A New Adaptive Neuro-Fuzzy Approach for On-Line Nonlinear System Identification

Morteza Hamzehnejad
Department of Automation and Instrumentation
Petroleum University of Technology
Tehran, Iran
hamzehnejad1982@yahoo.com

Karim Salahshoor
Department of Automation and Instrumentation
Petroleum University of Technology
Tehran, Iran
salahshoor@put.ac.ir

***Abstract*— A new on-line identification algorithm is presented in this paper based on a neuro-fuzzy model structure. The algorithm is developed based on the functional equivalence between a radial basis function (RBF) neural network and a fuzzy inference system (FIS). The developed algorithm utilizes a Weighted Rule Activation Record (WRAR) as a functional measure to monitor the modeling efficiency of the created rules. This measure evaluates the influence of each created rule with a time-based memory weight which puts more emphasis on the most recent input data. The proposed technique employs an Extended Kalman Filter(EKF) as a learning algorithm to adapt the antecedent and consequent parameters of the nearest rule. This algorithm benefits simple and understandable criteria to make it more attractive in practical applications. This leads to more efficient rule base with low created rules. Its low computational time makes it as an appropriate on-line identification approach. The performance of the proposed algorithm with some other new algorithms have been evaluated on a nonlinear dynamic system. Simulation results demonstrate the efficiencies of the proposed algorithm, resulting to the most simple rule structure with the lowest computational time.**

Keywords—*on-line identification, adaptive neuro-fuzzy, WRAR, WRMS, EKF*

I. Introduction

Considerable researches have explored the possibility of using different methodologies for modeling and identification of complex systems during the past decades. The conventional first principle modeling approach, which is based on mass and energy balances, often do not comply with many practical engineering problems, making its application a difficult or even impossible task. Furthermore, in most real-life situation, where the system dynamics are subject to perturbations, adaptive identification schemes have to be used to employ sequential input-output experimental data to adjust the estimated model automatically in real-time manner.

Fuzzy Rule-Based (FRB) models have a significant impact in the identification community to represent complex nonlinear systems due to their computational efficiency and transparency. The main obstacle in the design of classical FRB approaches, however, corresponds to a priori knowledge to generate a proper, adequate and expedient model structure in terms of the rule-base and membership functions and their relevant free parameters. Another stumbling block is that fuzzy models are not adaptive. This hinders their applications to practical problems with changing dynamics.

FRB approaches with on-line learning algorithm offer promising identification methodologies, incorporating inherent flexibility to adapt their consecutive model structure and parameters from experimental data, heuristic rules, or a combination of both. Many different FRB identification strategies have been proposed in the literature. Their learning algorithms can be broadly categorized into two general classes of batch and sequential learning schemes. In batch learning, the complete training data are assumed to be available a priori. Thus, the training involves cycling the data over a number of epochs to accurately adapt the identified model. However, this paper addresses on-line identification approaches in which the whole data are not available a priori, but arrive sequentially one by one in real-time manner. That is, learning should be able to start without a priori knowledge. This interesting feature makes the on-line learning approaches potentially very useful in autonomous and smart self-tuning control system applications.

A self-constructing neural-fuzzy inference network has recently been proposed by Juang and Lin [1]. In this approach, the fuzzy rules are created on-line from the training data based on distance criterion between the new input data and the center of Gaussian membership functions, relating to the existing fuzzy rules. The approaches, however, does not pass any monitoring scheme to evaluate any inefficient rule once created. This may lead to a large rule base structure with high number of insignificant rules. Wu and Er [2] proposed a hierarchical on-line self-organizing learning algorithm for dynamic fuzzy neural networks, inspired by the ideas of adding and pruning hidden neurons to form a minimal Radial Basis Function (RBF) network given by Yingwei et al.[3]. Leng et al. [4] presented another on-line self-organizing fuzzy neural network which includes a rule pruning method. These two algorithms, however, need all the past data received so far to process the pruning criteria, leading to a huge memory size requirement. Recently, Angelov and Filev [5] have proposed an on-line identification approach for an evolving Takagi-Sugeno (eTS) fuzzy model. The approach is based on a recursive potential clustering method responsible for model structure (rule-base) learning and an on-line recursive least square (RLS) algorithm for linear TS model parameter estimation. The algorithm utilizes two different potential monitoring criteria to check for a new rule addition or modification of an old nearby rule based on potential magnitude of the new data with respect to the other existing rules. The proposed algorithm, however, has not included the capability to ignore already created rules which may become insignificant in the future. Rong et al.[6] have recently presented a Sequential Adaptive Fuzzy Inference System (SAFIS) based on the functional equivalence of a RBF network and a fuzzy inference system (FIS). The proposed algorithm uses the concept of rule influence to check for any rule addition or deletion based on the input data received so far. The rule influence has been defined as its statistical contribution to the overall output of SAFIS. However, the exact calculation of this measure is not practically feasible in a truly sequential learning scheme. Thus, another simplified alternative approach has been adopted in [6] which requires the probability density function of the input data space. The algorithm, however, assumes a uniform distribution for the input data space to simplify the resulting formulation.

