Reasoning of Fuzzy Causality Diagram with Interval Probability

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***Abstract*—Causality Diagram is a probabilistic reasoning method. Fuzzy set theory was introduced to develop causality diagram methodology after discussing the development and the restriction of conventional causality diagram. The application of causality diagram is extended to fuzzy field by introducing fuzzy set theory. Fuzzy causality diagram can overcome the shortcomings that it is difficult to gain the accurate probability of the event in conventional causality diagram. Interval numbers can express all kinds of fuzzy number. So it is necessary to dicuss the reasoning of fuzzy causality diagram with interval probability. Based on the interval number, operator, fuzzy conditional probability and the normalization method were discussed in this paper. Then two reasoning algorithm of single-value fuzzy causality diagram is proposed, some remarks about these algorithms are given. The result of numerical simulating of a subsystem in nuclear plant is coincident with the fact, and it shows the normalizing method is effective. The research shows that Interval Fuzzy causality diagram is so effective in fault analysis, and it is more flexible and adaptive than conventional method.**

***Keywords*—Causality Diagram, Fuzzy Causality Diagram, Fuzzy Number, Interval Number, Interval Probability, Reasoning**

I. INTRODUCTION

Causality diagram (CD) is a method of reasoning and knowledge representation based on probability theory, which was developed from the belief networks theory and brought forward to express uncertain knowledge and reason in 1994 [1]. CD expresses knowledge by graph which node denotes the events, while arc denotes causalities as well as linkage intensity represents the intensity of causality. This kind of graph knowledge is natural and intuitional so it is easy to express dominant knowledge and benefit to designate knowledge for expert.

The main purpose of the reasoning with CD is to work out the posterior probability of a certain event with its known evidence, where evidence means that some basic events or middle events have happened. The probabilities of the basic events and the linkage events in CD are supposed to be known and independent. During the course of reasoning the middle event should be firstly converted into the logic expression of the basic events and linkage events, then the probabilities of the middle events can be worked out. In order to simplify reasoning computation, CD needs to be compiled to gain the expressions of all node events (i.e., the right of the logic expression is basic event and linkage event). The reasoning process of causality diagram is in four steps [1][4], i.e., calculating the CSs-1 (level-1 cut sets), calculating the CSs-f (final level cut sets), calculating the DCSs-f (final level disjoint cut sets), and calculating the posterior probability.

In recent years, the theory of causality diagram, which is mainly applied to fault diagnosis [1-8], has greatly developed in many domains[2-10], so that some models such as single-value causality diagram (SCD) [1][4], multi-value causality diagram (MCD) [3][5] and continuous-value causality diagram (CCD) [2] were proposed. Causality diagram is a probabilistic reasoning method that previous work mainly dealt with accurate probability. However, it is difficult to dispose in reality because there is fuzziness of failure probabilities of the basic events and the linkage events in large-scale complex system in some instance. It is necessary to introduce fuzzy set theory into CD because the former theory of CD is difficult to dispose of this problem. Fuzzy causality diagram (FCD) is causality diagram introduced into fuzzy set theory, and Fuzzy causality diagram, which disposes interval number, is referred to as interval fuzzy causality diagram (IFCD). FCD is more flexible and adaptive than conventional CD because it is able to handle the two uncertainties, i.e., randomness and fuzziness at the same time through replacing accurate probability value by fuzzy number.

Ref. [18-20] has discussed how to calculate the occurrence probability such as triangle fuzzy number (TNFN), trapezoidal fuzzy number (TPFN), normal fuzzy number (NFN), which are expressed by member -ship function. In this paper, we discuss interval number (IN) without membership function. Interval number is a common fuzzy number in fuzzy application [13]. In fact, the λ-level cut set of fuzzy number is interval number which is a closed set in the set of real numbers, so interval number is also fuzzy number. Interval numbers can express all kinds of fuzzy number. So it is necessary to dicuss the reasoning of fuzzy causality diagram with interval probability. The reasoning of CD needs the event probability while interval number can represent variable interval of the probability so that interval number can be used to denote probability, i.e., interval probability or probability interval [14][15]. Interval probability is the basis of this paper. FCD is caused due to the fuzziness of the event probability, so FCD need designate the fuzzy probabilities of the basic events and the linkage events at first; this shows FCD have sources-fuzziness [14]. Therefore, we regard interval probability as the extension of accurate probability, and the random variable of interval probability has probabilistic character and numerical characteristic with probability density function, distribution function math expectation, variance and so on [16] [17].

