Designing Non Constant Resolution Vision Sensors Via Photosites Rearrangement

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***Abstract*—Non conventional imaging sensors have been intensively investigate in recent works. Research is conducted to design devices capable to provide panoramic views with no need of mozaicing process. These devices combine optical lenses and/or non planar reflective surface with standard pinhole camera. We present in this paper an analysis of pixels rearranged sensor adapted to distortions induced by mirrors. Particularly, we aim to identify photosites distributions that compensate the non constant resolution for a given reflective surface. Our analysis is applied to a wide set of commonly used reflective surfaces in panoramic vision. Synthetic datas are produced in order to substantiate the geometric properties since such sensors do not exist yet.**

I. Introduction

Standard vision sensors provide almost constant spatial resolution image such that a wide range of linear operators can be applied for signal processing. Scene objects and their projections on image plane are linked only by the Thales relation, hence metrics of the scene can be computed. Straight lines are projected as straight lines on image plane and the projections are equiareal if distances to the sensor are constant.

If one is familiar with the perspective camera image formation, it is less trivial to apprehend sensor models that are intensively studied for their panoramic property. Non linear devices are used to enlarge the field of view : non planar reflective surfaces and/or wide angle lenses that do not comply with Gauss condition. The trade-off of the broadened field of view is the non linearity of the imaged signals. Resulting projections are aphylactic. The measures performed on images are highly complexified and most of the traditional image processing operators are no more appliable.

Intensive works are carried out on the design of omnidirectionnal vision imaging sytems and on their signal interpretation. Distortions induced by lenses or reflection from mirrors are described as ”deviation” from the perspective model and a metric to quantify them is introduced [1].

Reducing distortions may be non relevent especially when one does not need to synthetized correct prespective views. Many omnidirectional researches aim to design vision systems that possess only some specific geometric characteristics. This is specially true for mirror based systems and often, the concerned property is the spatial resolution preservation. Different constraint can be fixed for this purpose depending on how one interprets the meaning of ”constant resolution”. However, any of the proposed solution compute the mirror shape with differential methods [2], [3], [4], [5], [6]. Often an equi-areal sensor is also preserving angular resolution in the sense that two equal solide angles are mapped to equal surfaces on the image plane, but this is not always true. All proposed solutions to these problems are mirror shape design oriented, hence catadioptric sensor design is mainly a matter of mirror conception.

One other alternative to the problem it to focuse the analysis to the image plane. Because standard photosensitive sensors are coupled to a non linear device, the first reaction is to reshape the mirror (or the lens). Now, what if one can rearranged the photosites on the ccd/cmos sensor in order to fit it to the mapping? We explore here a generic method to determine how the photosensitive device should be designed, assuming the mirror shape is fixed, in order to fullfill a given imaging property. Commonly manufactured ccd/cmos sensors are designed as linear arrays, where photosites are regularly spaced. Up to our knowledge, there is no photosensitive sensor designed for distorsions compensation. However we can underline the existence of previous works on foveal sensors [7], [8]. These photosensors have their photocells rearranged like a retina, that is the resolution is decreasing from its center to its border. Though they are the only space variant resolution ccd/cmos sensors, they do not suit our purpose. However theses results give some proof of feasibility to our approach especially concerning the implementation part.

Our contribution is described in the context of catadioptric systems. The 2th section exposes the formalization of the problem for general case and for cases implying most commonly used mirror. The 3th section gives detailed calculus for commonly used mirror. In the 4th section, we synthetize the pixels arrangement for the area (respectively angular) preserving photosensor. Results and comments are discussed as conclusion in the last section.

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II. Formalization

A catadioptric sensor image formation is decomposed into two projections : a projection to a reflective surface followed by the projection to the focus plane. The main constraints we fixe for our work are the following : the mirror in use is a smooth surface with a revolution axis aligned with the optical axis of the camera. The first constraint is almost implicite since commonly used mirror is designed this way. If these requirements are satisfied, one can examine the projection by only considering the mirror profile i.e. intersection of the mirror and a plane containing the optical axis (see figure 1 for a generic scheme).