In this paper, a new on-line identification approach is presented based on a neuro-fuzzy model structure. In contrast to SAFIS, the proposed approach does not impose any restriction on the statistical nature of the input data space. It presents a new idea based on weighted rule significant measure, defined as Weighted Rule Activation Record (WRAR). The proposed measure incorporates a weighted forgetting factor to make a time-based discrimination between the activation effect of the individual created fuzzy rules in their past time record. This puts more emphasis on the most recent contribution of each existing rule to the modeling performance during the learning procedure. The algorithm presents another measure to evaluate the adequacy of the overall existing fuzzy rules to maintain a desired low model accuracy limit in terms of output error. However, to avoid from any inaccurate decision based on instantaneous model evaluation, resulting from impact of new input data, a Weighted Root Mean-Square (WRMS) error is proposed in this work. The proposed algorithm makes benefits of these two informative measures to decide on any model structure adaptation by adding or replacing rules during the learning phase. The identification procedure utilizes an EKF parameter learning algorithm to update the model parameters of the rule selected by the new input data on the basis of winner rule strategy as the closest rule.

The proposed identification methodology is evaluated on a nonlinear dynamic system as the simulation case study. The obtained results are compared with the several alternative well-known approaches. The results demonstrate the superiority of the proposed algorithm in providing acceptable training and testing accuracies with much simplicity in the identified model structure and computational complexity.

The paper contents are organized as follows. Section II presents a new adaptive neuro-fuzzy identification methodology, including the proposed model structure, learning procedure and the EKF parameter learning algorithm. The comparative performance evaluation of the proposed methodology is tested on a nonlinear dynamic system with other alternative well-known approaches in Section III. Section IV summarizes the resulting conclusions.

II. A New Adaptive Neuro-Fuzzy Identification Methodology

The nonlinear dynamic system which will be the focus on this research work can be described by the following frequently used Nonlinear Auto-Regressive with eXogenous input (NARX) regressive model:

$$y(k+1) = f[y(k),...,y(k-n+1);u(k),...,u(k-m+1)] \tag{1}$$

where $f(.)$ indicates a nonlinear functional relation, k denotes the discrete-time instant, m and n represent model orders, y is the system output and u is the system input.
Eqn. (1) can be written more conventionally as:

$$y(k+1) = f(X(k)) \tag{2}$$

where the regression vector, $X(k)$, is given by:

$$X(k) = [y(k),...,y(k-n+1);u(k),...,u(k-m+1)] \tag{3}$$

The main idea underlying the use of rule-based fuzzy models for identification of nonlinear systems is to approximate the system dynamics by a collection of local fuzzy rules of the following form:

$$ith\ rule: if (x_1 \ \ is \ \ A_{1i})...(x_N \ \ is \ \ A_{Ni}), then (\hat{y} \ \ is \ \ a_i) \quad i = 1,...,H \tag{4}$$

where $[x_1, x_2,..., x_N]^T$ represents the regression vector (X), $A_{ji}(j = 1,..., N)$ denotes the membership value of the jth input variable, a_i is the constant consequent parameter in the ith rule, H is the number of created rules and $\hat{y} = \hat{f}(x)$ indicates the identified model output.

The presented approach uses a neuro- fuzzy model structure, depicted in Fig.1, to realize the identification algorithm.

