A New Algorithm for Extracting Specific Signals with Temporal Structure

Min Wan, Shangming Yang and Hua Yan
Computational Intelligence Laboratory
School of Computer Science and Engineering
University of Electronic Science and Technology of China
Chengdu 610054, P. R. China
Email: wan_min@126.com, minn003@163.com, huayan@uestc.edu.cn

***Abstract*— In this paper we develop a batch learning algorithm for semi-blind extraction of a desired source signal with temporal structure from linear mixtures. By studying the discrete time algorithm, an invariant set is obtained so that the non-divergence of the algorithm can be guaranteed. In the invariant set, the local convergence of the algorithm is analyzed. It is proven that the trajectories of the algorithm starting from the invariant set will converge to the desired source signal which is the most autocorrelated for an specific delay. The simulations verified the results.**

I. Introduction

There has been increased interest in blind signal processing, with special attention to independent component analysis (ICA) and instantaneous blind source separation (BSS). Assuming that the source signals have been linearly mixed and that these mixed sensor signals are available, BSS finds in a blind manner a linear combination of the mixed signals that recovers the original source signals, possibly rescaled and randomly arranged in the outputs.

However, extracting all the source signals from a large number of sensors, for example, a magnetoencephalographic (MEG) measurement, which may output hundreds of recordings, could take a long time. Thus, it would be important for the user to extract only one or some desired signals instead of all sources. A similar problem arises in the cocktail party problem, where one needs to rearrange speech signals extracted in different frequency bands so that they could recover a given speech signal[1, 2]. This fact implies a semiblind kind of source separation.

Blind source extraction (BSE) is such a kind of technique that is able to extract only the signals of interest by using some *priori information*. Compared to BSS , BSE has many advantages and has received wide attention in various fields such as biomedical signal analysis, data mining, speech and image processing, and so on [3, 4, 5].

Many source extraction algorithms can extract a specific signal as the first output by using some *priori information* , such as non-Gaussianity [6], smoothness or linear predictability [5, 7], sparseness [8], generalized autocorrelations of the desired source signals [9], etc.

A generalized autocorrelation functions of the desired source is proposed and a fixed-point algorithm is derived in [9], the algorithm needs to be normalized after each step. In this paper, we transform the constrained optimum problem presented in [9] to a unstrained one and derive a new algorithm which need not be normalized. By studying the proposed algorithm, invariant set is obtained so that the non-divergence of the algorithm can be guaranteed. In the invariant set, the local convergence of the trajectories is analyzed.

This paper is organized as follows. Some preliminaries and the proposed algorithm will be presented in Section 2. In Section 3, invariant sets and convergence of the algorithm will be studied. Simulation results will be provided in Section 4. Finally, conclusions will be drawn in Section 5.

II. The algorithm

Assume that the observed signals $v(k)(k=1,2,\cdots)$ are described by the matrix equation

$$v(k) = As(k) \tag{1}$$

where A is an $n \times n$ unknown mixing matrix and $s(k) = (s_1(k),\cdots,s_n(k))^T$ is a vector of unknown zero-mean and unit-variance primary sources.

Provided that the signals v have already been followed by an $n \times n$ whitening matrix V such that the components of $x(k) = Vv(k)$ are unit variance and uncorrelated, i.e. $E\{x(k)x^T(k)\} = I$. It follows that $E\{x(k)x^T(k)\} = (VA)(VA)^T = I$, which means that the matrix VA is orthogonal, so $s(k) = (VA)^T x(k)$.

A linear single neuron network described by

$$y(k) = w^T x(k), y(k-\tau) = w^T x(k-\tau) \tag{2}$$

will be used to extract the desired source. Where w is the weight vector and τ is a delay in time. In [9], the following constrained maximization problem based on generalized autocorrelation functions of the desired source has been presented:

$$\begin{aligned}\max_{\|w\|=1} \Psi(w) &= \max_{\|w\|=1} E\{G(y(k))G(y(k-\tau)\} \\ &= \max_{\|w\|=1} E\{G(w^T x(k))G(w^T x(k-\tau)\}\end{aligned} \tag{3}$$

Where G is a differentiable function, examples of choices are $G(u) = u, G(u) = u^2, G(u) = u^3$ or $G(u) = \log\cosh(u)$.

