Error Equivalence Methodology for Dimensional Variation Control in Manufacturing

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Abstract—**The product dimensional quality could be affected by multiple error sources in manufacturing processes. One widely observed engineering phenomenon is that different error sources can result in identical error patterns on product features. For example, deviations in fixture locators and workpiece datum surfaces can produce the same type of dimensional variations in engine head machining. Such an "error equivalence" phenomenon often significantly increases the complexity of dimensional variation control such as root cause identification. By generalizing and extending the authors' previous work, the paper aims to establish error equivalence methodology which addresses issues of mathematical modeling of the error equivalence phenomenon in manufacturing, error equivalence analysis for root cause identification and automatic adjustment.**

Keywords—**error equivalence, manufacturing process**

I. INTRODUCTION

The manufacturing process usually involves multiple error sources that affect product dimensional quality. A widely observed engineering phenomenon is that individual error sources can result in identical error patterns on product features. For instance, a machining operation involves deviations from fixture locators, machine tool path movement, and workpiece surfaces. All three types of process deviations can generate the same amount of feature deviation y as shown in Fig. 1 [1-2]. This error equivalence phenomenon has also been noted in many other manufacturing processes, e.g., the automotive body assembly process ([3]).

The impact of such an "error equivalence" phenomenon is twofold. On one hand, it significantly increases the complexity of variation control. As an example, identifying the root causes becomes extremely challenging when different error sources are able to produce identical dimensional variations. On the other hand, the error equivalence phenomenon provides an opportunity for variation reduction. For instance, we could purposely use one error source to counteract the others and thereby reduce process deviation [2]. In both cases, a fundamental understanding of this engineering phenomenon will assist to achieve improved dimensional control.

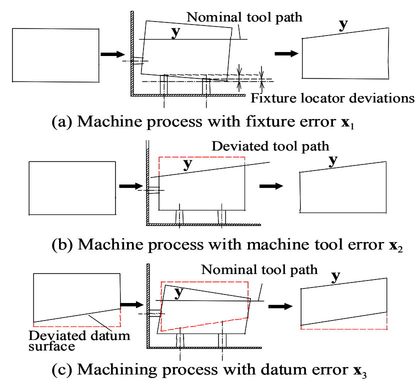

Figure 1. Error equivalence in machining [1][2]

The study on error equivalence is, however, very limited [1,2,4]. Most research has been focusing on the analysis of individual error sources, e.g., fixture errors [5-12] and machine tool errors [13-18]. Studies have been conducted to model the feature deviation y of a workpiece as a linear combination of multiple errors sources $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_p)^T$ [19-26],

$$\mathbf{y} = \Sigma_{i=1}^{p} \mathbf{\Gamma}_i \mathbf{x}_i + \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{\Gamma}_i$'s are sensitivity matrices determined by process and product design. $\boldsymbol{\varepsilon}$ is the noise term. As a quality measure, $\mathbf{y}$ can either represent the deviation of critical dimensions or the deviation of features represented by surface orientation n, position p, and size D, i.e., $\mathbf{y} = \left(\Delta\mathbf{n}^{\mathrm{T}} \quad \Delta\mathbf{p}^{\mathrm{T}} \quad \Delta\mathbf{D}^{\mathrm{T}}\right)^{\mathrm{T}}$ [23]. The major assumption is that process errors are relatively small and high order terms of process errors can be ignored. This line of research has made great achievements to understand the causal relationship between process errors and deviation of quality characteristics from design specification. It provides a foundation for conducting further analysis of the error equivalence.

The full utilization of the error equivalence for dimensional control requires research in the following aspects: (1) mathematical modeling of the error equivalence phenomenon in manufacturing, (2) error equivalence analysis for automatic process error compensation, and (3) error equivalence analysis for root cause identification. The three research aspects constitute the essential components of error equivalence methodology.

II. Mathematical Modeling of the Error Equivalence Phenomenon in Manufacturing

Suppose error source $\mathbf{x}_i$ leads to dimensional deviation $\mathbf{y}$ as $\mathbf{y} = \mathbf{f}_i(\mathbf{x}_i)$, i=1,2,…,p. A rigorous definition of error equivalence is given as follows.

Definition: Two error sources $\mathbf{x}_i$ and $\mathbf{x}_j$ are equivalent if expectation $E[\mathbf{f}_i(\mathbf{x}_i)]= E[\mathbf{f}_j(\mathbf{x}_j)]$. If function $\mathbf{f}_i$ could be approximated by a linear function $\mathbf{\Gamma}_i\mathbf{x}_i$ as shown in (1), the condition of error equivalence becomes $E[\mathbf{\Gamma}_i\mathbf{x}_i]= E[\mathbf{\Gamma}_j\mathbf{x}_j]$.