II. INTERVAL NUMBER

A. Interval Probability

Definition 1. Fuzzy set $\tilde{A}$ is continuous fuzzy subset of domain R in (-∞, +∞), and its membership function satisfy the following equation

$$\max_{x\in R} u_{\tilde{A}}(x) = 1 \tag{1}$$

In discrete probability space, $\Omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$ denotes the set of the basic event of probability.

Definition 2.[16][17] If n real number intervals $I_i = [L_i, U_i]$, $i = 1, 2, \ldots, n$, satisfy

$$0 \le L_i \le U_i \le 1 \qquad (i = 1, 2, \ldots, n) \tag{2}$$

They can be used to describe the probabilities of the basic events in Ω, and we refer to them as n dimension probability interval set, briefly as n-PRI. If symbol $(L, U) = \{ [L_i, U_i] / i = 1, 2, \ldots, n \}$ is introduced, the n-PRI can be denoted by n-PRI(L, U) as well as .

Definition 3. [16][17] Let $\Omega=\{w_1, w_2, \ldots, w_n\}$, the probability of basic event can be denoted by a n-PRI(L, U), we refer to $Q(w_i) = [L_i, U_i]$ as interval probability of basic event w_i $(i = 1, 2, \ldots, n)$.

$$Q(A) = \{\sum_{i=1}^{n} a_i p_i \mid a_i = \mathrm{Q}(\mathrm{w}_i), \\ (p_1, p_2, \cdots, p_n) \in S(L,U)\} \tag{3}$$

denotes interval probability of event A, and we refer to (n-PRI(L, U), Ω, S, Q) as n-sources interval probability space [14].

Here we define set $S(L, U)$ as

$$S(L,U) = \{(p_1, p_2, \cdots, p_n) \mid L_k \le p_k \le U_k, \\ k = 1,2,\cdots,n, \sum_{k=1}^{n} p_k = 1\} \tag{4}$$

Making a sign π_i

$$\pi_i = \Omega(\omega_i) = [L_i, U_i](i = 1,2,\cdots,n) \tag{5}$$

We can use following formula including π to express closing operation $Q(A)$.

$$\Omega(A) = \sum_{i=1}^{n} a_i \pi_i = \sum_{\pi_i \in A} \pi_i \tag{6}$$

Interval probability has following characters:

1) $Q(\varphi) = 0$ (φ denotes null set);

2) standardization, i.e. $Q(\Omega) = 1$;

3) if $[a, b] \le [c, d] \Leftrightarrow a \le c$ and $b \le d$, we have $A \subset B \Rightarrow Q(A) \le Q(B)$.

From above-mentioned definitions and characters, we can conclude that interval probability has probabilistic character and numerical characteristic so that we can adopt interval number to represent the fuzzy probability in causality diagram.

B. Algebraic Operation of Interval Nnumber

If the fuzzy interval probability $\tilde{A}_1$ and $\tilde{A}_2$ is expressed as $[L_1, U_1]$ and $[L_2, U_2]$ respectively, algebraic operation rules of fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ is as follow[11].