We examine here two kind of pixels rearrangement that are fundamentaly different and both can achieve interesting image properties. The first solution aims to affect a same quantity of pixels to image surfaces that have same area on the mirror, we refer to it as the "aeral" solution. The second solution is angular resolution preserving as we mentioned in the introduction, it is refered as the "angular" solution.

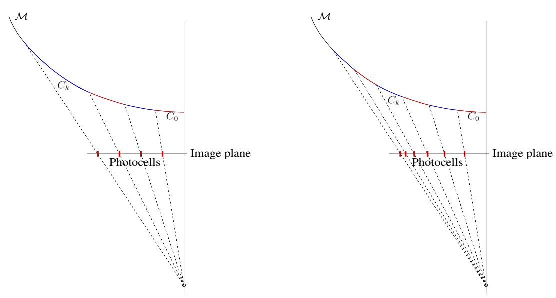

Fig. 1. Left figure : reflection sampling by standard ccd (regularly spaced pixels). The photocells projected to the mirror define non constant length arcs C_k for a given mirror shape. Right figure : arbitrarily sampled mirror surface. Rays intersections with the image plane define a new pixels rearrangement.

A. Aeral solution

1) Radial sampling: A standard ccd sensor has regularly spaced pixels, arranged in rows and columns but one can see that the samples back projected to the mirror define non equal length arcs hence same area surfaces is not being projected into same area surfaces on image plane. Conversly if we constraint the mirror to be sampled into constant length arcs, their projections produce a non regularly spaced pixels set.

We define ccd sensors with adapted pixels arrangement according to the observations. The mirror is first sampled into constant length curves, then the samples are perspectively projected to image plane. As the figure 2 shows, their coordinates along the x-axis (i.e. the x_k) set the photocells arrangement up to a scale.

For a better comprehension of the problem formalization, we introduce the following definitions (with figure 2 as illustration) :

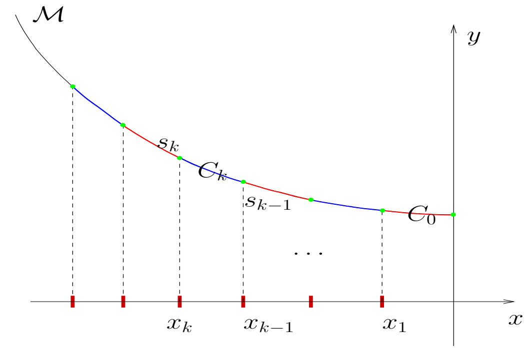

Fig. 2. The mirror is segmented into equal length arcs C_k. Each C_k is bounded by s_k and s_{k+1}. If s_k is the arc length of the $k$the sample, then x_k is the cartesian coordinate along the x-axis of the point associated to s_k

- Let $\mathcal{M}(s)$ be the arc length parametrization of the mirror profile.
- $\mathcal{M}$ is segmented into n curves of fixed length L_0. We note s_k the arc length of each sample. C_k is the kth arc and its length is $L_k = \int_{s_k}^{s_{k+1}} ds = L_0$.
- Since $ds = \sqrt{dx^2 + dy^2}$, we can write as a function of x :

$$L_k = \int_{x_k}^{x_{k+1}} \sqrt{1 + f(x)}dx \qquad (1)$$

  assuming $f = \frac{dy}{dx}$ and $x_k = x(s_k)$.
- If F is set as a primitive of $\sqrt{1+f(x)}$, we have $L_k = F(x_{k+1}) - F(x_k) = L_0$. Thus, one can see that $F(x_k)$ is the generic term of an arithmetic sequence satisfying :

$$F(x_k) = kL_0 \quad \Rightarrow \quad \frac{F(x_k)}{L_0} = k \qquad (2)$$

If equation(2) has solution, we can compute the x_k for k taking value in $\mathbb{N}$. The set $\{x_k\}$ gives the pixels arrangement up to a scale since the plane defined by $y = 0$ is parallel to the focus plane. This arrangement guarantees that arcs along radial direction are equivalently mapped.