In practice, engineering knowledge combined with parameter estimation can be used to verify the existence of error equivalence. If error sources $\mathbf{x}_i$ and $\mathbf{x}_j$ are equivalent, it is feasible to transform $\mathbf{x}_i$ into equivalent amount of error in terms of $\mathbf{x}_j$ without affecting the analysis of feature deviation $\mathbf{y}$. As shown later in this paper, the transformation can be linear, i.e., $\mathbf{x}_i^* = \mathbf{A}_i\mathbf{x}_i$. By definition, $E[\mathbf{f}_i(\mathbf{x}_i)]= E[\mathbf{f}_j(\mathbf{x}_i^*)]$. Hereafter, we simply denote it as $\mathbf{f}_i(\mathbf{x}_i)=\mathbf{f}_j(\mathbf{x}_i^*)$ or $\mathbf{\Gamma}_i\mathbf{x}_i=\mathbf{\Gamma}_j\mathbf{x}_i^*$ when linear approximation is appropriate.

As worthy of mention, errors might not be equivalent under all situations. For instance, the surface profile deviation caused by a machine tool might not be reproduced by a fixture. This study only focuses on the situations that error equivalence holds.

Figure 2 outlines the basic idea of mathematical modeling of the error equivalence phenomenon. If p process errors $\mathbf{x}_i$'s are equivalent, the first step of modeling is to transform $\mathbf{x}_i$'s into a base type error $\mathbf{x}_1$ through $\mathbf{x}_i^* = \mathbf{A}_i\mathbf{x}_i$. A significant advantage of this transformation is that the causal relationship between base error $\mathbf{x}_1$ and feature deviation, i.e., $\mathbf{y}=\mathbf{\Gamma}_1\mathbf{x}_1$, can be generally applied to other types of error sources. The remaining modeling steps can therefore be focused on the causal model $\mathbf{y}=\mathbf{\Gamma}_1\mathbf{x}_1$ because the transformed errors $\mathbf{x}_i^*$'s are to be grouped together into $\Sigma_{i=1}^{p}\mathbf{x}_i^*$ with $\mathbf{x}_1^* = \mathbf{x}_1$. The process model in (1) can be rewritten as $\mathbf{y} = \Sigma_{i=1}^{p}\mathbf{\Gamma}_i^*\mathbf{x}_i^* + \boldsymbol{\varepsilon}$. Since $\mathbf{x}_i^*$'s are treated as base error $\mathbf{x}_1$, we have $\mathbf{\Gamma}_i^* = \mathbf{\Gamma}_1$. The process model based on error equivalence modeling thus becomes

$$\mathbf{y}=\mathbf{\Gamma}_1\mathbf{u} + \boldsymbol{\varepsilon} \text{ with } \mathbf{u}=\Sigma_{i=1}^{p}\mathbf{x}_i^* \tag{2}$$

where $\mathbf{u}$ represents total amount of equivalent error.

The machining process is used as an example to illustrate the modeling procedure. Three major error sources are considered: fixture error $\mathbf{x}_1$, machine tool error $\mathbf{x}_2$, and datum surface error $\mathbf{x}_3$. The fixture error is chosen as the base error because of the following reasons:

(1) Fixture error is simply represented by the deviation of fixture locators, while machine tool error is relatively complicated. The datum error is usually caused by fixture or machine tool errors.

(2) Fixture error has been well studied. Methods are readily available for the analysis of workpiece positioning error [5-12], the induced feature deviation [19-26], and fixture error diagnosis [27-29].

(3) Flexible fixtures have been available whose locators are adjustable for accommodating a product family. It is possible to adjust the locator lengths for the purpose of error compensation.

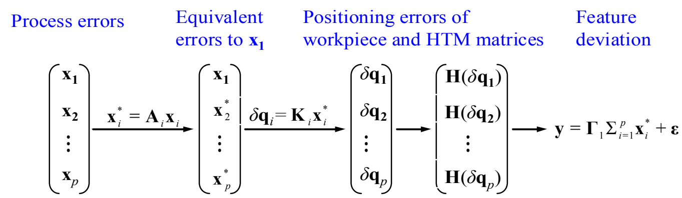

Figure 2. Mathematical modeling of error equivalence

Wang, Huang, and Katz (2005) [1] were the first to propose "Equivalence Fixture Error" (EFE) model and transform machine tool error $\mathbf{x}_2$ and datum error $\mathbf{x}_3$ into the equivalent amount of fixture locator error, $\mathbf{x}_2^*=\mathbf{A}_2\mathbf{x}_2$, and $\mathbf{x}_3^*= \mathbf{A}_3\mathbf{x}_3$. The machine tool error $\mathbf{x}_2$ was modeled as displacement error (x_m y_m z_m) and rotational error captured by Euler parameters i.e., $\mathbf{x}_2 =(x_m\ y_m\ z_m\ \delta e_{1m}\ \delta e_{2m}\ \delta e_{3m})^T$. The datum error $\mathbf{x}_3$ was modeled as the deviation of datum surfaces using $\left(\Delta\mathbf{n}^{\mathrm{T}}\quad \Delta\mathbf{p}^{\mathrm{T}}\quad \Delta\mathbf{D}^{\mathrm{T}}\right)^{\mathrm{T}}$. Matrices $\mathbf{A}_i$'s can be obtained using analytical geometry. For example, Fig. 3 show that the EFE for machine tool error or $\mathbf{x}_2^*$ equals $(\Delta m_1\ \Delta m_2\ \Delta m_3)^{\mathrm{T}}$, which can be found by exploring the geometrical relationship. For 3-2-1 fixture locating scheme, reference [1] has derived $\mathbf{A}_2$ and $\mathbf{A}_3$.