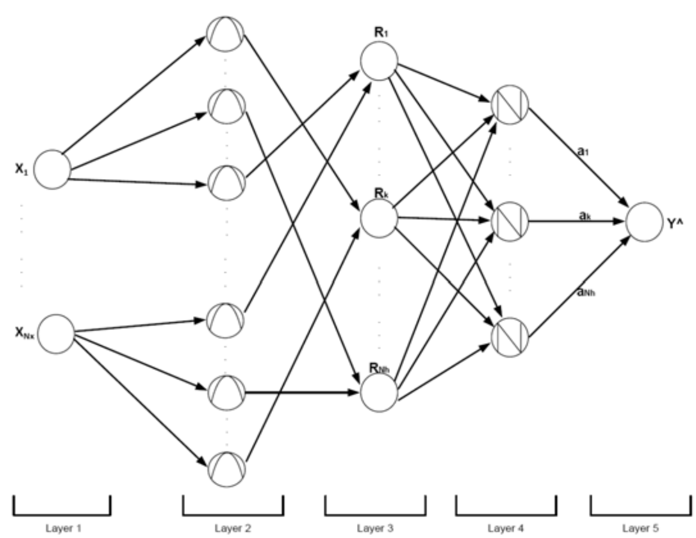


Fig.1. Neuro-fuzzy model structure

The main identification goal is to minimize the error between the actual output $y(k)$ and the estimated model output $\hat{y}(k)$ (i.e., $\|y(k) - \hat{y}(k)\|$).

A. Neuro-fuzzy model structure

The neuro-fuzzy model structure includes the following 5 layers to realize the set of identified fuzzy rules, described by Eqn. (4):

- *Layer 1*: This layer allocates one node to each input variable to transmit the incoming signal to the next layer.
- *Layer 2*: This layer implements the membership functions corresponding to individual input variables through its nodes. In this work, the Gaussian functions of the following form are utilized for this purpose:

$$A_{jp}(x_j) = \exp(-\frac{(x_j - \mu_{jp})^2}{\sigma_p^2}) ; \quad p = 1,2,...,H \tag{5}$$

  where μ_{jp} and σ_p indicate the center and the width of the pth Gaussian function for the jth input variable, respectively.
- *Layer 3*: Each node in this layer realizes the if part of the extracted if-then fuzzy rules which is based on the following sum-product composition computation:

$$R_p(X) = \coprod_{j=1}^{N} A_{jp}(x_j) = \exp(-\sum_{j=1}^{N} \frac{(x_j - \mu_{jp})^2}{\sigma_p^2})$$
$$= \exp(-\frac{\left\| X - \mu_{jp} \right\|^2}{\sigma_p^2}) \tag{6}$$

- *Layer 4*: This layer is used to compute the normalized contribution of each rule, given by:

$$\overline{R}_k = \frac{R_k(X)}{\sum_{p=1}^{H} R_p(X)} \tag{7}$$

- *Layer 5*: This layer consists of just one node to estimate the single system output as follows:

$$\hat{y} = \frac{\sum_{p=1}^{H} R_p(X) a_p}{\sum_{p=1}^{H} R_p(X)} \tag{8}$$

B. The proposed on-line identification algorithm

1) WRAR as a global rule significance measure : To assets the overall significance of a rule in fuzzy modeling performance, a collective time-based record is more beneficial. In this paper, a new WRAR measure is presented to accomplish this task. Each created rule has a significance factor, demonstrating its instantaneous modeling contribution at the current time (k), given by:

$$\lambda_l(k) = \frac{R_l(X)}{\sum_{i=1}^{H} R_i(X)} \quad ; \; l = 1,2,...,H \tag{9}$$

Thus, it is possible to assess the significance of a rule based on this measure. This procedure, however, emphasizes the contribution of each rule in instantaneous modeling effect at each time instant, leading to fluctuation in rule assessment judgment in the long term. A more efficient rule activation measure is based on a global time-weighted rule activation record which puts more weight on the most recent rule activation in modeling process, defined by the following recursive equation:

$$WRAR_l(k) = \gamma_1 (1-\gamma_1)^{k-1} \lambda_l(1) + ... + \gamma_1 \lambda_l(k)$$
$$\Rightarrow WRAR_l(k) = \frac{\gamma_1 \lambda_l(k)}{1-(1-\gamma_1)q^{-1}} \tag{10}$$

where γ_1 denotes forgetting factor ($0 < \gamma_1 \leq 1$) and $\lambda_l(i)$; $i = 1,...,k$ indicates the lth rule at the ith time instant.

