1}}{R_2} \end{aligned} \quad (3)$$

Substituting (3) into (2), $GARR_1$ is achieved.

$$a_1 \frac{E - E_{C1}}{R_1} - C_1 \dot{E}_{C1} - a_2 \frac{E_{C1}}{R_2} = 0 \Leftrightarrow GARR_1 \quad (4)$$

The global relation $GARR_1$ is mode-dependent, using $GARR_1$ for real time fault detection and isolation, requires knowledge of the system's current mode. This information comes from a mode-tracker, and the mode tracking principals are introduced in section III. GARRs can also be used for off-line monitoring ability analysis; this analysis is based on the Fault Signature Matrix (FSM). In the case of hybrid systems and the GARR concept, we derive a FSM to any system's mode. The result is the Mode-Dependent FSM (MD-FSM). At a first stage, a Mode-GARR Table (MGT) is generated, to present the GARRs at each system's mode.

TABLE I THE MGT OF THE ELECTRIC CIRCUIT (CASE 1)

Mode $[a_2 \ a_1]$	$GARR_1$
$[0 \ 0]$	$-C_1 \dot{E}_{C1} = 0$
$[0 \ 1]$	$\frac{E - E_{C1}}{R_1} - C_1 \dot{E}_{C1} = 0$
$[1 \ 0]$	$-C_1 \dot{E}_{C1} - \frac{E_{C1}}{R_2} = 0$
$[1 \ 1]$	$\frac{E - E_{C1}}{R_1} - C_1 \dot{E}_{C1} - \frac{E_{C1}}{R_2} = 0$

From TABLE I, it is clear that none of the component faults is isolable. The monitoring-ability is improved if more sensors are deployed in the system, e.g. an additional current sensor measures the current of the source E as given in the DHBG in Figure 6.

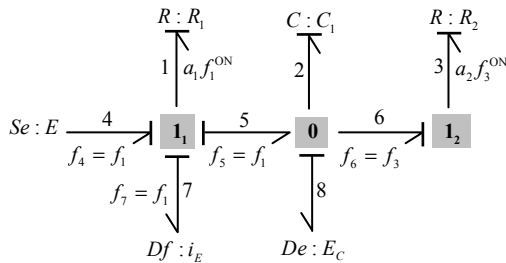


Figure 6. The electric circuit DHBG - case 2

We use the current sensor to derive an additional GARR. The controlled-junction constitutive relation is not recommended for GARR generation in this case, since it is valid only when the junction is ON. Instead we utilize the redundancy in bond-7, which leads to:

$$i_E = f_7 \Rightarrow i_E = a_1 \frac{E - E_{C1}}{R_1} \Leftrightarrow GARR_2 \quad (5)$$

Based on the two GARRs we form a MGT:

TABLE II THE MGT OF THE ELECTRIC CIRCUIT (CASE 2)

Mode $[a_2 \ a_1]$	$GARR_1$	$GARR_2$
$[0 \ 0]$	$-C_1 \dot{E}_{C1} = 0$	$i_E = 0$
$[0 \ 1]$	$\frac{E - E_{C1}}{R_1} - C_1 \dot{E}_{C1} = 0$	$i_E - \frac{E - E_{C1}}{R_1} = 0$
$[1 \ 0]$	$-C_1 \dot{E}_{C1} - \frac{E_{C1}}{R_2} = 0$	$i_E = 0$
$[1 \ 1]$	$\frac{E - E_{C1}}{R_1} - C_1 \dot{E}_{C1} - \frac{E_{C1}}{R_2} = 0$	$i_E - \frac{E - E_{C1}}{R_1} = 0$

III. USING GARRS FOR MODE TRACKING

In section II we presented the concept of Mode-GARR Table (MGT) which is an intermediate stage in the development of the MD-FSM. In the MGT each row represents a single mode and a set of ARR. Observing TABLE I and TABLE II, we understand that each system's mode is characterized by a unique set of ARRs. This observation suggests that system's mode can be identified by ARRs and we can design an ARR-based mode tracker.

Definition 3: A hybrid system is said to be *mode identifiable* if each one of its modes is characterized by a unique set of ARRs.