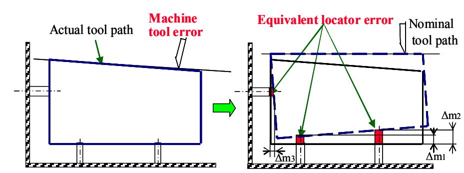

Figure 3. EFE for machine tool error [1]

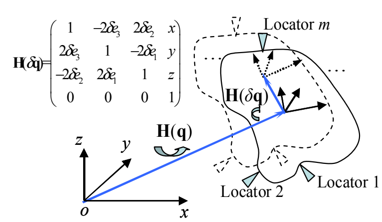

Figure 4. Modeling of workpiece positioning error [8]

Following the transformation $\mathbf{x}_i^* = \mathbf{A}_i\mathbf{x}_i$ is modeling of $\mathbf{y}=\mathbf{\Gamma}_1\mathbf{x}_1$, i.e., the effect of fixture error x1 on feature deviation y through improper positioning of workpiece. Workpiece positioning error has been investigated using kinematic analysis, especially the Homogeneous Transformation Matrix (HTM) method. The workpiece fixturing can be described using a HTM. The fixture locator deviation $\mathbf{x}_1$ leads to workpiece positioning error modeled by HTM $H(\delta\mathbf{q}_1)$ (Fig. 4), where parameter $\delta\mathbf{q}_1$ in $H(\delta\mathbf{q}_1)$ is determined by $\delta\mathbf{q}_1=\mathbf{K}_1\mathbf{x}_1$ [8].

For example, when the orientation vectors of datum surfaces are $(0\ 0\ \text{-}1)^{\mathrm{T}}$, $(0\ \text{-}1\ 0)^{\mathrm{T}}$, and $(\text{-}1\ 0\ 0)^{\mathrm{T}}$ in the FCS, $\mathbf{K}_1$ is [1]

$$\mathbf{K}_1=\begin{pmatrix}
\frac{f_{3x}f_{6y}(f_{4z}-f_{5z})+f_{2x}f_{6y}(-f_{4z}+f_{5z})+(f_{4x}-f_{5x})(f_{2y}-f_{3y})f_{6z}}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{f_{3x}f_{6y}(f_{4z}-f_{5z})+f_{1x}f_{6y}(-f_{4z}+f_{5z})+(f_{4x}-f_{5x})(f_{1y}-f_{3y})f_{6z}}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{f_{2x}f_{6y}(f_{4z}-f_{5z})+f_{1x}f_{6y}(-f_{4z}+f_{5z})+(f_{4x}-f_{5x})(f_{1y}-f_{2y})f_{6z}}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{f_{6y}}{f_{4x}-f_{5x}} & \frac{f_{6y}}{-f_{4x}+f_{5x}} & 1 \\
\frac{(f_{2x}-f_{3x})(-f_{5x}f_{4z}+f_{4x}f_{5z})}{(f_{4x}-f_{5x})(f_{3x}(-f_{y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{2x}(-f_{2y}+f_{3y}))} & \frac{(f_{x}-f_{3x})(f_{5x}f_{4z}-f_{4x}f_{5z})}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{(f_{1x}-f_{2x})(f_{5x}f_{4z}-f_{4x}f_{5z})}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{f_{5x}}{-f_{4x}+f_{5x}} & \frac{f_{4x}}{f_{4x}-f_{5x}} & 0 \\
\frac{-f_{3x}f_{2y}+f_{2x}f_{3y}}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & \frac{f_{3x}f_{1y}-f_{1x}f_{3y}}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & \frac{-f_{2x}f_{1y}+f_{1x}f_{2y}}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & 0 & 0 & 0 \\
\frac{f_{2x}-f_{3x}}{2(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{-f_{1x}+f_{3x}}{2(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{f_{1x}-f_{2x}}{2(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & 0 & 0 & 0 \\
\frac{f_{2y}-f_{3y}}{2(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{-f_{1y}+f_{3y}}{2(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{f_{1y}-f_{2y}}{2(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & 0 & 0 & 0 \\
-\frac{(f_{2x}-f_{3x})(f_{4z}-f_{5z})}{2(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{(f_{1x}-f_{3x})(f_{4z}-f_{5z})}{2(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{(f_{1x}-f_{2x})(f_{4z}-f_{5z})}{2(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{1}{-2f_{4x}+2f_{5x}} & \frac{1}{2f_{4x}-2f_{5x}} & 0
\end{pmatrix} \quad (3)$$