2) Rule addition criteria: The identification algorithm begins with no a priori fuzzy rule. As new data input pairs $\{X(k), Y(k)\}$ become available sequentially during learning, new rules can be added based on the following three criteria:

a) Distance criterion: The distance of any new input data is measured with respect to the nearest rule in the rule-base. This measure demonstrates the influence of the nearest rule on the new input data.

b) Modeling error criterion: using instant output error measure to make decision on the rule addition, can lead to undesirable rule creation. To improve this deficiency, a time-weighted root mean-squared (WRMS) error measure is presented as:

$$E_{WRMS}(k) = \sqrt{\sum_{j=k-(N-1)}^{k} \gamma_2^{\,j-k} \frac{\left\| e(j) \right\|^2}{N}} \tag{11}$$

where N indicates the length of time window, γ_2 represents the forgetting factor and $e(j) = y(j) - \hat{y}(j)$ denotes the instantaneous output error in the jth time iteration.

c) Rule influence criterion expressed in terms of WRAR measure: Therefore, the rule addition mechanism obeys the following three criteria:

$$\begin{aligned} &\left\| X(k) - \mu_{nr}(k) \right\| > \varepsilon_k \\ &WRAR_{Nr}(k) > WRAR_G \\ &E_{WRMS}(k) > E_G \end{aligned} \tag{12}$$

where ε_k , $WRAR_G$ and E_G denote the decision thresholds, $X(k)$ is the latest input data, and the subscript nr indicates nearest fuzzy rule to the input data (i.e., $\mu_{nr}(k)$ is the center of nearest fuzzy rule to $X(k)$). $WRAR_G$ and E_G represent the lowest acceptable model accuracy measures, specified in terms of the lowest accuracies expected from the nearest rule activation record and the WRMS output error, respectively. While, ε_k shows the lowest regional distance in which the nearest rule can have influence on the new input data. In this work, an exponential time varying relationship is presented for ε_k as :

$$\varepsilon_k = \varepsilon_{\max} - (\varepsilon_{\max} - \varepsilon_{\min})(1 - \exp(-(k-2)/\tau)) \quad (13)$$

where $\varepsilon_{\max}$ and $\varepsilon_{\min}$ are the largest and smallest distance of the interest and τ denotes the decay constant of the exponential function. This time-varying function allows the learning procedure to start adaptation coarsely in the initial stage and then lets the model to finely be tuned at the end.

3) The proposed Learning Algorithm : The proposed learning algorithm starts with no a priori rule and then continues with two parts of structural model improvement in terms of fuzzy rule addition or replacement and model parameter adjustment relating to the nearest rule premise and consequent parameters. Thus, given the threshold values (i.e., $WRAR_G$, E_G , $\varepsilon_{\max}$, $\varepsilon_{\min}$), the computational procedure can be summarized in the following steps for each sequential incoming data pair (x, y):

1. *Compute the overall estimated model output:*

$$\hat{y} = \frac{\sum_{i=1}^{H} R_i(X) a_i}{\sum_{i=1}^{H} R_i(X)} \quad (14)$$

2. Find the nearest rule to the new input data.
3. Calculate the measures relating to the rule addition criteria (i.e., E_{WRMS} , E_k , ε_k).
4. Check the rule addition criteria:

If $\|X(k) - \mu_{nr}(k)\| > \varepsilon_k$
  And if $E_{WRMS}(k) > E_G$
    And if $WRAR_{Nr}(k) > WRAR_G$
      Allocate a new rule ($H+1$) with the following coordinates:
      $a_{H+1} = e_k$
      $\mu_{H+!} = X_k$
      $\sigma_{H+1} = \kappa \|X_k - \mu_{nr}\|$; (κ is overlap constant) (15)
    Else
      Replace the nearest rule with the new rule, having the following coordinates:
      $a_{nr} = e_k$
      $\mu_{nr} = X_k$
      $\sigma_{nr} = \kappa \|X_k - \mu_{nr}\|$ (16)
    End if
  Else
    Adjust the nearest rule free parameters (a_{nr} , μ_{nr} , σ_{nr}) by an EKF estimation algorithm.
  End if
Else
  Adjust the nearest rule free parameters by an EKF estimation algorithm.
End if

C. Development of an EKF parameter estimation algorithm

The proposed identification approach uses an EKF estimation algorithm for only updating the free parameters of the winner rule. The winner rule is defined as the rule that is nearest to the new input data. As a consequence, there is no need to update other rules parameters. This decreases the required computational burden and hence leads to a fast identification procedure.