2) Azimuthal sampling: So far, we have analysed a method to reaffect the photosites radially according to our areal constraint. We have now to examine the rearrangement problem along directions orthogonal to the radial one i.e. the azimuthal directions (defined from the rotation along the optical axis). Obviously, constant angular sampling is not suitable for the purpose. We rather fix the sampling rule as follows : the number of pixels used to image circles centered at the optical axis should be proportionnal to their perimeters. Hence each arc of length ds is imaged by n pixels and pixels size is constant in azimuthal directions (figure 3).

B. Angular solution

1) Radial sampling: Early used mirror shapes are designed with easy-to-handle geometric properties, especially the one preserving a unique center of projection. In the

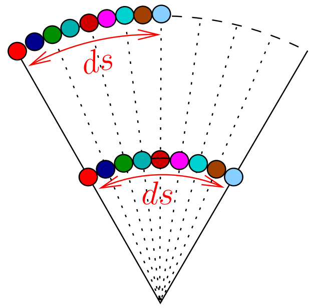

Fig. 3. The mirror is sampled azimuthally since the catadioptric device has an axis of revolution. In order to comply with the areal constraint, arcs of circles centered to the optical axis of same length ds are imaged by a same number of pixels n.

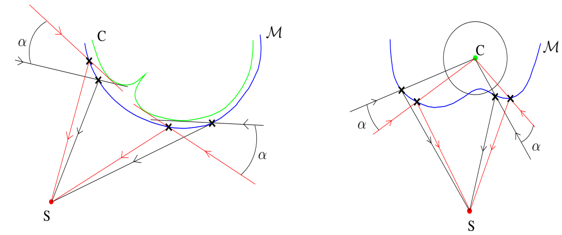

Fig. 4. Left figure shows the most general case where incident rays are tangent to the caustic surface. We can determine how pixels sould be placed in order to satisfy the constant angular resolution by sampling the caustic such that the angle α defined by the tangents associated to two consecutive samples are constant. Right figure shows a special case where the caustic is reduced to a point : the catadioptric device has a single viewpoint. The same method is still appliable, but there is an easier solution. We set a unit sphere centered on the viewpoint then it is sampled with a constant angular sampling. We trace lines that pass through the center and through each sample. The intersections of these lines with the mirror give the solution.

cases of catadioptric sensors, this consists to insure that the projection to the mirror has a unique center and that center is mapped to the camera's one. Only a small class of mirrors is able to satisfy this preservation [9], such mirrors are conics of revolution with the pinhole camera coaxially placed on one of their foci.
These configurations are said single viewpoint constrained and one can easily apprehend the constant angular resolution property with them. However this turns to be much harder to define when the imaging device has no more a unique center of projection. Light rays incident to the mirror are not converging in a single point, they follow paths that are tangent to a virtual surface that is known as the caustic. This surface depends only on the position of the light source and on the mirror geometry. Now, if we consider the ccd sensor as the light source, the caustic can be estimated if both the relative position between the mirror and the camera and the shape of the mirror are known. Readers should refer to [10], [11] to determine the analytic equation of this surface. However, if no analytic solution is available (for whatever the reason), numeric estimation is still available via method described in [12].
Figure 4 shows a mirror profile and its caustic C with respect to the camera position. Each point on the mirror is mapped to only one element of the caustic, hence a sampling of it is mapped to a sampling of the mirror. Incident rays to $\mathcal{M}$ are tangent to C then we sample C such that the angle defined by the tangents of two consecutive samples remains constant. This gives us the radial rearrangement by projecting the sampling of the mirror to the image plane.

2) Azimuthal sampling: Since we have to insure constant angular resolution and given the fact that we set the whole system with an axis of revolution, it will be sufficient to sample the mirror with a constant angle interval. The number of allocated pixels is then proportionnal to the angular section (see figure 5).