The maximization can be easily transformed to the following unconstrained optimization:

$$\min\{-\Psi(w) + (\|w\|^2 - 1)^2\}. \tag{4}$$

From (4), the following gradient descent algorithm can be easily derived:

$$\begin{aligned} w(k+1) &= w(k) + \eta a(1 - \|w(k)\|^2)w(k) \\ &+ \eta b E\{g(y(k))G(y(k-\tau))x(k) \\ &+ G(y(k))g(y(k-\tau))x(k-\tau)\}, \end{aligned} \quad (5)$$

where η is a constant learning rate, $a > 0, b > 0$ are constants to adjust the learning parameters more conveniently.

In the following, we will study the behavior of (5) when $G(u) = u$. i.e. the source signals have linear autocorrelation.

Assume the ith source signal is the desired source signal to be extracted from the observed signals, it is the most linear autocorrelated signal for a specific time delay τ, i. e.

$$E\{s_i(k)s_i(k-\tau)\} > E\{s_j(k)s_j(k-\tau)\} \geq 0,$$

for all $j \neq i$, where k is the time index. $\{s_p(k), s_p(k-\tau)\}$ and $\{s_q(k), s_q(k-\tau)\}(\forall p \neq q)$ are mutually independent, i. e. $E\{s(k)s(k-\tau)^T\}$ is diagonal.

If $G(u) = u$, (5) can be transformed as

$$\begin{aligned} w(k+1) &= w(k) + \eta a(1 - \|w(k)\|^2)w(k) \\ &+ \eta b E\{y(k-\tau)x(k) + y(k)x(k-\tau)\}. \end{aligned} \quad (6)$$

Before the convergence analysis, a transformation is given first. Let $z(k) = (VA)^T w(k)$, it follows that $y(k) = z(k)^T s(k)$ and

$$\begin{aligned} z(k+1) &= [1 + \eta a - \eta a\|z(k)\|^2 + \eta b(E\{s(k)s(k-\tau)^T\} \\ &+ E\{s(k-\tau)s(k)^T\})]z(k). \end{aligned} \quad (7)$$

That is, the convergence analysis of algorithm (6) can be transformed to an analysis of the convergence of the algorithm (7).

III. ALGORITHM ANALYSIS

In this section, we will study the behavior of (7).

Some trajectories of (7) may diverge. Consider a one-dimensional example. From (7), it follows that

$$z(k+1) = [1 + \eta a - \eta a z^2(k) + 2\eta b E\{s(k)s(k-\tau)\}]z(k).$$

If

$$z^2(k) > \frac{2 + \eta a + 2\eta b E\{s(k)s(k-\tau)\}}{\eta a},$$

it holds that

$$[1 + \eta a - \eta a z^2(k) + 2\eta b E\{s(k)s(k-\tau)\}]^2 > 1.$$

Consequently,

$$z^2(k+1) > z^2(k).$$

So, if

$$z^2(0) > \frac{2 + \eta a + 2\eta b E\{s(k)s(k-\tau)\}}{\eta a},$$

it holds that

$$\lim_{k\to\infty} z^2(k) = \infty.$$

The example above shows (7) may diverge. A problem to address is therefore to find out the conditions under which the algorithm can be bounded. Next, we will prove an interesting theorem, which gives an invariant set of (7). The trajectories, thus, can be guaranteed to be bounded.

A. Invariant sets

Definition 1: A compact set $S \subset R^n$ is called an invariant set of (7), if for any $z(0) \in S$, the trajectory of (7) starting from $z(0)$ will remain in S for all $k \geq 0$.

An invariant set is interesting since it provides a method to guarantee non-divergence of trajectories.

The following lemma [10] will be useful.

Lemma 1: Suppose that $D > 0$, $E > 0$, it holds that

$$[D - Eh]^2 h \leq \frac{4D^3}{27E}$$

for all $0 \leq h \leq \frac{D}{E}$.