To develop the EKF parameter estimation algorithm, the gradients of the nonlinear identified model, i.e. $\hat{y} = \hat{f}(X)$, should be derived with respect to the free parameters of the nearest fuzzy rule as follows:

$$\dot{a}_{nr} = \frac{\partial \hat{y}_k}{\partial a_{nr}} = \frac{R_{nr}}{\sum_{i=1}^{H} R_i} \quad (17)$$

$$\dot{\mu}_{nr} = \frac{\partial \hat{y}_k}{\partial \mu_{nr}} = \frac{\partial \hat{y}_k}{\partial R_{nr}} \frac{\partial R_{nr}}{\partial \mu_{nr}} = a_{nr} \left[\frac{\sum_{i=1}^{H} R_i - R_{nr}}{(\sum_{i=1}^{H} R_i)^2} \right] \frac{\partial R_{nr}}{\partial \mu_{nr}} \quad (18)$$

$$\dot{\sigma}_{nr} = \frac{\partial \hat{y}_k}{\partial \sigma_{nr}} = \frac{\partial \hat{y}_k}{\partial R_{nr}} \frac{\partial R_{nr}}{\partial \sigma_{nr}} = a_{nr} \left[\frac{\sum_{i=1}^{H} R_i - R_{nr}}{(\sum_{i=1}^{H} R_i)^2} \right] \frac{\partial R_{nr}}{\partial \sigma_{nr}} \quad (19)$$

where

$$\frac{\partial R_{nr}}{\partial \mu_{nr}} = 2R_{nr} \frac{X_k - \mu_{nr}}{\sigma_{nr}^2} \quad (20)$$

$$\frac{\partial R_{nr}}{\partial \sigma_{nr}} = 2R_{nr} \frac{\|X_k - \mu_{nr}\|^2}{\sigma_{nr}^3} \quad (21)$$

The gradient vector is then obtained as $\dot{\theta}_{nr} = [\dot{a}_{nr}, \dot{\mu}_{nr}, \dot{\sigma}_{nr}]^T$. Thus, the EKF algorithm can be formulated as follows to update the nearest rule parameters:

$$K(k) = P_{nr}(k-1)\dot{\theta}_{nr}(k)[R\ (k) + \dot{\theta}_{nr}^T(k) P_{nr}(k-1)\dot{\theta}_{nr}(k)]^{-1} \quad (22)$$

$$\theta_{nr}(k) = \theta_{nr}(k-1) + K(k)e(k) \quad (23)$$

$$P_{nr}(k) = [I - K(k)\dot{\theta}_{nr}^T(k)]P_{nr}(k-1) \quad (24)$$

where K is the Kalman gain, R denotes the modeling error covariance and P_{nr} indicates the error covariance matrix of the nearest rule.

III. SIMULATION STUDY

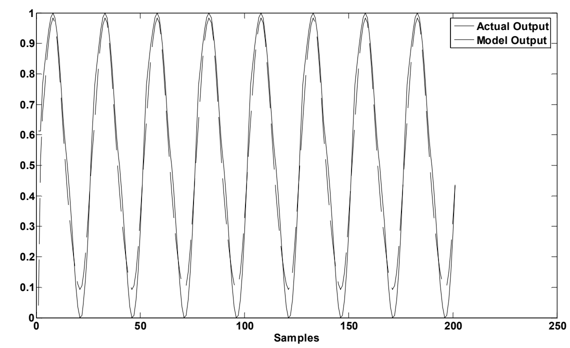
The performance of the proposed on-line identification methodology is evaluated on a nonlinear dynamic system. The resulting performance will be compared with some other well-known identification algorithms such as Minimum Resource Allocation Network (MRAN) studied in [3], eTS presented in [5] and SAFIS presented in [6].

The nonlinear dynamic system, considered in this simulation study, is given by [7] as:

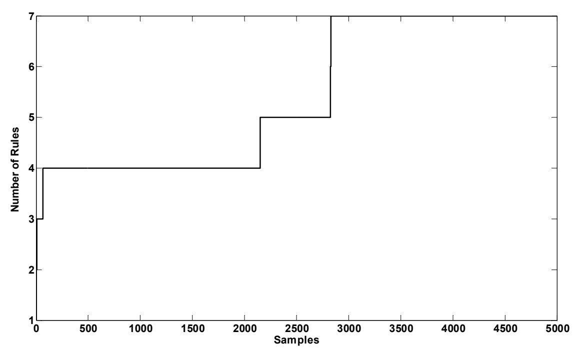
$$y(k) = \frac{y(k-1)y(k-2)(y(k-1)-0.5)}{1 + y^2(k-1) + y^2(k-2)} + u(k-1) \quad (25)$$