With only fixture error, the resulting feature deviation y after an ideal cutting operation on surface $(\mathbf{n}^{\mathrm{T}}, \mathbf{p}^{\mathrm{T}})^{\mathrm{T}}$ is $\mathbf{y}=\mathbf{\Gamma}_1\mathbf{x}_1 + \boldsymbol{\varepsilon}$, where $\mathbf{\Gamma}_1$ is, e.g.,

$$\mathbf{\Gamma}_1=\begin{pmatrix}
\frac{f_{3x}(f_{4z}-f_{5z})n_y+f_{2x}(-f_{4z}+f_{5z})n_y+(f_{4x}-f_{5x})(f_{2y}-f_{3y})n_z}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{x}(-f_{2y}+f_{3y}))} & \frac{f_{3x}(f_{4z}-f_{5z})n_y+f_{1x}(-f_{4z}+f_{5z})n_y+(f_{4x}-f_{5x})(f_{1y}-f_{3y})n_z}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{f_{2x}(f_{4z}-f_{5z})n_y+f_{1x}(-f_{4z}+f_{5z})n_y+(f_{4x}-f_{5x})(f_{1y}-f_{2y})n_z}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{n_y}{-f_{4x}+f_{5x}} & \frac{n_y}{f_{4x}-f_{5x}} & 0 \\
\frac{(f_{2x}-f_{3x})(f_{4z}n_x-f_{5z}n_x+(-f_{4x}+f_{5x})n_z)}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{x}(-f_{2y}+f_{3y}))} & \frac{(f_{x}-f_{3x})(f_{4z}n_x-f_{5z}n_x+(-f_{4x}+f_{5x})n_z)}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{(f_{x}-f_{2x})(f_{4z}n_x-f_{5z}n_x+(-f_{4x}+f_{5x})n_z)}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{n_x}{f_{4x}-f_{5x}} & \frac{n_x}{-f_{4x}+f_{5x}} & 0 \\
\frac{f_{2y}n_x-f_{3y}n_x+(-f_{2x}+f_{3x})n_y}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & \frac{-f_{1y}n_x+f_{3y}n_x+(f_{1x}-f_{3x})n_y}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & \frac{f_{1y}n_x-f_{2y}n_x+(-f_{1x}+f_{2x})n_y}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & 0 & 0 & 0 \\
\frac{-f_{2x}(f_{4z}-f_{5z})(f_{6y}+p_y)+f_{3x}(f_{4z}-f_{5z})(f_{6y}+p_y)+(f_{4x}-f_{5x})(f_{2y}-f_{3y})(f_{6z}+p_z)}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{-f_{1x}(f_{4z}-f_{5z})(f_{6y}+p_y)+f_{3x}(f_{4z}-f_{5z})(f_{6y}+p_y)+(f_{4x}-f_{5x})(f_{1y}-f_{3y})(f_{6z}+p_z)}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{-f_{1x}(f_{4z}-f_{5z})(f_{6y}+p_y)+f_{2x}(f_{4z}-f_{5z})(f_{6y}+p_y)+(f_{4x}-f_{5x})(f_{1y}-f_{2y})(f_{6z}+p_z)}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{f_{6y}+p_y}{-f_{4x}+f_{5x}} & \frac{f_{6y}+p_y}{f_{4x}-f_{5x}} & -1 \\
\frac{(f_{2x}-f_{3x})((f_{4z}-f_{5z})p_x+f_{5x}(f_{4z}+p_z)-f_{4x}(f_{5z}+p_z))}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{(f_{1x}-f_{3x})((f_{4z}-f_{5z})p_x+f_{5x}(f_{4z}+p_z)-f_{4x}(f_{5z}+p_z))}{(f_{4x}-f_{5x})(f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y}))} & \frac{(f_{1x}-f_{2x})((f_{4z}-f_{5z})p_x+f_{5x}(f_{4z}+p_z)-f_{4x}(f_{5z}+p_z))}{(f_{4x}-f_{5x})(f_{3x}(-f_{1y}+f_{2y})+f_{2x}(f_{1y}-f_{3y})+f_{1x}(-f_{2y}+f_{3y}))} & \frac{f_{5x}+p_x}{f_{4x}-f_{5x}} & \frac{f_{4x}+p_x}{-f_{4x}+f_{5x}} & 0 \\
\frac{(f_{2y}-f_{3y})p_x+f_{3x}(f_{2y}+p_y)-f_{2x}(f_{3y}+p_y)}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & \frac{(-f_{1y}+f_{3y})p_x-f_{3x}(f_{1y}+p_y)+f_{1x}(f_{3y}+p_y)}{f_{3x}(f_{1y}-f_{2y})+f_{x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & \frac{(f_{y}-f_{2y})p_x+f_{2x}(f_{1y}+p_y)-f_{x}(f_{2y}+p_y)}{f_{3x}(f_{1y}-f_{2y})+f_{1x}(f_{2y}-f_{3y})+f_{2x}(-f_{1y}+f_{3y})} & 0 & 0 & 0
\end{pmatrix} \quad (4)$$

If $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ co-exist in the machining process, $\mathbf{y}=\mathbf{\Gamma}_1(\mathbf{x}_1+\mathbf{x}_2^*+\mathbf{x}_3^*)+\boldsymbol{\varepsilon}$.

Remark 1: Since $\Sigma_{i=1}^{p}\mathbf{x}_i^* \le \Sigma_{i=1}^{p}|\mathbf{x}_i^*|$, model $\mathbf{y}=\mathbf{\Gamma}_1\Sigma_{i=1}^{p}\mathbf{x}_i^* + \boldsymbol{\varepsilon}$ therefore indicates the possibility that multiple process errors can cancel out one another and thereby reduce process variation. Error equivalence modeling thus provides a basis for error compensation.

Remark 2: The process errors can be either static or dynamic. The error equivalence modeling still applied to dynamic error sources [4]. The difference is that time index t will be introduced to the model, i.e., $\mathbf{x}_i^*(t)=\mathbf{A}_i\mathbf{x}_i(t)$ with transformation $\mathbf{A}_i$ remaining the same as before.

