Optimized Joint-Torques Trajectory Planning for Bipedal Walking Robots

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***Abstract*—This paper proposes a new method of trajectory planning for biped robots walking on flat terrain. In this approach, the hip and foot trajectories are designed in Cartesian space using polynomial interpolation. The key parameters which define the hip and foot trajectories are searched by genetic algorithm. The objective is to obtain stable walking trajectory with minimized joint-torques requirement. ZMP stability criterion is used to ensure physically realizable walking motion. The effectiveness of our method is verified by simulation of a humanoid robot named NUSBIP-II.**

I. Introduction

In recent years, walking gait synthesis has become one of the most interesting and challenging research areas in robotics. Many on-going research projects have been conducting around the world to tackle this problem [1] - [15].

Stability is a critical issue in bipedal walking. The zero-moment-point (ZMP) [16] has been widely used as a criterion to ensure stability of bipedal walking in many studies. Takanishi *et al.* [6], Hirai *et al.* [7], Kajita *et al.* [1], Shih *et al.* [11], and Erbatur *et al.* [8] have used the ZMP as the stability index to design the walking pattern for bipedal robots. Basically, in these methods, the desired ZMP trajectory is first prescribed and then the hip or trunk motion is derived to obtain the ZMP trajectory. However, it is impossible to determine which ZMP trajectory is good to generate a smooth and energy efficient motion. In addition, the hip acceleration may need to be very large to achieve a desired ZMP trajectory and this will lead to the increase of applied joint torques which is not desirable. To solve this problem, some researchers have proposed methods of gait synthesis without first prescribing the desired ZMP trajectory. Huang *et al.* [10] proposed a method in which the foot and hip trajectories are planned beforehand in Cartesian space using cubic spline interpolation. The hip trajectory is affected by two parameters, these parameters are determined by iterative computation to obtain largest stability margin for the biped. The advantage of this method is that the motion of the hip or trunk is very smooth which make the control of the upper limbs easier. However, since the characteristic of the hip trajectory is only determined by two parameters and the range of these parameters are quite limited therefore the diversity of the hip trajectory is limited. Hence, the optimal trajectory obtained may not be the best trajectory.

In this study, we propose a method of trajectory generation in which the foot and hip trajectories are planned in the Cartesian space using polynomial interpolation. The characteristics of the trajectories are governed by the 7 key parameters whose optimal values are searched by genetic algorithm (GA). The objective is to find an optimal trajectory resulting in stable walking behavior with minimized torque at the joints. In this method, ZMP is used as a stability criterion to check whether the trajectory is physically realizable. However, no prescribed ZMP trajectory is needed. The good thing about this approach is that by using the 7 key parameters a big set of trajectories are generated which gives a good space to finding the optimal trajectory.

This paper is organized as follows. Section II describes the life-size humanoid robot NUSBIP-II whose specifications are used in the simulation. Section III presents the generation of foot and hip trajectories and the choice of key parameters. Genetic algorithm implementation is described in Section IV. Section V shows the simulation results and conclusion is made in Section VI.

II. THE BIPED ROBOT

The robot considered in this study is a 7-link biped robot named NUSBIP-II, which was built and developed in our lab (see Fig. 1). The total weight is about 22.2 kg. The biped has 12 DOFs. Each leg has six active DOFs of which three DOFs are at the hip (pitch, yaw, roll), one at the knee (pitch) and two at the ankle joint (pitch, roll). Each DOF is driven by a DC motor and integrated with an angular position sensor to measure the relative angle between two consecutive links. Each of the feet is equipped with four force sensors (two at the toe and two at the heel) to sense the contact forces between the feet and the ground.

The model of the robot is depicted in Fig. 2. The mass of link ith is m_i and the moment of inertia around its COG is I_i.

III. TRAJECTORY GENERATION

A. *Foot Trajectory*

The walking parameters that define the foot trajectory is shown in Fig. 3 where S is the half walking step length, S_p is the horizontal distance from the start of the step (foot lift-off) to the place where the swing-foot at its highest position,

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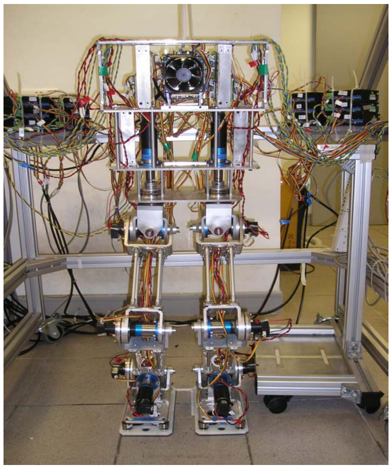

Fig. 1. Picture of NUSBIP-II.