Theorem 1: Denoted by

$$S_1 = \left\{ z | z \in R^n, \|z\|^2 < \frac{1 + \eta a}{\eta a} \right\}.$$

If

$$\eta b \|E\{s(k)s(k-\tau)^T\}\| \leq \frac{1}{4}(1 + \eta a), \eta a \leq 0.32,$$

then S_1 is an invariant set of (7).

Proof: From (7), it follows that

$$\begin{aligned} &\|z(k+1)\| \\ = &\|[1 + \eta a - \eta a\|z(k)\|^2 + \eta b(E\{s(k)s(k-\tau)^T\} \\ &+ E\{s(k-\tau)s(k)^T\})] \cdot z(k)\| \end{aligned} \quad (8)$$

for all $k \geq 0$. The norm of the vector and matrix in (8) are $\|\cdot\|_2$ and $\|\cdot\|_F$ respectively. According to the compatibility of the two norms, it can be easily derived that

$$\begin{aligned} \|z(k+1)\| \leq &[|1 + \eta a - \eta a\|z(k)\|^2| \\ &+ 2\eta b\|E\{s(k)s(k-\tau)^T\}\|] \cdot \|z(k)\|. \end{aligned}$$

Thus, there must exist $\alpha(k)$ so that

$$\begin{aligned} \|z(k+1)\| = &\alpha(k)[|1 + \eta a - \eta a\|z(k)\|^2| \\ &+ 2\eta b\|E\{s(k)s(k-\tau)^T\}\|] \cdot \|z(k)\| \end{aligned}$$

for all $k \geq 0$, where $0 < \alpha(k) \leq 1$. If

$$\|z(k)\|^2 < \frac{1 + \eta a}{\eta a},$$

it holds that

$$\begin{aligned} &\|z(k+1)\|^2 \\ = &[\alpha(k)(1 + \eta a + 2\eta b\|E\{s(k)s(k-\tau)^T\}\|) \\ &- \alpha(k)\eta a\|z(k)\|^2]^2 \cdot \|z(k)\|^2 \\ \leq &\max_{0 \leq \|z(k)\|^2 \leq \xi} \big\{[\alpha(k)(1 + \eta a + 2\eta b\|E\{s(k)s(k-\tau)^T\}\|) \\ &- \alpha(k)\eta a\|z(k)\|^2]^2 \cdot \|z(k)\|^2\big\}, \end{aligned}$$

where

$$\xi = \frac{1 + \eta a + 2\eta b\|E\{s(k)s(k-\tau)^T\}\|}{\eta a}.$$

Let $D = \alpha(k)(1 + \eta a + 2\eta b\|E\{s(k)s(k-\tau)^T\}\|)$, $E = \alpha(k)\eta a, h = \|z(k)\|^2$, from Lemma 1, it follows that

$$\begin{aligned} &\|z(k+1)\|^2 \\ \leq\ & \frac{4[\alpha(k)(1 + \eta a + 2\eta b\|E\{s(k)s(k-\tau)^T\}\|)]^3}{27\alpha(k)\eta a} \\ \leq\ & \frac{4(1 + \eta a + 2\eta b\|E\{s(k)s(k-\tau)^T\}\|)^3}{27\eta a}. \end{aligned}$$

If

$$\eta b\|E\{s(k)s(k-\tau)^T\}\| \leq \frac{1}{4}(1 + \eta a), \eta a \leq 0.32,$$

it follows that

$$\begin{aligned} \frac{4(1 + \eta a + 2\eta b\|E\{s(k)s(k-\tau)^T\}\|)^3}{27} &\leq \frac{1}{2}(1 + \eta a)^3 \\ &< 1 + \eta a. \end{aligned}$$

Consequently,

$$\|z(k+1)\|^2 < \frac{1 + \eta a}{\eta a}.$$

So, S_1 is an invariant set of (7). ∎

Since $\|w(k)\| = \|z(k)\|$ for all $k \geq 0$, the following theorem can be easily obtained.

Theorem 2: Denoted by

$$S = \left\{ w | w \in R^n, \|w\|^2 < \frac{1 + \eta a}{\eta a} \right\}.$$

If

$$\eta b\|E\{s(k)s(k-\tau)^T\}\| \leq \frac{1}{4}(1 + \eta a), \eta a \leq 0.32,$$

then S is an invariant set of (6).

