Modeling and H∞ Robust Control for Mobile Robot

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Abstract—**Based on kinematic and kinetic analyses of the 3-Degree of Freedom differentially driven wheels mobile robot, we proposed a nonlinear mathematical model with uncertain disturbance. This model was transformed into linear control systems through an approximate linearization algorithm, which is based on balanced flow pattern. After that, we designed a partial feedback H_∞ robust controller based on LMI for the robots. The efficiency of our method received supports from the simulation results.**

Keywords—**Mobile Robot, Linearization, LMI, H_∞ robust control**

I. INTRODUCTION

Since 1980's, mobile robots are widely used in lots of dangerous and heavy jobs, including the transportation of nuclear waste, fire control, lunar exploration etc. They are becoming more and more important and valuable for us. Therefore, considerable research attention has been attracted to this area. In control field, researchers have focused on establishing mathematical models (including kinematics models and kinetics models) and designing appropriate controllers. The kinetics models proposed in the literature [1][2] are too complicated and it is very difficult to implement effective control. Literature [3] applied UKF arithmetic in mobile robot control and gained better control effect, but its complexity limits its practical application. Literature [4][5] introduced intelligent arithmetic in this filed and avoided establishing the precise mathematical models, but the design of control rules are highly dependent on personal experience, and its preciseness and stability are not satisfactorily addressed and need to be further examined.

In this paper, based on kinematic and kinetic analysis for the AS-R 3-Degree of Freedom differential wheeled mobile robot, a nonlinear mathematical model with uncertain disturbance is established. An approximate linearizing algorithm based on balanced flow pattern is proposed to transform the model to linear control system. Then, based on LMI, a partial feedback H_∞ controller is designed. Finally, simulation is performed and the result shows the efficiency of the method.

II. PROBLEM DESCRIPTION

It is composed of three modules, including drive module, control module, and sensor module. The three components are connected and fixed through bolts. The robot keeps its balance by three wheels: two front wheels (body 1 and 2) are driving wheels and one rear wheel (body 3) is steering wheel. The geometrical model is schematically depicted in Fig.1.

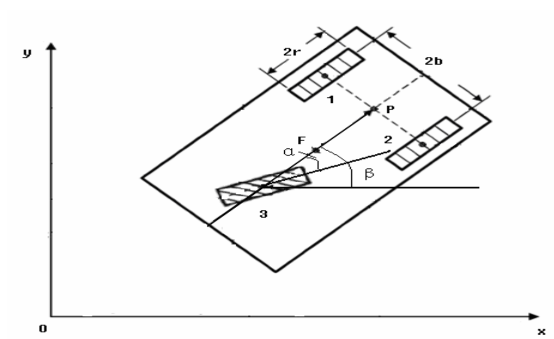

Fig. 1. Geometrical model of the mobile robot

Point F is the projection of the mass center C of the robot; Point P is the center of the two front wheels axle; 2b denotes the distance between the two front wheels; r is the radius of the wheels; l is the distance between point P and point F; β is the heading angle of the robot; θ_1 and θ_2 are the angles of the right and left driving wheels respectively; V_p denotes the speed of point P. Then kinematics equations can be expressed as:

$$\begin{cases} \upsilon_1 = r_1\dot{\theta}_1 & (1) \\ \upsilon_2 = r_2\dot{\theta}_2 & (2) \\ r_1 = r_2 = r & (3) \end{cases}$$

$$\begin{cases} \upsilon_P = \frac{r}{2}(\dot{\theta}_1 + \dot{\theta}_2) = \frac{\upsilon_1 + \upsilon_2}{2} & (4) \\ \dot{\beta} = \frac{r(-\dot{\theta}_1 + \dot{\theta}_2)}{2b} = \frac{-\theta_1 + \theta_2}{2b} & (5) \end{cases}$$

$$\begin{cases} \dot{x}_P = \upsilon_P \cos\beta & (6) \\ \dot{y}_P = \upsilon_P \sin\beta & (7) \end{cases}$$

$$\begin{cases} x_F = x_P - l\cos\beta & (8) \\ y_F = y_P - l\sin\beta & (9) \end{cases}$$

T_1 and T_2 are driving torque of the left and right front wheels respectively. f_1, f_2 and f_3 denote the friction pressed on body 1, 2, 3 respectively. α is spin angle of the robot. J and J_m are the moment of inertia of the wheels and the robot respectively. M and m are the mass of the robot and the wheels respectively; h denotes the distance between point F and steering wheel. Then kinetics equations can be expressed as:

$$\begin{cases} T_1 - f_1 r_1 = J_1\ddot{\theta}_1 & (10) \\ T_2 - f_2 r_2 = J_2\ddot{\theta}_2 & (11) \\ J_1 = J_2 = J & (12) \end{cases}$$

$$\begin{cases} f_1\cos\beta + f_2\cos\beta - f_3\cos(\beta-\alpha) = M\ddot{x}_F & (13) \\ f_1\sin\beta + f_2\sin\beta - f_3\sin(\beta-\alpha) = M\ddot{y}_F & (14) \\ -f_1 b + f_2 b - f_3 h\sin(\alpha) = J_m\ddot{\beta} & (15) \end{cases}$$

The parameters of the mobile robot are showed in Table 1.