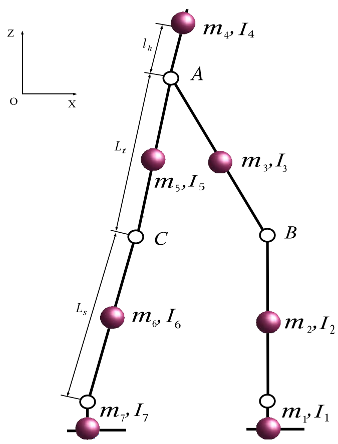

Fig. 2. Model of the robot.

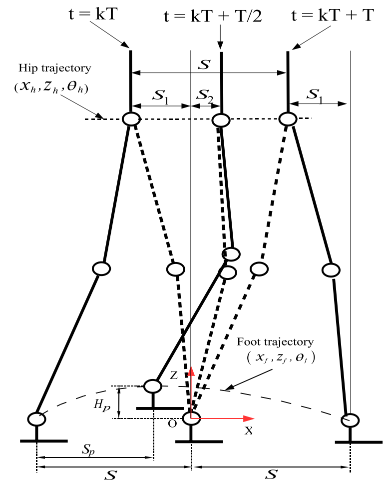

Fig. 3. Parameters defining the walking trajectory of the robot. The stance-leg and swing-leg are represented by dashed lines and solid lines, respectively.

H_p is the highest position of the foot in vertical axis. In this work, the foot is constrained to move such that it is always parallel to the ground. Let T be the period of one walking step, the interval of the k^{th} step (k = 1, 2, ...) is from kT to $(k+1)T$. The corresponding time instant when the foot is at its highest position is $kT + t_p$. Since the walking motion is repeated periodically, we only need to plan the motion in one period. The horizontal motion of the foot trajectory must satisfy the position constraints as in (1), with the coordinate system placed at the stance ankle:

$$x_f(t) = \begin{cases} -S, & t = kT \\ -S + S_p, & t = kT + t_p \\ S, & t = (k+1)T \end{cases} \tag{1}$$

Since the feet fully contact with the ground at $t = kT$ and $t = kT + T$, the following velocity constraints must be included:

$$\begin{cases} \dot{x}_f(kT) = 0 \\ \dot{x}_f(kT + T) = 0 \end{cases} \tag{2}$$

Totally, the horizontal motion of the swing foot must satisfy 5 constraints (position and velocity constraints), a fourth-order polynomial is enough to describe the motion:

$$x_f(t) = a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \tag{3}$$

Similarly, the position and velocity constraints of the vertical motion of the swing foot are defined in (4) and (5), respectively.

$$z_f(t) = \begin{cases} 0, & t = kT \\ H_p, & t = kT + t_p \\ 0, & t = (k+1)T \end{cases} \tag{4}$$

$$\begin{cases} \dot{z}_f(kT) = 0 \\ \dot{z}_f(kT + T) = 0 \end{cases} \tag{5}$$

Again, a fourth-order polynomial is used to describe the vertical motion:

$$z_f(t) = b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 \tag{6}$$

In this study, the walking step length S and the step period T are given. H_p and S_p are important parameters that have significant influence on the swing leg dynamics. Therefore, we leave H_p, S_p as free variables and they will be searched by GA to find the optimal value.

B. Hip Trajectory

Hip motion is very critical to stability of the biped because a little change in hip motion may cause the whole dynamics of the biped to change dramatically and thus significantly affect the stability of the biped. Therefore, in this work the hip trajectory will be carefully treated such that stable walking could be achieved. Since the vertical motion of the hip is quite limited, it doesn't affect much on the stability of the biped. For simplification sake , the hip height is constrained to be constant and the pitch angle of the trunk is kept upright (pitch angle

$\theta_h = 0$). The horizontal motion $x_h(t)$ of the hip is the main factor that affects the stability of the biped in sagittal plane. In this method, instead of using the cubic spline interpolation as in [10] we use the high-order polynomial interpolation to generate the smooth hip trajectory. The order of the polynomial is chosen to be larger than the number of constraints such that we can have more variations for the hip trajectory. The higher the order of the polynomial is, the more diverse the set of hip trajectories could be obtained. Let's call the difference between the order of the polynomial and the number of constraints the number of redundant-coefficients. Diverse hip trajectories can be achieved by using different set of redundant-coefficients.