1)fuzzy number addition

$$\tilde{A}_1 + \tilde{A}_2 = [L_1 + L_2, U_1 + U_2] \tag{7}$$

2) fuzzy number subtraction

$$\tilde{A}_1 - \tilde{A}_2 = [L_1 - L_2, U_1 - U_2] \tag{8}$$

3) fuzzy number multiplication

$$\tilde{A}_1 \times \tilde{A}_2 = [L_1 L_2, U_1 U_2] \tag{9}$$

4) fuzzy number devision

$$\tilde{A}_1 \div \tilde{A}_2 = [L_1 / U_2, U_1 / L_2], \text{ where, } 0 \notin \tilde{A}_2 \tag{10}$$

III. REASONING OF IFCD

In this paper, the causality diagram bases on the following assumptions

1) Independent assumption means that the relation between basic events and linkage events and the relation among basic events are independent;

2) Two-value assumption denotes that every event has only two states, i.e., normal state and failure state, which is expressed by 0 and 1 respectively;

3) Probability assumption means that probability can totally portray the system and the trouble behavior of the unit.

A. Computing Interval Probability

Based on algebraic operation of interval number and reasoning method of causality diagram, the We get following operation of fuzzy number.

The Reasoning of causality diagram need compute the probability of complement event and the AND-gate probability.

Theorem 1. Complement fuzzy operation is following

$$\widetilde{P}_{\overline{X}} = 1 - \widetilde{P}_X = 1 - [L, U] = [1-U, 1-L] \tag{11}$$

Theorem 2. AND-gate fuzzy operation is as follow

$$\begin{aligned}\widetilde{P}_X &= \widetilde{P}_{X_1 X_2 \cdots X_n} = \prod_{i=1}^{n} \widetilde{P}_{X_i} \\ &= \prod_{i=1}^{n} [L_i, U_i] = [\prod_{i=1}^{n} L_i, \prod_{i=1}^{n} U_i]\end{aligned} \tag{12}$$

So the fuzzy probability of the cut set $C_i = \bigcap_{j=1}^{n_i} V_{ij}$ is as follows

$$\widetilde{P}_{C_i} = \prod_{j=1}^{n_i} [L_j^{C_i}, U_j^{C_i}] = [\prod_{j=1}^{n_i} L_j^{C_i}, \prod_{j=1}^{n_i} U_j^{C_i}] \tag{13}$$

We needn't gain OR-gate fuzzy operation because the OR relations of causality diagram are expressed as DCSs-f. In fact, we can gain disjoint fuzzy operation based on the DCSs-f formal.

Theorem 3. Disjoint fuzzy operation is as follows

$$\begin{aligned}\widetilde{P}_X = [&\prod_{j=1}^{n_1} L_j^{C_1} + \prod_{j=1}^{n_2} L_j^{C_2} (1 - \prod_{k=1}^{n_1} U_j^{C_1}) \\ &+ \cdots + \prod_{j=1}^{n_m} L_j^{C_m} \prod_{i=1}^{m-1} (1 - \prod_{k=1}^{n_i} U_j^{C_i}), \\ &\prod_{j=1}^{n_1} U_j^{C_1} + \prod_{j=1}^{n_2} U_j^{C_2} (1 - \prod_{k=1}^{n_1} L_j^{C_1}) \\ &+ \cdots + \prod_{j=1}^{n_m} U_j^{C_m} \prod_{i=1}^{m-1} (1 - \prod_{k=1}^{n_i} L_j^{C_i})]\end{aligned} \tag{14}$$

B. Fuzzy Conditional Probability of Interval Number

It is necessary to calculate the posterior probability under known evidence in causality diagram. If E is evidence, $P(E) \neq 0$ and $P(V_iE)$ is accurate probability,We can get the following definition of fuzzy conditional probability.

$$P(V_i / E) = \frac{P(V_i E)}{P(E)} \tag{15}$$

When the event probabilities are interval numbers, the calculating result of $P(E)$ and $P(V_iE)$ using (7), (8) and (9) is interval number. Therefore, by virtue of Eq (10), we can get the fuzzy conditional probability while the event probabilities are interval numbers.