If the system is *mode identifiable* then the ARR-based mode-tracker observes the complete set of ARRs of all modes, simultaneously. In FDI framework, each ARR forms a residue. If the residue's absolute value is below a threshold, then we say that the ARR is consistent with the monitored system. If only a set of ARRs, which describes only one of the system's modes, is consistent with the monitored system, then we deduce that the system operates in that mode. A simple configuration is given as follows:

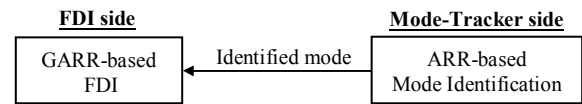


Figure 7. A simple configuration for health monitoring of hybrid system

The simple configuration (Figure 7) suffers from several drawbacks. The system's mode is identified by ARRs and the mode information is fed to the GARRs. The GARRs at the FDI side with the mode information is equivalent to the set of ARRs which is used for the mode identification (at the mode-tracker side). Consequently, identical set of ARRs is used in both sides. Moreover, as long as the GARRs at the FDI side are consistent with the monitored system it is clear that the system is normal and its current mode is known; therefore mode identification is not required at that stage. Mode identification is necessary only when GARRs at the FDI side show inconsistency with the monitored system. We use the notation $GAAR_i \uparrow$ to describe an event in which $GAAR_i$ at

the FDI side is crossing a threshold. The event $GAAR_i \uparrow$ is an indication to a discrepancy between the model used for FDI and the monitored system. Two scenarios can explain this discrepancy, one is a components fault and the other one is a change of mode. In this work we consider only the second scenario and the mode-tracker goal is to provide the system current mode while the system is normal (in a more general scenario, a fault is detected if a new mode is not identified).

The ARR-based mode identification is useful and has an important role in the proposed health monitoring strategy, but its continuous running is inefficient. Important information for mode-tracking is hidden in the $GAAR \uparrow$ events.

In HBG, system's modes are represented by controlled-junction's state. The GARRs are mode dependent, and controlled-junctions' states are part of the GARR expression. This suggests that we can explore the GARR structural-properties, to deduce on the GARR response to a change of mode. Consider for example $GAAR_1$ in (4), the two variables a_1 and a_2 represent the state of the two controlled-junctions 1_1 and 1_2 . Any inconsistency between the actual state of the controlled-junctions and the values of a_1 and a_2 (that is used in the GARR) is reflected by the GARR. If a sudden change of controlled-junctions state causes inconsistency, an event $GAAR_1 \uparrow$ is expected. We utilize these events for mode tracking; the process is based on the Mode-Change Signature Matrix (MChSM).

Definition 4: A Mode-Change Signature Matrix (MChSM) is a matrix that represents cause-effect relations between mode-changes and GARRs.

The MChSM is analogous to the Fault Signature Matrix (FSM) which is used for FDI of continuous system, but instead of representing cause-effect relations between component faults and ARRs, it represents cause-effect relations between mode-changes and GARRs. In this work we assume that any change of mode is due to a single controlled-junction change of state. Using this assumption, any row of the MChSM is a mode-change signature and all mode-changes are covered by the matrix (some of the signatures may be null).

The concept of monitoring ability analysis, based on the FSM, is well known in the context of continuous systems FDI. In the same spirit, the MChSM represents the mode-change monitoring ability of the hybrid system by its GARRs. Mode-change detection-ability (D_b) and mode-change isolation-ability (I_b) are represented at the last two columns of the MChSM. If the row of a_i is not null, then any mode-change, due to a change of a_i , can be detected by at least one of the GARRs. If the signature of a_i is unique, then the mode-change is isolable and can be uniquely identified by GARR events (i.e. $GAAR_i \uparrow$). As an example, TABLE III presents the MChSM of the electric circuit example (Figure 4).