As stated earlier, the reference coordinate is placed at the stance ankle. The constraints for hip motion are:

$$x_h(t) = \begin{cases} -S_1, & t = kT \\ S_2, & t = kT + T/2 \\ S - S_1, & t = (k+1)T \end{cases} \tag{7}$$

In order for the periodic trajectory $x_h(t)$ to be smooth, the following continuous condition must be satisfied:

$$\dot{x}_h(kT) = \dot{x}_h(kT + T) \tag{8}$$

In the process of walking, according to biomechanic studies, the body must slow down and then speed up again during each step [18]. Therefore, the body's acceleration immediately before the touch down of the swinging foot (end of walking step) is positive and reaches maximum value. Whereas, immediately after the touch down of the swinging foot (start of walking step) the body's acceleration is a negative, minimum value. In addition, the body motion can be reasonably described as an inverted pendulum whose acceleration at the beginning and end of the walking step are equal in magnitude but opposite in sign [19]. Based on these studies, we can impose the acceleration constraints on the body motion as in (9),

$$\ddot{x}_h(kT) = -\ddot{x}_h(kT + T) \tag{9}$$

Although there are only five constraints from (7), (8) and (9), we found that a fourth-order or fifth-order polynomial can not describe the hip trajectory having these constraints. At least a sixth-order polynomial is required. We wish to have a set of diverse trajectories from which we can find the best trajectory. To obtain this we add higher order coefficients to the polynomial and by changing these coefficients we can get various trajectories instead of only one. In this work, we found that a seventh-order polynomial is good enough to provide various hip trajectories. We also found that higher order polynomials (8, 9, ...) don't make a significant improvement. Therefore, the hip trajectory can be described as:

$$x_h(t) = a_7t^7 + a_6t^6 + a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \tag{10}$$

A seventh-order polynomial has 8 coefficients. Since there are five constraints, five coefficients are used to comply with the constraints. Let's call these five coefficients the constrained-coefficients, the other three coefficients will be the redundant-coefficients. a_0, a_1, a_3, a_4, and a_6 are constrained-coefficients and a_2, a_5 and a_7 are redundant-coefficients. a_2, a_5 and a_7 will be searched by GA and the constrained-coefficients are determined as in (11).

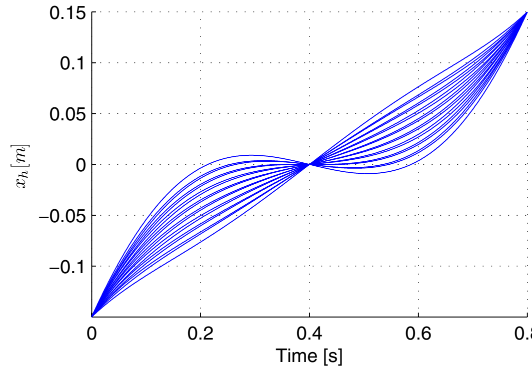

Fig. 4. Diverse hip trajectories generated when $a_2 = \{-2; -1.5; \}$, $a_5 = a_7 = \{-1; 0; 1\}$ and $S_1 = 0.15$, $S_2 = 0$.

$$\begin{aligned} &a_6 = 64[S/2 - S_1 - S_2 - 7a_7(T/2)^7]/T^6, \\ &a_4 = (M_4 - 2M_3/T - 10T^3a_5)/4T^2, \\ &a_3 = (M_3 - 5T^4a_5 - 4T^3a_4)/3T^2, \\ &a_1 = (M_1 - T^5a_5 - T^4a_4 - T^3a_3)/T, \quad a_0 = -S_1 \end{aligned} \tag{11}$$

where

$$\begin{aligned} &M_1 = S - a_7T^7 - a_6T^6 - a_2T^2 \\ &M_2 = S_2 + S_1 - a_7(T/2)^7 - a_6(T/2)^6 - a_2(T/2)^2 \\ &M_3 = -7a_7T^6 - 6a_6T^5 - 2a_2T \\ &M_4 = -42a_7T^5 - 30a_6T^4 - 4a_2 \end{aligned} \tag{12}$$

Fig. 4 shows a bunch of trajectories generated by choosing deferent sets of (a_2, a_5, a_7). From the figure it can be seen that by using different sets of the redundant-coefficients we can achieve a set of diverse trajectories.