Theorem 4. If E is evidence, and $\widetilde{P}$ denotes fuzzy probability, Let $\widetilde{P}(V_iE) = [L_{V_iE}, U_{V_iE}]$, $\widetilde{P}(E) = [L_E, U_E]$, $0 \notin \widetilde{P}(E)$, the fuzzy conditional probability is as follows

$$\begin{aligned}\widetilde{P}(V_i / E) &= \frac{\widetilde{P}(V_i E)}{\widetilde{P}(E)} = \frac{[L_{V_iE}, U_{V_iE}]}{[L_E, U_E]} \\ &= [\frac{L_{V_iE}}{U_E}, \frac{U_{V_iE}}{L_E}]\end{aligned} \tag{16}$$

C. Normalization of Interval Number

The probability value should fall in interval [0, 1]. However, directly applying the definition of fuzzy conditional probability will cause that the calculating result is beyond the interval [0, 1]. Especially, this case is easy to happen when the probability value of basic events and linkage events largely vary. This shows that it is necessary to modify fuzzy numbers so that FCD can apply in the system with largely varying probabilities of the events. Here, we adopt the normalizing method, that is, $\widetilde{P}(X(z))$ is divided by its supper bound $\sup(\widetilde{P}(X(z)): z \in R$. Therefore, we get the following algorithm to normalize the interval number.

Algorithm 1. Normalizing Algorithm of Interval Number

Input: The k unnormalized interval numbers of events $\widetilde{P}(X_i) = [L_i, U_i]$ $(i = 1, 2, \ldots, k)$.

Output: Normalized interval numbers.

The algorithm steps is as follows:

1) Gaining t value in virtue of following formular

$$t = \sup_{i=1,..,k} (L_i, U_i) = \sup_{i=1,..,k} (U_i) \tag{17}$$

Let $t = U_s (1 \le s \le k)$.

2) If $t \le 1$, the algorithm end up;

3) Calculating the mean value of all interval numbers.

a) Gaining mean value of interval numbers of basic events and linkage events m_j . If the interval number of the ih basic event or linkage event is $\widetilde{A}_i = [a_i, b_i], i = 1,2,..,k$, we gain $m_i = (a_i + b_i)/2, i = 1,2,..,k$;

b) Gaining mean value M_i (i = 1, 2, . . ., k) of interval number of observed event through the method of triangular fuzzy number.

4) Computing normalization factor w in virtue of following

$$w = \frac{1 - M_s}{\max(1, U_s) - M_s} \tag{18}$$

where M_s is mean value of $\widetilde{P}(X_s) = [L_s, U_s]$;

5)Normalizing interval number $\widetilde{P}(X_i) = [L_i, U_i]$ through following formula

$$\widetilde{P}_i = [M_i - (M_i - L_i)w, M_i + (U_i - M_i)w] \tag{19}$$

Theorem 5. The normalizing result of interval number certainly falls in interval [0, 1];

Therefore, the normalized interval number is fuzzy probability.

D. Reasoning Algorithm of Single-value Fuzzy Causality Diagram

In fuzzy causality diagram, the probabilities of the basic events and the linkage events are fuzzy numbers. Single-value causality diagram with accurate probability has a set of reasoning methodology[1][4]. However, the reasoning method causality diagram with fuzzy probability is different from the convential causality diagram. So it is necessary to propose an reasoing algorithm for single-value fuzzy causality diagram. In fact, the core of reasoning for causality diagram is to compute the conditional probability of observed node event. From above-mentioned analysis, we can calculate the fuzzy probability of obsered node event by the fuzzy operators, the definition of the fuzzy conditional probability and normalization method. However, it is hard to compare the result so as to realize the reasoning of causality diagram because a fuzzy number may represent many real numbers with different member function. So, it is necessary to convert a fuzzy number into an accurate number, that is, de-fuzzy operating. Here, we adopt middle-value method for de-fuzzy operating, which need calculate middle-value of a fuzzy number. Based above-mentioned discussion, we give the reasoning algorithm of single-value fuzzy causality diagram as follows. Here, we give two selective reasoning algorithms.

Algorithm 2. Reasoning Algorithm of Single-value Fuzzy Causality Diagram Based on Confidence Level

Input: The n fuzzy numbers of probabilties of the basic events and linkage events.