The theorem above shows any trajectory of algorithm (6) starting from $w(0)$ will remain in S. This guarantees the non-divergence of the algorithm. In next section, we will study the convergence behavior of (7) in the invariant set.

B. Local analysis

Lemma 2: Assume the desired source signal s_i satisfies
1) $E\{s_i(k)s_i(k-\tau)\} > E\{s_j(k)s_j(k-\tau)\} \geq 0, \forall j \neq i$,
2) $E\{s(k)s^T(k)\} = I, E\{s(k)s(k-\tau)^T\}$ is diagonal.
If

$$\eta b\|E\{s(k)s(k-\tau)^T\}\| \leq \frac{1}{4}(1 + \eta a), \eta a \leq 0.32, z(0) \in S_1,$$

then it holds that

$$\lim_{k\to\infty} z_j(k) = 0, \forall j \neq i.$$

Proof: Since $E\{s(k)s(k-\tau)^T\}$ is diagonal, from (7), it holds that for all q,

$$z_q(k+1) = [1 + \eta a - \eta a\|z(k)\|^2 + 2\eta bE\{s_q(k)s_q(k-\tau)\}]z_q(k). \tag{9}$$

From Theorem 1, it follows that

$$\|z(k)\|^2 < \frac{1 + \eta a}{\eta a}$$

for all $k \geq 0$. Thus,

$$1 + \eta a - \eta a\|z(k)\|^2 + 2\eta bE\{s_q(k)s_q(k-\tau)\} > 0, \forall q, k.$$

Since

$$\begin{aligned} &\frac{z_j^2(k+1)}{z_i^2(k+1)} \\ =\ & \left[\frac{1 + \eta a - \eta a\|z(k)\|^2 + 2\eta bE\{s_j(k)s_j(k-\tau)\}}{1 + \eta a - \eta a\|z(k)\|^2 + 2\eta bE\{s_i(k)s_i(k-\tau)\}}\right]^2 \\ & \times\frac{z_j^2(k)}{z_i^2(k)}, \end{aligned}$$

and $E\{s_i(k)s_i(k-\tau)\} > E\{s_j(k)s_j(k-\tau)\}, \forall j \neq i$, it follows that

$$\frac{z_j^2(k+1)}{z_i^2(k+1)} < \frac{z_j^2(k)}{z_i^2(k)}.$$

Consequently,

$$\lim_{k\to\infty} \frac{z_j^2(k)}{z_i^2(k)} = 0, \forall j \neq i.$$

Since

$$\|z(k)\|^2 < \frac{1 + \eta a}{\eta a}$$

for all $k \geq 0$, it follows that

$$\lim_{k\to\infty} z_j^2(k) = 0, \forall j \neq i.$$

From (9), it follows that the sign of the $z_q(k)(\forall k \geq 0)$ is the same as that of $z_q(0)$ in the invariant set S_1, thus

$$\lim_{k\to\infty} z_j(k) = 0, \forall j \neq i.$$

∎

Definition 2: A point $z^* \in R^n$ is called an equilibrium point of (7), if and only if

$$\begin{aligned} z^* = & [1 + \eta a - \eta a\|z^*\|^2 + \eta b(E\{s(k)s(k-\tau)^T\} \\ & + E\{s(k-\tau)s(k)^T\})] \cdot z^*. \end{aligned} \tag{10}$$

Theorem 3: Assume the desired source signal s_i satisfies
1) $E\{s_i(k)s_i(k-\tau)\} > E\{s_j(k)s_j(k-\tau)\} \geq 0, \forall j \neq i$,
2) $E\{s(k)s^T(k)\} = I, E\{s(k)s(k-\tau)^T\}$ is diagonal.
If

$$\eta b\|E\{s(k)s(k-\tau)^T\}\| \leq \frac{1}{4}(1 + \eta a), \eta a \leq 0.32, z(0) \in S_1,$$

then it holds that the trajectories of (7) will converge to

$$z^* = \left(0, \cdots, \pm\sqrt{\frac{\eta a + 2\eta bE\{s_i(k)s_i(k-\tau)\}}{\eta a}}, \cdots, 0\right)^T.$$