In order to give more freedom to the hip trajectory, the parameters S_1 and S_2 are decided to be free variables. These parameters, along with the coefficients a_2, a_5, a_7 and H_p, S_p will be searched by GA to obtain the optimal values resulting in stable and minimized joint-torque walking trajectory. Compared to [10], our formulation of hip trajectory can produce a much bigger set of hip trajectories to be selected by GA.

IV. GENETIC ALGORITHM IMPLEMENTATION

Genetic Algorithm (GA) [17] is a famous search algorithm developed by John Holland and his colleagues at the University of Michigan. The basic idea of these algorithms is borrowed from the natural selection process in which the fittest individuals have highest chance to survive in the next generation and vice versa. GA have proved to be very robust, efficient search algorithms and been being applied successfully in a wide range of fields from biology, medicine, computer science, engineering, etc.

A. GA's parameters

As mentioned in the earlier sections, the interested parameters in this study are the maximum height H_p of the swinging foot, the coefficients a_2, a_5, a_7 of the polynomial describing the horizontal hip trajectory and the parameters S_1, S_2. These parameters will be searched by GA to find the best trajectory that has large stability margin and low energy consumption. All the genetic operations (reproduction, crossover and mutation) will be performed on a string of 60 bits binary number. The real value of the parameters a_2, a_5, a_7 and H_p, S_1, S_2 are extracted from this string.

B. The Fitness Function

In genetic algorithm, fitness function is the core of the searching mechanism. It has the role of guiding the search such that desired effects could be achieved. The objective that we want to achieve is reflected through the fitness value. Therefore, choosing the correct fitness function is necessary otherwise GA may never converge or even if GA converges, desired performance may not be obtained.

In this work, our objective is to search for the interested parameters a_2, a_5, a_7 and H_p, S_p, S_1, S_2 such that *the applied torques on the joints of the biped are minimized*. For such, the cost function can be expressed as follows:

$$P = \frac{1}{n}\int_0^{t_s} \tau^T \tau dt \tag{13}$$

where n is the number of integration steps, τ is the vector of all the joint-torques of the biped, and t_s is the time of one walking trial.

To achieve the above mentioned objective, the cost function must be minimized. As a convention, in GA the fitness function is always being maximized. Therefore, the fitness function can be chosen as the inverse of the cost function:

$$F = \begin{cases} \frac{1}{P} & \text{if ZMP stays inside the stable region} \\ 0 & \text{Otherwise} \end{cases} \tag{14}$$

By formulating as in (14) we can incorporate the ZMP stability criterion into the fitness function to make sure that the resulting walking trajectory is feasible. The flowchart of the proposed method is shown in Fig. 5 where GN is the generation number and GN_{max} is the maximum number of generation.

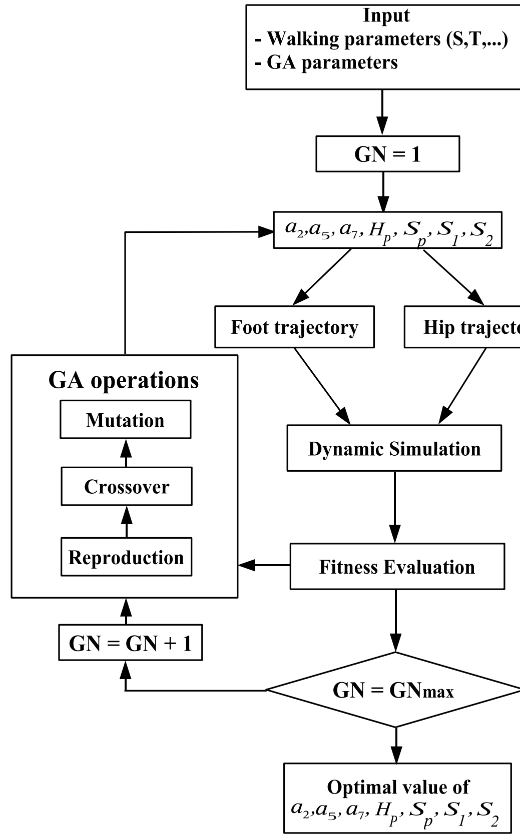

Fig. 5. Flowchart of the proposed method.

TABLE I
SPECIFICATIONS OF NUSBIP-II.