Output: Gaining the fuzzy probabilities of the observed events and finding out the failure cause.

The algorithm steps is as follows:

1) Giving a set of confidence levels λ_i ($\lambda_i \in [0,1]$, i=1, 2, …, n), and calculating the confidence intervals of the basic events and linkage events corresponding to the confidence levels;

2) Based on the fuzzy operation rules and fuzzy operators, the confidence intervals of the observed events corresponding to the different confidence levels λ_i are calculated applying reasoning method of single-value causality diagram with accurate probability;

3) If there exists confidence interval beyond [0,1], we should normalize these fuzzy numbers adopting normalizing algorithm, that is, the fuzzy probabalities are gained through normalizing all the gained fuzzy numbers of the obsered events using normalizing algorithm.

4) Gaining mean values of normalized fuzzy numbers of the events;

5) Based on comparing the mean values, we can find the failure cause because the more larger mean values, the more possibility to cause failure.

6) The reasoning ends up.

Algorithm 3. Reasoning Algorithm of Single-value Fuzzy Causality Diagram Based on Fuzzy Number Paramater

Input: The n fuzzy numbers of probabilties of the basic events and linkage events.

Output: Gaining the fuzzy probabilities of the observed events and finding out the failure cause.

The algorithm steps is as follows:

1) Computing the parameters of the fuzzy numbers of the basic events and linkage events;

2) Based on the fuzzy operation rules and fuzzy operators, the parameters of the fuzzy numbers of the observed events are calculated applying reasoning method of single-value causality diagram with accurate probability;

3) Gaining the most deviation of the fuzzy numbers of obersed events from its mean values;

4) If there exists most deviation beyond [0,1], we should normalize these fuzzy numbers adopting normalizing algorithm, that is, the fuzzy probabalities are gained through normalizing all the gained fuzzy numbers of the obsered events using normalizing algorithm.

5) Gaining mean values of normalized fuzzy numbers of the events;

6) Based on comparing the mean values, we can find the failure cause because the more larger mean values, the more possibility to cause failure;

7) The reasoning ends up.

Remark 1. Algorithm 2 needs to compute the condidence interveal of fuzzy number in different confidence levels λ_i , and it's result of reasoning is more accurate. Its computing time complexity is $O(n^2)$. Algorithm 3 doesn't need to compute the condidence interveal of fuzzy number in different confidence levels λ_i, and it directly deals with the paramater of fuzzy number. It's reasoning is more simple, and its computing time complexity is $O(n^2)$.

Remark 2. The reasoning method of single-value causality diagram with accurate probability denotes the conventional single-value reasoning algorithm in [1][4].

Remark 3. The normalizing algorithm of different type fuzzy numbers, such as triangle fuzzy number (TNFN), trapezoidal fuzzy number (TPFN), normal fuzzy number (NFN), and interval number(IN), is different. So, we should adopt different normalizing algorithm in the reasoning process of fuzzy causality diagram with different fuzzy numbers. We can get corresponding reasoning algorithm fuzzy causality diagram with different fuzzy numbers through corresponding normalizing algorithm.

Remark 4. Algorithm 2 and algorithm 3 only disposes single-value value fuzzy causality diagram. Multi-value fuzzy causality diagram is different from single-value value fuzzy causality diagram, and its reasoning algorithm will be discussed in another paper.

IV. EXAMPLE

In order to further illustrate the methodology of this paper, we consider a simple example of causality diagram [12] as shown in Fig.1. Fig.1. is causality diagram of the subsystem of steam reactor in nuclear power plant. There are three node events X_1, X_2 and X_3. The input data are given in Tab.I with the interval numbers of failure probability of basic event and linkage event.