Proof: From Theorem 1, S_1 is an invariant set of (7). From Lemma 2, it follows that in S_1,

$$\lim_{k\to\infty} z_j(k) = 0, \forall j \neq i.$$

So we only need to discuss the stability of the equilibrium points of (7) such as $z^* = (0, \cdots, z_i^*, \cdots, 0)^T$ in S_1 . From (10), it can be easily derived that all the kind of equilibrium points are:

$$z^* = (0, \cdots, 0, \cdots, 0)^T,$$
$$z^* = \left(0, \cdots, \pm\sqrt{\frac{\eta a + 2\eta b E\{s_i(k)s_i(k-\tau)\}}{\eta a}}, \cdots, 0\right)^T.$$

Clearly, these three points all belong to the invariant set S_1.

In the following, we will compute the eigenvalue of Jacobian matrix at each equilibrium point.

The Jacobian matrix of (7) at $z^* = (0, \cdots, z_i^*, \cdots, 0)^T$ is

$$\nabla|_{(0,\cdots,z_i^*,\cdots,0)^T} = diag(J_1, \ldots, J_n),$$

where

$$J_i = 1 + \eta a + 2\eta b E\{s_i(k)s_i(k-\tau)\} - 3\eta a(z_i^*)^2,$$

$$J_j = 1 + \eta a + 2\eta b E\{s_j(k)s_j(k-\tau)\} - \eta a(z_i^*)^2, \forall j \neq i.$$

If $z^* = (0, \cdots, 0, \cdots, 0)^T$, it follows that $|J_p| > 1, p = 1, \ldots, n$. Thus, this equilibrium point is unstable.

If

$$z^* = \left(0, \cdots, \pm\sqrt{\frac{\eta a + 2\eta b E\{s_i(k)s_i(k-\tau)\}}{\eta a}}, \cdots, 0\right)^T,$$

from the conditions in this theorem, it can be easily derived that

$$|J_i| = |1 - 2\eta a - 4\eta b E\{s_i(k)s_i(k-\tau)\}| < 1,$$

$$|J_j| = |1 + 2\eta b(E\{s_j(k)s_j(k-\tau)\} - E\{s_i(k)s_i(k-\tau)\})| < 1,$$

$\forall j \neq i$. So, these two equilibrium points are stable.

From above discussion, we can conclude that in the invariant set S_1, (7) will converge to

$$z^* = \left(0, \cdots, \pm\sqrt{\frac{\eta a + 2\eta b E\{s_i(k)s_i(k-\tau)\}}{\eta a}}, \cdots, 0\right)^T.$$

■

After convergence, the output of the network is $y(k) = (z^*)^T s(k) = cs_i(k)$, where

$$c = \pm\sqrt{\frac{\eta a + 2\eta b E\{s_i(k)s_i(k-\tau)\}}{\eta a}}.$$

So the ith source signal is extracted. That is, (7) can extract the most linear autocorrelated source signal.

Fig. 1. Original images 1-3 and sources 1-3

Fig. 2. Mixtures 1-3

IV. Experimental Results

In this section, an experiment will be carried out to confirm the convergence analysis derived in Theorem 3.

In the experiment, we have selected three statistically independent 128×128 images shown in Fig.1, with two natural images and one i.i.d Gaussian noise. After normalization, the three images are used as source signals. Randomly mixed images are shown in Fig.2.