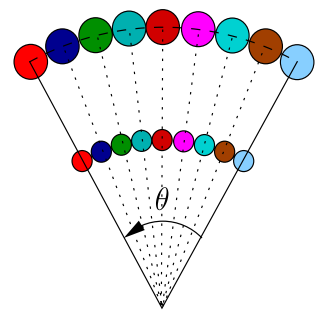

Fig. 5. A constant angular sampling will satisfy the equi-angular property, thus the number of pixels along azymuthal directions are directly proportionnal to the angular section θ.

III. Photosites arrangement for commonly used mirror

The systems that have a unique center of projection are intensively used and designed, thus the most commonly used mirror are conics of revolution. The pixels arrangements for these configurations are analysed and we focuse only on the radial sampling since our hypotheses imply that the optical axis is also the revolution axis, the azimuthal sampling is the same for each case.
We also give here analytic formulation only for the areal solution. The angular solution will be computed numerically according to section II, since C has often a non easy to handle equation.

A. Parabola mirror

This kind of profile is widely used because the unicity of the center of projection is preserved if the mirror focus is placed at infinity. This is realised by inserting an orthographic lens between the ccd sensor and the mirror. If the mirror axis is set as the y-axis, its profile has the generic form :

$$y = ax^2 + c$$

According to previous statements (equation 1), the arc length measured between x_k and x_{k+1} is given as :

$$L_k = \int_{x_k}^{x_{k+1}} \sqrt{1+4a^2x^2}dx.$$

After rearrangement and simplification, a primitive F of $\sqrt{1+4a^2x^2}$ is :

$$F(x_k) = \frac{1}{4a}\left(2ax_k\sqrt{1+4a^2x_k^2} + \ln\left(2ax_k + \sqrt{1+4a^2x_k^2}\right)\right)$$

Equation (2) gives x_k as fonction of k, we solve it numerically for $k \in \mathbb{N}$. Figure 6 shows as exemple the mirror profile sampling for fixed $L0$ and parameters a and c. The projection of the samples on the x-axis is the pixels arrangement i.e. the set of x_k.

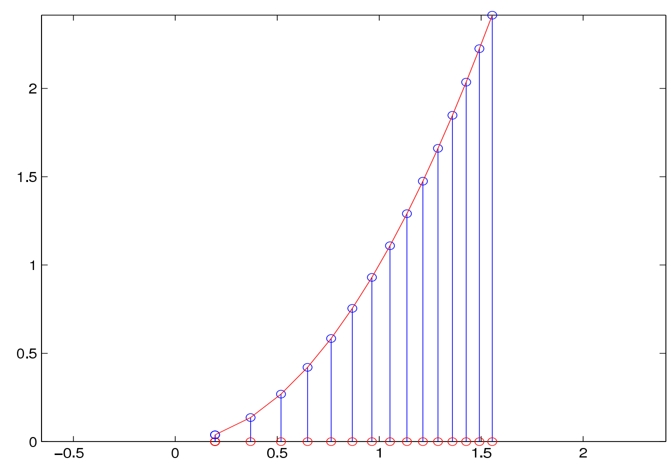

Fig. 6. Constant length arc segmentation of a parabola mirror profile (red curve). Parameters are fixed $a = 1$, $c = 0$ and $L_0 = 0.2$. Mirror samples are marked by blue circles and the x_k is marked by red circles. The length of the projection of each arc decreases as k increases.

B. Hyperbola mirror

Mirror with hyperbola profile is also widely used in panoramic imaging. The single projection center is insured if the camera is placed at one of the mirror foci. Under similar assumptions stated above, the mirror profile generic forme is:

$$\frac{x^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$$

Thus, if we consider only the upper half of the curve :

$$y = y_0 + |b|\sqrt{\frac{x^2}{a^2} - 1}$$

However for the hyperbola curve, the primitive $F(x_k)$ can only be numerically computed because of the elliptic nature of L_k. Expressing x_k with regard to k becomes much more complexe.