TABLE I. THE PARAMETERS OF THE MOBILE ROBOT

Symbol	Parameter	Value	Unit
m	mass of front wheel	0.5	kg
r	radius of front wheel	0.105	m
J	moment of inertia of front wheel	0.0014	$kg.m^2$
M	mass of robot	25	kg
J_m	moment of inertia of robot	0.5512	$kg.m^2$
2b	distance between the two front wheels	0.41	m
h	distance between F and steering wheel	0.1965	m
L	distance between point P and point F	0.09	m

Driving motors of the AS-R robot are RE36 Model made by the Maxon Company. They are hollow-cup-rotor DC electrical machines. u_1 and u_2 are input voltage of the left and right driving motors respectively. The parameters of the driving motors are showed in Table 2.

TABLE II. THE PARAMETERS OF THE DRIVING MOTORS

Parameter	Value	Unit
Rated power	70	W
Rated voltage	24	V
No-load speed	6610	rpm
Block torque	730	mNm
Speed/torque slope	9.23	rpm/mNm
Resistance	0.628	Ω
Rated efficiency	84	%
Speed constant	375	rpm/V
Rotor inertia	60.2	$g.cm^2$

According to the mechanical characteristic plot, we can obtain the following formula[6]:

$$\begin{cases} \upsilon_1 = pu_1 - qT_1 & (16) \\ \upsilon_2 = pu_2 - qT_2 & (17) \end{cases}$$

The state variable

$$x = (x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6)^T$$

$$= (x_F \quad y_F \quad \beta \quad \dot{x}_F \quad \dot{y}_F \quad \dot{\beta})^T$$

is used to describe the position and stance of the mobile robot. The six variables denote the X axis displacement, Y axis displacement, angular displacement, X axis speed, Y axis speed and angular speed of the mobile robot in the world coordinates. Input vector $u=[u_1\ u_2]^T$ denotes input voltage of the left and right driving motors respectively; Uncertain disturbance vector $\omega=[\omega_1\ \omega_2\ \omega_3]^T$ denotes the friction between the mobile robot and ground. Let $A = \frac{r}{Jq}$, mathematical model with uncertain disturbance is established as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ -Ax_4 - x_6x_5 + Al\sin x_3 x_6 - l\cos x_3 {x_6}^2 \\ x_4x_6 - Ax_5 - \cos x_3 x_6 - l\sin x_3 {x_6}^2 \\ -Ax_6 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{Ap}{2}\cos x_3 & \frac{Ap}{2}\cos x_3 \\ \frac{Ap}{2}\sin x_3 & \frac{Ap}{2}\sin x_3 \\ -\frac{Ap}{2b} & \frac{Ap}{2b} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \quad (18)$$

III MODEL APPROXIMATE LINEARIZATION

From Section 1, we obtained the mathematical model of AS-R mobile robot. It is a complicated nonlinear coupling system, so that we cannot easily analyze and control the robot. In this section, we introduced an approximate linearizing algorithm[7] based on balanced flow pattern to transform the model to linear control system.

$$\begin{cases} \dot{x} = Ax + E\omega + Bu \\ y = Cx \end{cases} \quad (19)$$

where，

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -739 & 0 & 0 \\ 0 & 0 & 0 & 0 & -739 & -66.5 \\ 0 & 0 & 0 & 0 & 0 & -739 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.01478 & 0.01478 \\ 0 & 0 \\ -0.07209 & 0.07209 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

IV. PARTIAL FEEDBACK H_∞ CONTROLLER DESIGN

For the facility application in engineering situation, we reduced the order of original system to 4 by the method introduced by [8](Stability is also proved in [8]).

$$\begin{cases} \dot{x} = A_1 x + B_1\omega + B_2 u \\ z = C_1 x \end{cases} \quad (20)$$

Where,

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -739 & 0 & 0 \\ 0 & 0 & -739 & -66.5 \\ 0 & 0 & 0 & -739 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0.01478 & 0.01478 \\ 0 & 0 \\ -0.07209 & 0.07209 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If there exists matrix $X=X^T$ and matrix W, satisfied the following linear matrix inequality:

$$\begin{cases} \begin{bmatrix} A_1X+B_2W+(A_1X+B_2W)^T & B_1 & (C_1X)^T \\ B_1^T & -I & 0 \\ C_1X & 0 & -I \end{bmatrix} < 0 \\ X > 0 \end{cases} \quad (21)$$

Then, a state feedback robust H_∞ controller $u=WX^{-1}x$ can be obtained to guarantee the local feedback system(as shown in Fig 2.) progressive stability[9].