TABLE III THE ELECTRIC CIRCUIT MChSM - CASE 1

C-J	$GARR_1$	D_b	I_b
a_1	1	1	0
a_2	1	1	0

From the table we understand that all mode-changes are detectable by $GARR_1$, but none of these changes is isolable. Nevertheless, this MChSM (TABLE III) is still very useful for mode tracking; the principles are described as follows:

The hybrid system mode evolution is represented by a series of controlled-junctions' states, and is notated by (6) (where the upper index k represents mode evolution):

$$\begin{bmatrix} a_1^0 & a_2^0 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^1 & a_2^1 \end{bmatrix} \rightarrow \dots \rightarrow \begin{bmatrix} a_1^k & a_2^k \end{bmatrix} \quad (6)$$

Assume the initial mode of the circuit is known and the mode information is fed to $GARR_1$. As long as $GARR_1$ is consistent with the monitored system, the system's mode is $\begin{bmatrix} a_1^0 & a_2^0 \end{bmatrix}$ (and hence, mode-tracking is not required at that stage). In a case of $GAAR_1 \uparrow$ event, we use the MChSM (TABLE III). From the matrix we understand that $GARR_1$ is sensitive to changes of a_1 and a_2 , and the next mode is one of two mode-hypotheses, as given in (7) (where the upper line represents a binary complement)

$$\begin{aligned} \text{mode-hypothesis 1: } & \begin{bmatrix} a_1^1 & a_2^1 \end{bmatrix} = \begin{bmatrix} \overline{a_1^0} & \overline{a_2^0} \end{bmatrix} \\ \text{mode-hypothesis 2: } & \begin{bmatrix} a_1^1 & a_2^1 \end{bmatrix} = \begin{bmatrix} a_1^0 & \overline{a_2^0} \end{bmatrix} \end{aligned} \quad (7)$$

Using this information and the MGT (TABLE I), we run two ARRs simultaneously to identify the hybrid system's new mode (e.g. if the circuit initial mode is $[0 \ 1]$, then we will run simultaneously the ARRs of modes $[0 \ 0]$ and $[1 \ 1]$). When the new mode is identified, the mode information is fed to $GARR_1$, and the role of the mode-tracker is terminated until the next $GAAR_1 \uparrow$ event. If the hybrid system initial mode is unknown then the first step is to run all ARRs of all modes simultaneously to identify the system's initial mode; such identification process is also utilized if the MChSM based Mode-change Isolation has failed (e.g. lost of mode tracks).

The GARR-based mode tracking principles of the electric circuit are generalized to a mode tracking method of hybrid system in FDI framework, and its scheme is presented in Figure 8. This mode tracking process is in-use only while the system is normal and $GAAR \uparrow$ events are due to mode-changes. The Unsuccessful Mode Identification arrow at the bottom of the scheme triggers fault isolation and fault identification processes which are supported by a different mode tracking strategy. In a wider framework an additional role of the mode tracker is to distinguish between a mode-change and a parametric-fault.

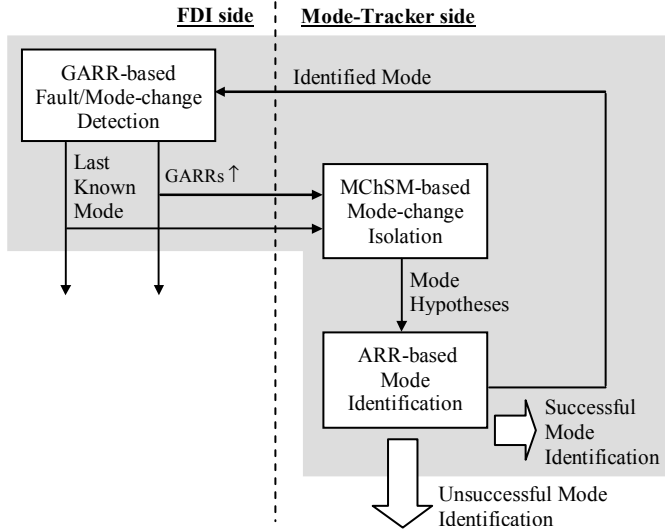


Figure 8. Mode tracking of hybrid system in FDI framework

In section II we added a current sensor, to improve the fault monitoring-ability of the circuit, which led to $GARR_2$. In the same spirit, mode-change monitoring-ability is improved if more sensors are deployed in the system. The MChSM of the electric circuit with its two sensors is presented in TABLE IV. Observing TABLE IV, we understand that all mode-changes are isolable.