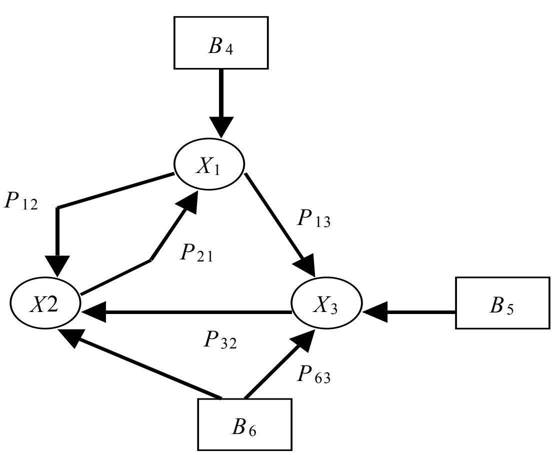

Figure 1. Causality diagram of the subsystem of steam reactor.

What is the possibility of events accident when X_1 and X_2 is failure? What is the cause of X_1 and X_2 failure?

This is an inverse reasoning of causality diagram, i.e. the goal of reasoning is to compute the posterior probability of H ($\Pr\{H/E\}$) under the given evidence $E = X_1 X_2$.

Under this assumption that X_1 and X_2 is failure, we can compute the posterior failure probability of the event, i.e. $\Pr\{H/E\}$, where $H = X_3, B_4, B_5$, or B_6.

TABLE I. FUZZY FAILURE PROBABILITY OF BASIC EVENT AND LINKAGE EVENT

Event	*L*	*M*	*U*
B_4	0.4	0.5	0.6
B_5	0.35	0.5	0.65
B_6	0.3	0.5	0.7
P_{12}	0.4	0.5	0.6
P_{13}	0.3	0.5	0.7
P_{21}	0.35	0.5	0.65
P_{32}	0.35	0.5	0.65
P_{63}	0.35	0.5	0.65

TABLE II. FUZZY CONDITONAL PROBABILITY (FCP) OF EVENTS CONDITIONED WITH EVIDENCE

Event	*L*	*M*	*U*
$\Pr(X_6/E)$	0.847	0.865	1.000
$\Pr(B_4/E)$	0.716	0.730	0.800
$\Pr(B_5/E)$	0.530	0.541	0.613
$\Pr(B_6/E)$	0.637	0.649	0.704

We can calculate the posterior failure probability of the events as showed in Tab.II through algorithm 1 and algorithm 3. From Tab.II, We can obtain a total order comparing the fuzzy posterior probability in based on the dominance criteria. The total order is $\Pr(X_3/E) > \Pr(B_4/E) > \Pr(B_6/E) > \Pr(B_5/E)$.

Then we can draw the conclusion that X_3 has the most possibility of malfunction when X_1 and X_2 have failures and it is most possible that the cause of X_1 and X_2 failures is B_4 because B_4, B_5 and B_6 are basic events, i.e. final failure causes. The reasoning result is consistent with [12], so the reasoning method is effective and reliable when the events probabilities are interval numbers, that is, causality diagram under interval numbers can rightly reason. Under the circumstance that the probability value of the event is interval number, the probability value of every event is a range that can be changed, so IFCD has more flexible and adaptable than causality diagram in the past.

V. CONCLUSIONS

Causality Diagram is a probabilistic reasoning method. We propose a fuzzy causality diagram with possibility concepts in real systems to overcome the weak points of the conventional causality diagram and fault analysis, i.e. they do not give the tolerances of probability values of hazards. Our approach using fuzzy causality diagram can give a picture concerning the qualitative expression of hazards.

This research considers the relative frequencies of the basic events and linkage event as interval numbers. Because hazard events are rare and the frequencies of hazard events show large spreads generally, the interval numbers is also considered to provide a precise description of hazard events.

Based on the interval number, operator, fuzzy conditional probability and the normalization method were discussed in this paper. The result of numerical simulating of a subsystem in nuclear plant is coincident with the fact, and it shows the normalizing method is effective.

Such studies can be of considerable importance for fault analysis of complex and hazardous industrial systems such as the Chernobyl nuclear plant.

This research has posed a necessity that may be a fruitful area for future research.

The method proposed here may need a large amount of computation; thus, a computer code for solving this fuzzy causality diagram could be developed.

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