III. ERROR EQUIVALENCE ANALYSIS FOR ROOT CAUSE IDENTIFICATION

The ultimate goal of root cause identification is to estimate all process errors $\mathbf{x}_i$'s, i=1,…,p. The most commonly used approach is parameter estimation based on (1) [27-30]. With error equivalence phenomenon in manufacturing processes, the diagnosis issues include (1) diagnosability analysis, (2) identification of error occurrence, (3) decision-making on measuring certain process errors, and (4) error decomposition and individual error identification. Figure 5 shows the causal model based root cause identification through (1) least square estimation ($\hat{\mathbf{u}}^{(n)}$ for the nth sample) and (2) statistical test on the effect of six equivalent fixture errors using an F test statistic (F_i, i=1,2,…6) that in fact indicates signal (process error) to noise ratio [29] .

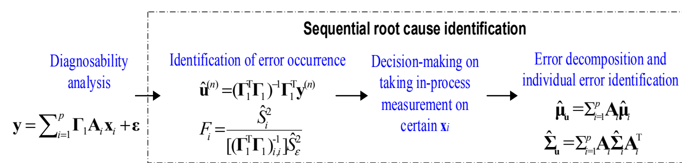

Figure 5. Error equivalence analysis for root cause identification

Diagnosability analysis of a manufacturing process with error equivalence phenomenon: From parameter estimation point of view, a manufacturing process is diagnosable if all the process errors are estimable. We can prove an intuitive belief that a manufacturing process with error equivalence phenomenon is not diagnosable with measurement on $\mathbf{y}$. In fact, (2) can be rewritten as $\mathbf{y}=[\mathbf{\Gamma}_1 \mid \mathbf{\Gamma}_1\mathbf{K}_2 \mid \cdots \mid \mathbf{\Gamma}_1\mathbf{K}_p][\mathbf{x}_1^{\mathrm{T}} \mid \mathbf{x}_2^{\mathrm{T}} \mid \cdots \mid \mathbf{x}_p^{\mathrm{T}}]^{\mathrm{T}}+\boldsymbol{\varepsilon}$. It is clearly that columns in $[\mathbf{\Gamma}_1 \mid \mathbf{\Gamma}_1\mathbf{K}_2 \mid \cdots \mid \mathbf{\Gamma}_1\mathbf{K}_p]$ are dependent because columns of $\mathbf{\Gamma}_1\mathbf{K}_i$ are the linear combination of columns in $\mathbf{\Gamma}_1$. Therefore, the least square estimate of $\mathbf{x}_i$ does not exist and the manufacturing process is not diagnosable.

Sequential root cause identification: The diagnosability conclusion indicates that measurement other than quality characteristics $\mathbf{y}$ is necessary to distinguish error sources. However, it is not economical to take the additional measurement if no process error occurs. Therefore, a sequential procedure becomes more desirable, that is, first identify existence of errors based on $\mathbf{y}$, and then discriminate error sources using additional measurement only if process error is detected (Fig. 5).

Error equivalence model in (2) makes it feasible to detect error occurrence only using data $\mathbf{y}$. The LSE of total amount of equivalent error is $\hat{\mathbf{u}}=(\mathbf{\Gamma}_1^{\mathrm{T}}\mathbf{\Gamma}_1)^{-1}\mathbf{\Gamma}_1^{\mathrm{T}}\mathbf{y}$ with matrix $\mathbf{\Gamma}_1$ full rank.

The physical reason of full rank is that each column of $\mathbf{\Gamma}_1$ corresponds to an independent error component of base error $\mathbf{x}_1$, e.g., one of the six fixture locators. The method proposed by Apley and Shi (1998) [29] can be applied to test the mean shift and variance change in the total amount of equivalent error $\mathbf{u}$.

Once process change is detected, decision has to be made on how to measure process errors inline at lower cost. Theoretically (p-1) out of p process errors $\mathbf{x}_i$'s have to be checked for complete diagnosis. The error not to be measured should be the one that costs the most. For example, machine tool error $\mathbf{x}_2$ is relatively costly to be measured. Once the total amount of equivalent error $\mathbf{u}$ is estimated, we can measure fixture locator error $\mathbf{x}_1$ and datum error $\mathbf{x}_3$ (datum error is usually part of quality characteristics $\mathbf{y}$). The machine tool error $\mathbf{x}_2$ can be estimated through decomposition approach (to be discussed next). Furthermore, we only need to measure the error components that are significant. For instance, if component j of $\mathbf{u}$ is significant in the machining process, the occurrence of fixture error can be determined by only measuring locator j of that fixture. This sequential procedure is expected to dramatically reduce the measurement cost.