Length	Thigh		Shank		Foot Length		
(m)	0.32		0.32		0.21		
Weight	m_1	m_2	m_3	m_4	m_5	m_6	m_7
(kg)	1.52	1.68	2.68	10.45	2.68	1.68	1.52
Inertia	I_{1y}	I_{2y}	I_{3y}	I_{4y}	I_{5y}	I_{6y}	I_{7y}
(kgm^2)	0.1040	0.2739	0.3458	0.2426	0.3458	0.2739	0.1040

V. SIMULATION RESULTS

The specifications of the simulated biped are taken from a real biped, which was named NUSBIP-II and developed in our laboratory (Fig. 1). Table I summarizes the specifications of the biped robot NUSBIP-II. The simulation is done in Yobotics,[1] a dynamic simulation software which allows the running of batches of simulation.

Table II shows the initial conditions for GA where IndivNo is the number of individuals and GenNo is the number of generations that GA will do the search. In this simulation, the step length is chosen to be 0.3 m ($S = 0.3$ m), and the walking step period is $T = 0.8$ s. As mentioned in the previous sections, the interested parameters that are searched by GA are H_p, S_p (determining the maximum height position of the swinging foot during walking), the coefficients a_2, a_5, a_7 and the parameters S_1, S_2 which constitute the hip trajectory. These parameters will be determined when the GA converges.

In this study, GA converges after 450 generations. The converged values of the interested parameters are $a_2 = -1.5102$,

TABLE II
INITIAL CONDITIONS FOR GA.

GA parameters			
IndivNo	GenNo	Crossover rate	Mutation rate
1000	500	80%	2%

[1] http://www.yobotics.com/simulation/simulation.htm

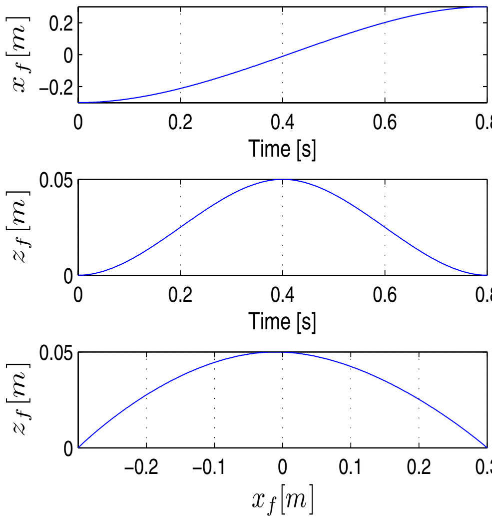

Fig. 6. The resulting foot trajectory. From top to bottom are the respective horizontal (X) trajectory, vertical (Z) trajectory and the path of the foot motion.

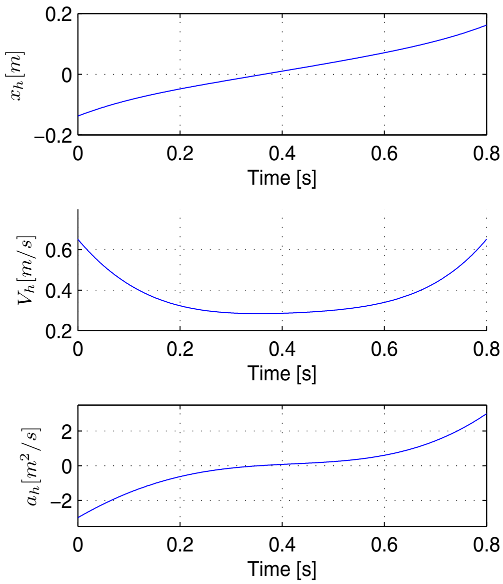

Fig. 7. From top to bottom are the resulting position, velocity and acceleration hip trajectories of the robot,respectively.

$a_5 = -0.8013$, $a_7 = -0.5200$ and $H_p = 0.050$, $S_p = 0.291$, $S_1 = 0.138$, $S_2 = 0.013$. Since these parameters are known, the foot trajectory and the hip trajectory are determined. Fig. 6 depicts the optimal foot trajectory of the bipedal robot. The resulting optimal hip trajectory is shown in Fig. 7.