C. Spherical mirror

The Mirror with circle profile does not comply with the unique center of projection constraint unless the camera is placed into the center of the mirror. Such configuration is not relevant for omnivision purpose as the camera will see the inside of sphere. We give a detailed explanation here for both areal and angular solutions. This will give a concrete exemple of the method that one can apply for the two previous mirror.

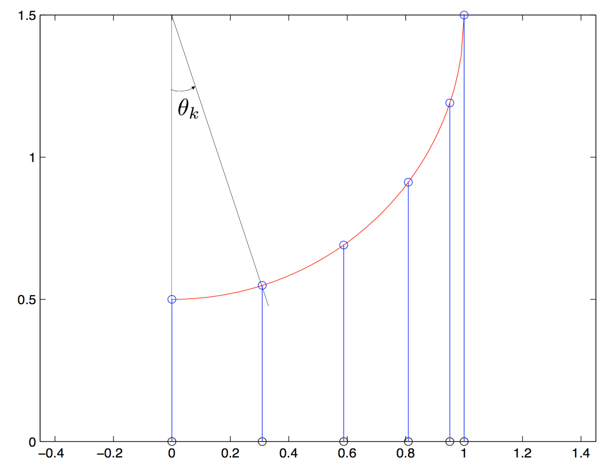

Fig. 7. Circle is sample into same length arcs. For spherical mirror, the sampling can be done without computing the integral since the measured lengths are proportionnal to the angle of the sector defined by two consecutive samples and $x_k = r\cos(\theta_k)$ if θ_k is the angle measured between the kth sample and the y-axis.

1) Areal solution: With the optical axis set as the y-axis, the profile has the generic form :

$$y = a - \sqrt{r^2 - x^2}$$

The sphere of radius r is centered at $[0, a]^t$. This equation gives only its lower half as we assume the camera is placed below the mirror. Equation(1) becomes :

$$L_k = \int_{x_k}^{x_{k+1}} \frac{r}{\sqrt{r^2 - x^2}}dx$$

and

$$F(x_k) = r \arctan\left(\frac{x_k}{\sqrt{r^2 - x_k^2}}\right)$$

For this profile, equation (2) has explicit analytic solution :

$$x_k = \sqrt{\frac{r^2\tan^2\left(\frac{kL_0}{r}\right)}{1+\tan^2\left(\frac{kL_0}{r}\right)}}$$

Negative solutions are symetrics of the positive ones since the y-axis acts as the optical axis. Figure 7 shows the mirror sampling. Distances between x_k decrease as k increases.

2) Angular solution: The analytic parametrization of the caustic $C(x)$ is determined by applying the definition given in [10]. The tangent vectors can then be extracted from it : $T(x) = \frac{\dot{C}}{|\dot{C}|}$. Then we choose an angular sampling interval α and an arbitrary first sample (for example the bottom of the mirror). The other samples are then determined iteratively such that the angle formed by the tangent of the current sample and the tangent of the next one is equal to α.

The set of samples on the mirror is finally projected to

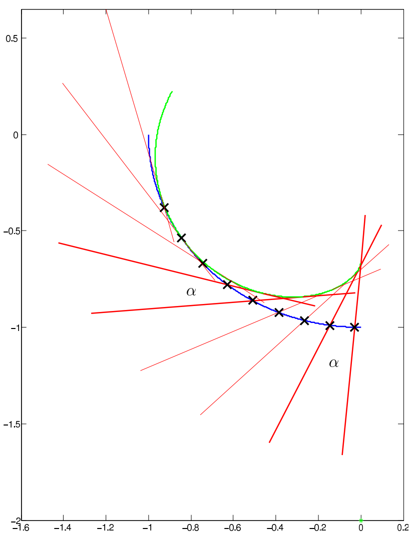

Fig. 8. It is equivalent to sample the caustic C (top curve) or the mirror $\mathcal{M}$ (quarter circle) and the equi-angular preservation can be achieved if we chose the points on C such that the angle α defined by the corresponding tangents of two consecutive samples are constant (red lines).

the image plane, producing the pattern that photosites have to satisfy. Figure 8 illustrates the whole operation for a spherical mirror of unit radius facing a the CCD camera placed at it bottom.