To decompose $\hat{\mathbf{u}}$ with measurement of (p-1) process errors, we first assume error sources $\mathbf{x}_i$'s are independent from one another with mean $\boldsymbol{\mu}_i$'s and variance-covariance matrices $\boldsymbol{\Sigma}_i$'s. Since $\mathbf{x}_i^* = \mathbf{A}_i\mathbf{x}_i$ and $\mathbf{u}=\Sigma_{i=1}^{p}\mathbf{x}_i^*$, we have

$$\hat{\boldsymbol{\mu}}_{\mathbf{u}} = \Sigma_{i=1}^{p}\mathbf{A}_i\hat{\boldsymbol{\mu}}_i \text{ and } \hat{\boldsymbol{\Sigma}}_{\mathbf{u}} = \Sigma_{i=1}^{p}\mathbf{A}_i\hat{\boldsymbol{\Sigma}}_i\mathbf{A}_i^T \text{ with } \mathbf{A}_1=\mathbf{I} \tag{5}$$

With measurement of N workpiece, the sample of $\hat{\mathbf{u}}$ will be $\hat{\mathbf{u}}^{(n)} = (\mathbf{\Gamma}_1^T\mathbf{\Gamma}_1)^{-1}\mathbf{\Gamma}_1^T\mathbf{y}^{(n)}$, n=1, 2, …, N. $\hat{\boldsymbol{\mu}}_{\mathbf{u}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}$ are obtained by computing the sample mean and covariance of $\hat{\mathbf{u}}$. $\hat{\boldsymbol{\mu}}_i$ and $\hat{\boldsymbol{\Sigma}}_i$ are estimated with measurement of (p-1) process errors $\mathbf{x}_i$'s. Using (5), the remaining un-measured process error can be obtained through decomposing $\hat{\boldsymbol{\mu}}_{\mathbf{u}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}$. Under normality assumption, confidence intervals can be established to diagnose individual process errors.

IV. Error Equivalence Analysis for Automatic Process Error Compensation - A System Approach

For a manufacturing process with input-output relationship $\mathbf{y}=\mathbf{f}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_p)+\boldsymbol{\varepsilon}$, the traditional error compensation strategy is to minimize individual process errors $\mathbf{x}_i$'s so as to reduce output deviation $\mathbf{y}$ [17]. Since error equivalence also implies the cancellation among process errors, this allows us to develop a new compensation strategy, i.e., treating all process error sources as a system and using one error to compensate for the others. For instance, with the development of flexible fixture whose locator length is adjustable through a control system, it is feasible to compensate for the overall process errors in the machining process by changing locator length. In this new strategy, the outputs of the controller and process will be monitored using methods developed in Statistical Process Control (SPC) (Fig. 6). The main purpose is to monitor unexpected events such as controller failure.

Applicable conditions: The first open issue is, however, under what condition this "error canceling error" strategy will be effective. Let $\mathbf{G}_{\mathbf{x}1}$ be the controller adjusting base error source $\mathbf{x}_1$ (e.g., fixture locators). The base error $\mathbf{x}_1$ is not random anymore because of the controlled adjustment. Denote by $\mathbf{z}$ the controller output or the amount of adjustment applied to base error source. Although the adjustment $\mathbf{z}$ is expected to compensate for the remaining process errors $\Sigma_{i=2}^{p}\mathbf{x}_i^*$, it becomes a new random "error source" because of the variability in the actuator. Therefore, the adjusted total process error $\mathbf{u_a}$ has

$$\hat{\boldsymbol{\mu}}_{\mathbf{u}_a} = \boldsymbol{\mu}_{\mathbf{z}} + \Sigma_{i=2}^{p}\mathbf{A}_i\hat{\boldsymbol{\mu}}_i \text{ and } \hat{\boldsymbol{\Sigma}}_{\mathbf{u}_a} = \hat{\boldsymbol{\Sigma}}_{\mathbf{z}} + \Sigma_{i=2}^{p}\mathbf{A}_i\hat{\boldsymbol{\Sigma}}_i\mathbf{A}_i^T \tag{6}$$

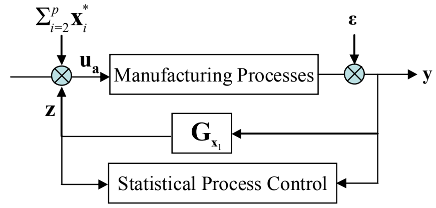

Figure 6. A new process error compensation strategy using error equivalence phenomenon

The controller $\mathbf{G}_{\mathbf{x}1}$ normally aims to keep the process output $\mathbf{y}$ on the target and with minimum variation. The commonly used control algorithm is to let $\boldsymbol{\mu}_{\mathbf{z}} = -\Sigma_{i=2}^{p}\mathbf{A}_i\hat{\boldsymbol{\mu}}_i$ or $\hat{\boldsymbol{\mu}}_{\mathbf{u}_a} = 0$ [31-33]. However, the generalized variance of error $\mathbf{u_a}$ or $|\hat{\boldsymbol{\Sigma}}_{\mathbf{u}_a}|$ is not necessary to be smaller than the one without adjustment. Cleary, if $|\hat{\boldsymbol{\Sigma}}_{\mathbf{z}}| \leq |\hat{\boldsymbol{\Sigma}}_1|$, the new compensation strategy will uniformly reduce process variation. If $|\hat{\boldsymbol{\Sigma}}_{\mathbf{z}}| > |\hat{\boldsymbol{\Sigma}}_1|$ but the increase of total process variation $(|\hat{\boldsymbol{\Sigma}}_{\mathbf{u}_a}|-|\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}|)/|\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}|$ is insignificant, the compensation might be acceptable as well. For instance, the precision of fixture is usually much higher than the workpiece and machine tool. A fixture equipped with a controller could have lower precision or larger $|\hat{\boldsymbol{\Sigma}}_{\mathbf{z}}|$. The minor percentage of fixture variation in the tool process errors might justify the application of error compensation because it brings the process on the target. Compensation is normally not effective if $|\hat{\boldsymbol{\Sigma}}_{\mathbf{z}}| > |\hat{\boldsymbol{\Sigma}}_1|$ and $(|\hat{\boldsymbol{\Sigma}}_{\mathbf{u}_a}|-|\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}|)/|\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}|$ is appreciable.