Fig. 8 shows the resulting ZMP trajectory of the biped. It can be seen that, the ZMP is always inside the stable region which means the bipedal walking is physically feasible. In addition, *the ZMP stays very close to the middle of the foot* which is desirable because the closer the ZMP to the middle

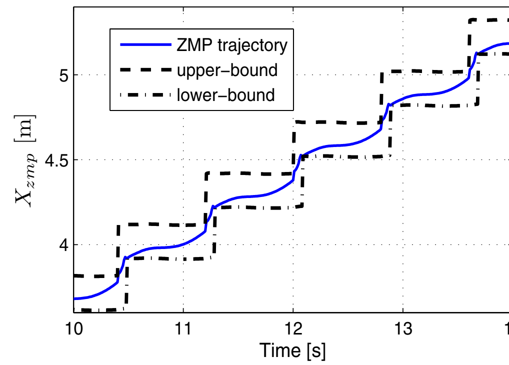

Fig. 8. The resulting ZMP trajectory.

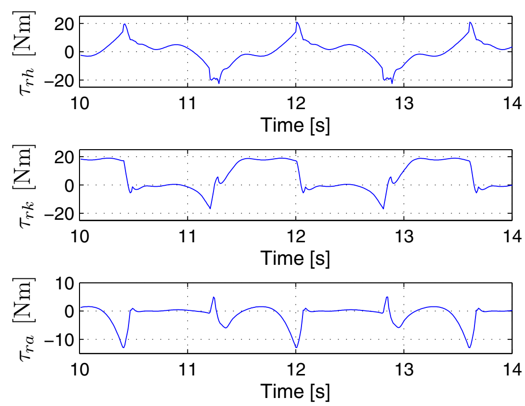

Fig. 9. Torques to be applied at right hip, knee and ankle joints.

of the foot, the more stable the bipedal walking is. The stable region is defined by the upper-bound and the lower-bound. When only one foot is supporting, the upper-bound and the lower-bound are the toe and the heel locations of the supporting foot, respectively. When two legs are touching the ground, the upper-bound is the toe of the front foot and the lower-bound is the heel of the behind foot.

Fig. 9 shows the torques needed to be applied at the right joints (hip, knee and ankle) of the biped. The maximum joint torque for all the joints is about 20 Nm. The maximum torque applied at the ankle joint is about half that of the hip joint. The hip, knee and ankle joint angle trajectories are shown in Fig. 10. The walking motion images (in stick diagram) of the biped captured at 0.05s apart is shown in Fig. 11.

VI. CONCLUSION

In this study, we proposed a method of optimal trajectory generation for bipedal robot. The foot and hip trajectories in sagittal plane are planned in the Cartesian space using polynomial interpolation. The contribution of our work is the proposal of the new way of designing the hip trajectory such that a big set of smooth hip trajectories can be generated by choosing different sets of the 7 key parameters (a_5, a_5, a_7, H_p, S_p, S_1, S_2). The best trajectory with minimized joint torques

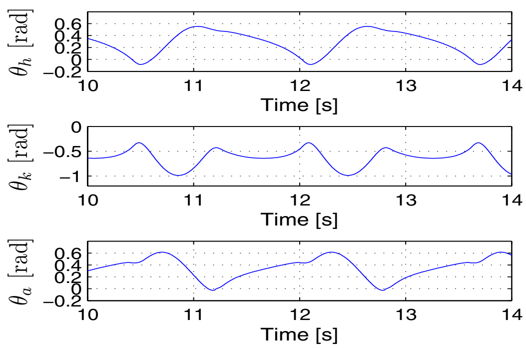

Fig. 10. The hip, knee and ankle joint angle trajectories.

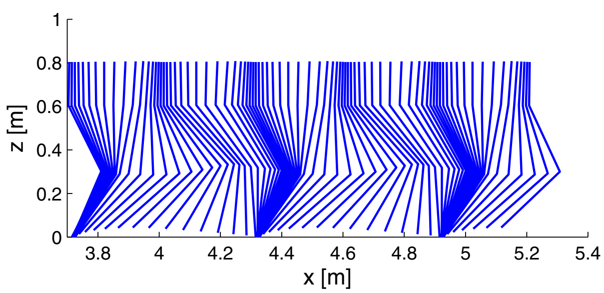

Fig. 11. The stick diagram of the walking robot (showing the right leg only). Images are captured at 0.05s apart starting from time $t_1 = 10s$ to $t_2 = 14s$.

is then searched by GA. It is obvious that the bigger the set of hip trajectories is generated, the better the resulting optimal trajectory can be obtained. In this study, by using the 7 key parameters as free variables we can create a much bigger set of hip trajectories (compared to other related works) for GA to search. According to our knowledge, this is the first work to consider finding the optimal foot trajectory by finding the optimal key parameters H_p and S_p. The simulation results show that the achieved walking motion is very stable with very large stability margin. The maximum joint torque is only about 20Nm.

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