The ability to isolate any change of mode is significant to the mode tracking process. In such systems, if the initial mode is known and the system is normal, then mode tracking process can carry out based on $GARR \uparrow$ events and the MChSM only; ARR-based mode identification is not necessary. We say the electric circuit with its two sensors is *mode-change identifiable*.

TABLE IV THE ELECTRIC CIRCUIT MChSM - CASE 2

C-J	$GARR_1$	$GARR_2$	D_b	I_b
a_1	1	1	1	1
a_2	1	0	1	1

Definition 5: A hybrid system is said to be *mode-change identifiable* if every possible mode-change has a unique signature.

Although *mode-change identifiability* is sufficient for mode tracking (under restrictive conditions, e.g. knowledge of initial mode), it is a weaker property than the *mode identifiability*. The *mode-change identifiability* guaranties mode-identification only with respect to a known previous mode. The ARR-based mode identification is stronger, because its ability to identify a new mode is based the mode continuous dynamics (and knowledge of previous modes is not

required). Using the analogy of encoder position sensor, we say that the ARR-based mode identification is *absolute*, while the MChSM-based mode identification is *incremental*. The drawback of the ARR-based mode identification, if is used by itself, is its high demand for computational resources (especially, if the number of system's modes is large). To summarize, our method utilizes the benefits of both mode tracking techniques (MChSM-based and ARR-based) to achieve more efficient (in terms of computational resources) and more robust (in terms of mode identification) mode tracking.

IV. CONCLUSIONS

In this work we developed a mode tracking method, to support health monitoring of hybrid systems. The method is based on two main processes; the first is the mode-change detection and isolation which is based on the Mode-Change Signature Matrix (MChSM), and the second is the ARR-based mode identification.

The mode-tracking is efficiently integrated into the health monitoring process and the mode tracker is invoked only when inconsistency between the monitored system and its model is detected by the FDI module.

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REFERENCES

- [1] A. Balluchi, L. Benvenuti, M.D. Di Benedetto, A. Sangiovanni-Vincetelli, "Design of observers for hybrid systems," in: C.J. Tomlin, M.R. Greenstreet (Eds.), *Hybrid Systems: Computation and Control (HSCC'02)*, in: LNCS, vol. 2289, Springer-Verlag, Berlin, 2002. , pp. 76–89
- [2] B.O. Bouamama, A. K. Samantary, M. Staroswiecki, G. D. Tanguy, "Derivation of constraint relations from bond graph model for fault detection and isolation," *International Conference on Bond Graph Modeling (ICBGM'03)*, pp 104-109, 2003.
- [3] C. B. Low, D. Wang, S. Arogeti, J. B. Zhang, "Causality assignment and model approximation for quantitative hybrid bond graph-based fault diagnosis," *The 17th IFAC World Congress*, Seoul, 2008.
- [4] C. B. Low, D. Wang, S. Arogeti, J. B. Zhang, "Monitoring ability and quantitative fault diagnosis using hybrid bond graph," *The 17th IFAC World Congress*, Seoul, 2008.
- [5] D. C. Karnopp, D. L. Margolis, R. C. Rosenberg, *System Dynamics Modeling and Simulation of Mechatronic Systems*, John Wiley & Sons Inc., New Jersey, 2006.
- [6] Pieter J. Mosterman and Gautam Biswas, "Behavior generation using model switching: a hybrid bond graph modeling technique," *Proceedings of the International Conference on Bond Graph Modeling '95*, vol. 27, no. 1, Society for Computer Simulation, San Diego, CA, 1995, pp. 177-182.
- [7] S. Arogeti, D. Wang, C. B. Low, J. B. Zhang, "Energy based mode tracking of hybrid systems", *The 17th IFAC World Congress*, Seoul, 2008.
- [8] S. Narasimhan, G. Biswas, "Model-based diagnosis of hybrid systems", *IEEE Transactions on System, Man and Cybernetics – Part A: Systems and Humans*, Vol. 37, No. 3, May 2007, pp. 348-361.
- [9] W. Wang, L. Li, D. Zhou, K. Liu, "Robust State estimation and fault diagnosis for uncertain hybrid nonlinear systems", *Nonlinear Analysis: Hybrid Systems*, 1, 2007, pp. 2-15.