The conventional compensation strategy aims to offset $\hat{\boldsymbol{\mu}}_i$ and reduce $\hat{\boldsymbol{\Sigma}}_i$ individually. It will be effective if there is only a limited few process errors dominating in $\hat{\boldsymbol{\mu}}_{\mathbf{u}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}$. Otherwise, it has to develop controllers for all error sources in order to keep the process output $\mathbf{y}$ on the target. In that case, the two compensation strategies can be applied complementarily. The error sources with the largest variations can be counteracted using conventional methods to reduce $\hat{\boldsymbol{\Sigma}}_{\mathbf{u}}$, while the new compensation strategy is to achieve $\hat{\boldsymbol{\mu}}_{\mathbf{u}_a} = 0$.

Controller design and performance evaluation: Using the observed feature deviation $\mathbf{y}^{(n)}$ at time period n as input, controller $\mathbf{G}_{\mathbf{x}1}$ generates adjustment $\mathbf{z}^{(n)}$ to counteract

$\Sigma_{i=1}^{p}\mathbf{x}_i^{*(n+1)}$ for the (n+1)th time period. Let $\mathbf{Z}^{(n)}$ be the cumulative amount of adjustment, i.e., $\mathbf{Z}^{(n)} = \mathbf{z}^{(0)} + \cdots + \mathbf{z}^{(n)}$. $\mathbf{y}^{(n+1)}$ is given as

$$\mathbf{y}^{(n+1)} = \sum_{l=0}^{p_0}\mathbf{\Gamma}_1^{(l)}\mathbf{Z}^{(n-l)} + \mathbf{\Gamma}_1\Sigma_{i=1}^{p}\mathbf{x}_i^{*(n+1)} + \boldsymbol{\varepsilon}^{(n+1)} \text{ and } \mathbf{\Gamma}_1^{(0)} = \mathbf{\Gamma}_1 \quad (7)$$

where p_0 matrices $\{\mathbf{\Gamma}_1^{(l)}\}_{l=0}^{p_0}$ depict how the adjustment $\{\mathbf{Z}^{(n-l)}\}_{l=0}^{p_0}$ affect $\mathbf{y}^{(n+1)}$. The goal of controller $\mathbf{G}_{\mathbf{x}1}$ design is to cancel $\boldsymbol{\mu}_{\mathbf{y}}$ and minimize the process variation. A commonly adopted approach is Minimum-Mean-Square-Error (MMSE) control, i.e., to choose controller parameters so as to minimize the mean square error of the controlled process output $\boldsymbol{\mu}_{\mathbf{y}^2}$. Since the process errors include the quasi-static components which are relatively constant within each period whereas are time-varying between time periods, it is important to establish a dynamic model for these slow-varying errors. Denote $\mathbf{P}_d$ and $\mathbf{P}_s$ as the sets of subscripts for all quasi-static errors and static errors, respectively. Assume that quasi-static errors $\{\mathbf{x}_i^{*(n)}\}_{i\in\mathbf{P}_d}$ are modeled by

$$\mathbf{x}_i^{*(n)} = -\sum_{l=1}^{p_1}\mathbf{W}_i^{(l)}\mathbf{x}_i^{*(n-l)} + \sum_{l=1}^{p_2}\mathbf{V}_i^{(l)}\mathbf{t}_i^{(n-l)} + \mathbf{D}_i^{(n)} + \mathbf{e}_i^{(n)} \qquad (8)$$

where $\mathbf{D}_i^{(n)}$ is the intercept term, $\mathbf{t}_i$ includes the measured variables that impact $\mathbf{x}_i^{*(n)}$ (e.g., temperatures of machine tool), $\mathbf{e}_i^{(n)}$ is the noise term, p_1 and p_2 represent the maximum time lags, $\mathbf{W}_i^{(l)}$ and $\mathbf{V}_i^{(l)}$ are the coefficient matrices. Following the similar derivation in [4] and [34], the MMSE adjustment for both static and quasi-static errors can be

$$\mathbf{Z}^{(n)} = -\hat{\mathbf{u}}^{(n+1)} = \sum_{i\in\mathbf{P}_d}\sum_{l=0}^{p_0}\mathbf{W}_i^{(l)}\mathbf{x}_i^{*(n-l)} - \sum_{i\in\mathbf{P}_d}\sum_{l=0}^{p_2}\mathbf{V}_i^{(l)}\mathbf{t}_i^{(n-l)} - \mathbf{Z}_0^{(n)} - \sum_{j\in\mathbf{P}_s}\mathbf{x}_j^{*}, \quad (9)$$