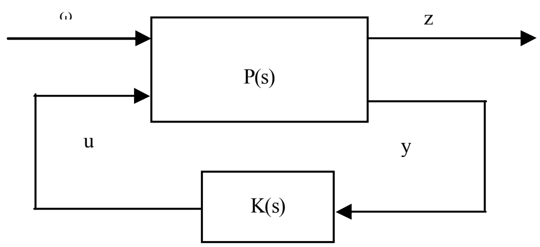

Fig.2. The frame of local feedback control system

Use LMI tool box in Matlab, we can get suited matrix X and matrix W:

$$X = \begin{bmatrix} 0.6292 & -0.0731 & 0.0829 & -0.9216 \\ -0.0731 & 4.5364 & -0.0226 & 0.1838 \\ 0.0829 & -0.0226 & 0.0786 & -0.3261 \\ -0.9216 & 0.1838 & -0.3261 & 5.7816 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.0289 & 1.1045 & -0.0007 & -0.2897 \\ -0.0655 & 1.1406 & -0.0140 & 0.2972 \end{bmatrix}$$

Then, a proper local feedback H_∞ controller is obtained:

$$K = \begin{bmatrix} 0.0156 & 2.4521 & -2.3412 & -0.7087 \\ -0.2188 & 2.4975 & 1.0861 & 0.4610 \end{bmatrix}$$

V. SIMULATION RESULT

Applying the local feedback H_∞ controller represented in Section 4 to the target mobile robot, a local closed loop control system is formed, as shown in Fig.2. When providing a unit step signal, we generated the simulation results shown in Fig.3.

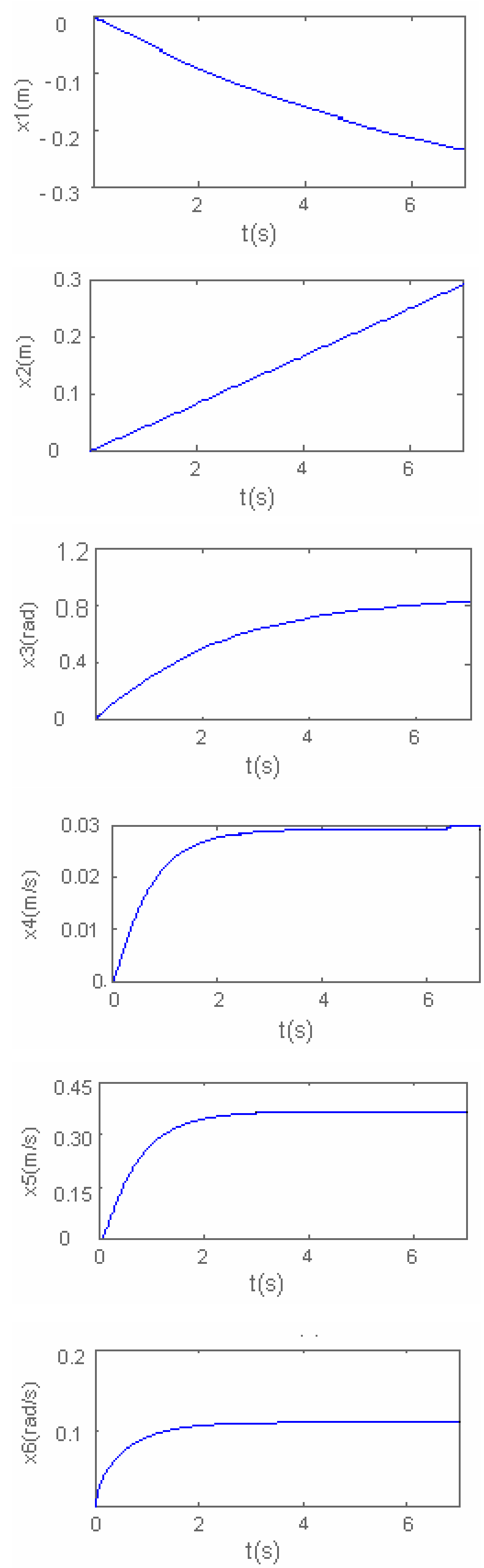

Fig.3.Unit step response graph

From the simulation results, the mobile robot keeps constant speed or approximate constant speed after an accelerated process in the initial 3 seconds. And the whole system is stable and robust with the uncertain disturbance.

VI. CONCLUSIONS

In this paper, based on the kinematic and kinetic analysis for the AS-R 3-DOF differential wheeled mobile robot, a mathematical model for such robots is proposed. Choosing voltage as inputs made the control more direct and simple. Based on model reduction, a partial output feedback H_∞ robust controller is designed via LMI. Simulation results shows the whole system is reasonably stable. Our results can be extended to nonlinear systems widely existing in engineering situations.

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