The algorithm (6) is used to extract the most autocorrelated source signal. For finding the appropriate delay τ, we calculate the autocorrelation $\zeta(f) = E\{x_j(t)x_j(t-f)\}$ of the signals

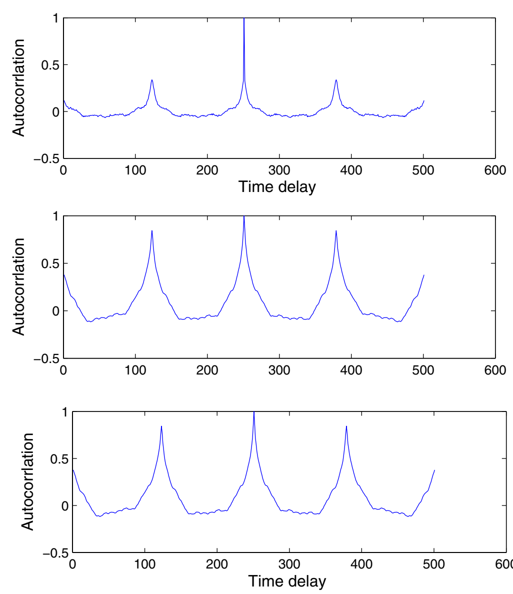

Fig. 3. Autocorrelation functions of the mixtures 1-3

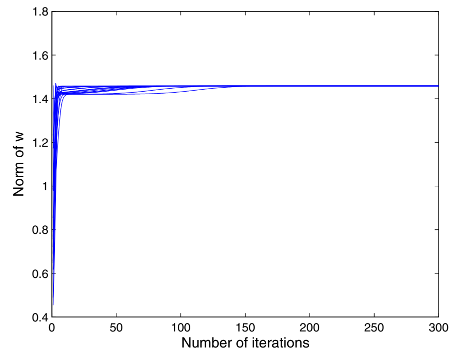

Fig. 4. Evolution of norm of w

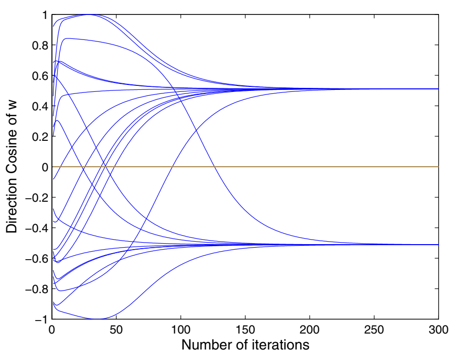

Fig. 5. Evolution of direction of w

x as a function of the time delay f and find the optimal time delay at which the autocorrelation function has a peak [9]. The corresponding autocorrelation functions of the three mixed images are shown in Fig.3. It can be easily seen that they present several peaks and the peaks are similar. So, the optimal time delay is chosen as $\tau = 128$. The other parameters of the algorithm are set as follows: $\eta a = 0.32$, $\eta b = \frac{1}{4}(1 + \eta a)/\sqrt{3}$, $w(0) \in S$. The evolution of the norm of w is shown in Fig.4. Clearly, the norm changes in a certain range and will not go to infinity.

To verify whether the direction of w converge or not, we will compute the direction cosine between w and a reference vector $r = [1, 1, 1]^T$ at each k as [11]:

$$DirectionCosine(k) = \frac{w^T(k) \cdot r}{\|w(k)\| \cdot \|r\|}$$

for all $k \geq 0$. Fig.5 shows the evolution of $DirectionCosine$ from different initial values. Clearly, the curves eventually converge to two points. It is consistent with the results derived in Theorem 3.

Fig. 6. Extracted lenna

The converged signal corresponding to the positive value is shown in Fig.6 (left). Fig.6 (middle) shows the normalized signal. In order to measure the accuracy of extraction, we adopt the performance index(PI) as [9]:

$$PI = -10Elog(s(t) - s'(t))^2.$$

Where s is the source signal, s' is the extracted image(both are normalized to be zero-mean and unit-variance). The higher PI is , the better the performance is. The experiment is independently repeated 50 times using different random mixing matrix A, the averaged PI of the extracted image is 73.5. Finally, we transform the normalized extracted signal to original image as follows. Suppose y is the normalized extracted image, m and d are the expectation and variance of the original lenna image respectively. Let $y' = y \times \sqrt{d} + m$, the image corresponding to y' is shown in Fig.6 (right).

V. Conclusion

In this paper, we have transformed the constrained optimum problem presented in [9] to a unconstrained one and obtained a new algorithm. The invariant set of the proposed algorithm has been found and the dynamical behavior of the trajectories in the invariant set has been studied. The experiments have verified the results of Theorem 3 and the desired source signal has been successfully extracted.

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