The input to the system is uniformly selected in the range [-1.5 , 1.5] and the test input is chosen as $u(k) = \sin(2\pi k/25)$, as recommended in [7]. 5000 and 200 observation data are produced to be used for the purpose of training and testing, respectively. Practical observations showed that the identification performance is not sensitive to the initial setting of some parameters. That is those parameters are not critically problem dependent. These parameters were predefined as $\tau = 5000$ (i.e., the number of training data), overlap factor $\kappa = 1.75$, length of time window $N = 10$, forgetting factors $\gamma_1 = \gamma_2 = 0.997$ and modeling error covariance=1. Other parameter values were set, based on the problem, to $\varepsilon_{\max} = 0.7$, $\varepsilon_{\min} = 0.1$, $WRAR_G = 0.1$ and $E_G = 0.1$ to accomplish a desired model accuracy.

In all the studies, the tuning parameters of eTS, SAFIS and MRAN algorithms were set to the values recommended in [6] to obtain their best possible performances.



(a)



(b)

Fig.2. (a) Model testing result (b) Rule updating profile

Fig.2 illustrates the resulting identification outcomes for the proposed algorithm in terms of the output response evaluation for the actual testing data set and the fuzzy rules adaptation profile corresponding to training experiment. The performance comparison of the proposed algorithm with the other alternative algorithms is summarized in Table. I. As shown, the proposed algorithm is able to demonstrate acceptable RMSE accuracies with superiority in model simplicity (i.e., lower number of created rules) and lower computational complexity.

TABLE I. RESULTS FOR DIFFERENT ALGORITHMS

Methods	Number of Rules	Training RMSE	Testing RMSE
WRAR	7	0.0506	0.0742
SAFIS	17	0.0539	0.0221
MRAN	22	0.0371	0.0271
eTS($r = 1.8, \Omega = 10^6$ **)**	49	0.0292	0.0212

IV. CONCLUSION

In this paper, a new on-line identification methodology has been presented based on a neural-fuzzy structure. It proposes an innovative idea of WRAR measure for evaluating fuzzy rule significance. The method utilizes a weighted RMS measure of output error to evaluate identified model accuracy. The proposed method makes the benefits of these two informative new measures to decide on model structure adaptation via rule addition or replacing an old inefficient rule. The identification procedure utilizes an EKF parameter learning algorithm to update the parameters of the nearest rule to the input data to fulfill the model parameter adaptation. The simulation results on a nonlinear dynamic system benchmark demonstrated its effectiveness compared to the other alternative well-known identification algorithms in terms of the identified model simplicity and its lower computational complexity.

REFERENCES

[1] C.-F. Juang, C.-T. Lin, An on-line self-constructing neural fuzzy inference network and its applications, IEEE Trans. Fuzzy Systems 10 (2) (2002) 144–154.

[2] S.Wu, M.J. Er, Dynamic fuzzy neural networks—a novel approach to function approximation, IEEE Trans. Systems Man Cybernet.—Part B: Cybernet. 30 (2) (2000) 358–364.

[3] L.Yingwei, N. Sundararajan, P. Saratchandran, A sequential learning scheme for function approximation using minimal radial basis function (RBF) neural networks, Neural Comput. 9 (1997) 461–478.

[4] G. Leng, T.M. McGinnity, G. Prasad, An approach for on-line extraction of fuzzy rules using a self-organising fuzzy neural network, Fuzzy Sets and Systems 150 (2) (2005) 211–243.

[5] P.P. Angelov, D.P. Filev, An approach to online identification of Takagi–Sugeno fuzzy models, IEEE Trans. Systems Man Cybernet.—Part B: Cybernet. 34 (1) (2004) 484–498.

[6] Hai-Jun Rong, N. Sundararajan∗, Guang-Bin Huang, P. Saratchandran. Sequential Adaptive Fuzzy Inference System (SAFIS), Fuzzy Sets and Systems 157 (2006) 1260 – 1275

[7] L. Wang, J. Yen, Extracting fuzzy rules for system modeling using a hybrid of genetic algorithm and Kalman filter, Fuzzy Sets and Systems 101 (1999) 353–362.