$$\mathbf{Z}^{(n_0)} = 0, \text{ and}$$

$$\hat{\mathbf{x}}_i^{*(n-l)} = -\mathbf{Z}^{(n-l-1)} + (\mathbf{\Gamma}_1^{\mathrm{T}}\mathbf{\Gamma}_1)^{-1}\mathbf{\Gamma}_1^{\mathrm{T}}[\mathbf{y}^{(n-l)} - \sum_{l=1}^{p_0}\mathbf{\Gamma}_1^{(l)}\mathbf{Z}^{(n-l)}] - \sum_{j\in\mathbf{P}_s}\mathbf{x}_j^{*},$$

$$\sum_{j\in\mathbf{P}_s}\mathbf{x}_j^{*} = (\mathbf{\Gamma}_1^{\mathrm{T}}\mathbf{\Gamma}_1)^{-1}\mathbf{\Gamma}_1^{\mathrm{T}}\mathbf{y}^{(n_0)} - \mathbf{x}_i^{*(n_0)}, \quad n \geq n_0,$$

where n_0 is the starting period to apply adjustment, and $\mathbf{Z}_0^{(n)}$ is the intercept term. In $\mathbf{Z}^{(n)}$, the first three terms compensate for the quasi-static components while the last term $-\sum_{j\in\mathbf{P}_s}\mathbf{x}_j^{*}$ is to cancel the static errors. Equation (9) predicts the total equivalent errors at time n+1 based on the historical information on process equivalent errors and adjustment. It should be noted that the constraints on the adjustable variables have to be considered. For example, a stopping criterion to a small adjustment must be incorporated into the control rule since tiny adjustment exceeding the device accuracy limits can only increase the process fluctuation [4].

Since MMSE control may have unstable modes, it is necessary to estimate the controller performance such as stability and sensitivity. The stability of a controller means that an error in the output can be cancelled by an adjustment sequence that converges to zero. From control theory [35], one can obtain the controller stability by inspecting the poles of the transfer function of (9). Sensitivity refers to how the quality could be affected whenever moderate changes occur in the controller parameters. This can be analyzed by differentiating (9) with respect to $\mathbf{W}_i^{(l)}$ and $\mathbf{V}_i^{(l)}$.

Integration of Statistical Process Control with error compensation: On some occasions, unexpected process errors (e.g., variation of adjustable fixture locator, hot chips during machining) have not been considered in $\{\mathbf{x}_i\}$ and thus the controlled process could show a large variation. Furthermore, the cost of frequent process adjustments might be substantial. Integration of SPC and APC [36-39] is an economic way to reduce the variation of controlled process though it has been rarely applied in a discrete part manufacturing process. The samples of outputs $\{\mathbf{y}^{(n)}\}$ of the manufacturing process can be monitored by control charts. Within the device constraints, the incremental adjustment $\mathbf{z}^{(n)}$ should be applied (due to the quasi-static errors) only when $\mathbf{y}^{(n)}$ exceed certain range, which , together with device constraints, defines a dead band for the adjustment. Monitoring the noise, i.e., $\hat{\boldsymbol{\varepsilon}}^{(n)} = \mathbf{y}^{(n)} - \Sigma_{i=1}^{p}\mathbf{x}_i^{*(n)} - \sum_{l=1}^{p_0}\mathbf{\Gamma}_1^{(l)}\mathbf{Z}^{(n-l)}$ can help to detect if unexpected errors impact the process output. When the unexpected errors take place, we can update the process error model to track the latest information about errors and make a closer prediction. With the updating scheme, the fitted coefficient matrices $\{\mathbf{Z}_0^{(n)}\}$, $\{\mathbf{W}_i^{(l)}\}_{l=1}^{p_1}$ and $\{\mathbf{V}_i^{(l)}\}_{l=0}^{p_2}$ in (9) also change with period n. So, it is reasonable to denote them as $\{\mathbf{Z}_{0,n}^{(n)}\}$, $\{\mathbf{W}_{i,n}^{(l)}\}_{l=1}^{p_1}$ and $\{\mathbf{V}_{i,n}^{(l)}\}_{l=0}^{p_2}$. We have demonstrated [4] that updating scheme can quickly reduce the variation caused by the controller parameter fluctuation in a milling process.

V. CONCLUSIONS

Error equivalence is a fundamental engineering phenomenon concerning the mechanism that different error sources result in identical dimensional variation patterns. This study aims to establish error equivalence methodology and obtain insights into this fundamental phenomenon for improved dimensional variation control. The methodology in this paper addressed the following four issues related to error equivalence.

- Mathematical modeling of the error equivalence phenomenon: Definition of error equivalence was introduced by measuring the impact of two error sources on feature deviations. A modeling approach was developed to transform error sources into a base type error and group errors into a total amount of equivalent error.
- Root cause identification: The error equivalence model clearly shows that a process is not diagnosable with measurement on product feature alone. A sequential root cause identification procedure was thus proposed to identify existence of errors and then discriminate error sources.
- Automatic process error compensation: The error equivalence also implies a new compensation strategy, i.e., treating all process error sources as a system and using the base error source to compensate for the others.

This study serves as an initial attempt to systematically analyze error equivalence phenomenon. Applications could be extended to processes other than machining.

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