The Azimuthal sampling is not specific to the mirror shape providing the whole system has a symetry axis, which is the case according the hypotheses. For both areal and angular problems, we sample the mirror according to the methods presented in section II.

For any given mirror profile, we can see that the determination of a correct pixels arrangement is equivalent to "rectify" a smooth curve, in the sense of measuring the length of an arc. This approach does not distinguish between central (i.e. single center preserving) or non central imaging system.

IV. Simulation for the spherical mirror

In this section we simulate the photocells rearrangement for both areal and angular solutions. The synthetic images are produced in order to show how pixels size change along radial and azimuthal directions. We assume that the mirror is spherical and its axis is aligned with the camera's one. The choice of this configuration is due to the isotropic geometry of the sphere that makes the alignement easier. For the areal sampling, we have segmented the mirror profile into 11 arcs of same length, and each circle is also segmented into arcs of same length ds, choosen arbitrarily (here $\frac{1}{96}$th of the greatest circle). The samplings in both radial and azimuthal directions is the mapped to the image plane, thus this gives the photocells arrangement. The figure 9 reproduces it. Each (almost) square represents a pixel and as one can see, the farer we are from the center, the higher is the pixel density. The angular sampling

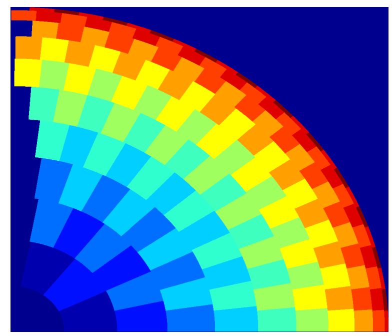

Fig. 9. This figure shows how photocells have to be placed and shaped in order to image fairly (i.e. with same number of pixel) any parts of the mirror that have equal area and of course that is visible to the camera.

is easier to reproduce because of the constant angular sampling. For the radial part, we segment the mirror profile via the caustic with the value of α set arbitrarily (here $\alpha = \frac{\pi}{18}$rad $= 10$ degree). Finally the azimuthal sampling is performed with a constant angle of $\frac{\pi}{46}$. The figure 10 shows of to place and shape the photocells. Each square represents such a cell and one can see how its size change with respect to its position. The pixel density change less quickly than the previous case.

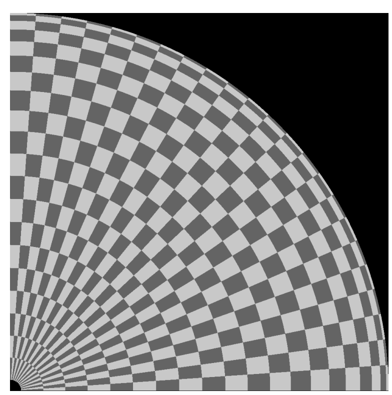

Fig. 10. The figure shows how to place and shpae the pixel in order to affect the same quantity of pixels to equal solide angles. One can see that the cells are thinner in the radial direction near the center. They become squares in the middle and thin back (perpendicularly) at the border.

V. Conclusion

Previous works on catadioptric views are only focused on mirror design to manufacture omnidrectionnal systems imbued with specific geometric properties. However no works are proposed to study the ccd/cmos sensors that can provide equivalent solutions. As an exemple of what can one do with pixel arrangement, we present a photosensor rearrangement that allocates the same numbers of pixels for a same area surface on the mirror. We also present the solution to get a constant angular projection in the sense of a fair pixel allocation for equal solid angles.

Through the examples, we can see that the hardest part of the problem is to sample correctly the mirror profile. This problem is equivalent "rectify" smooth curves, in the sense of measuring the length of an arc. This approach does not distinguish central (i.e. single viewpoint) configurations from the non central ones, hence we can apply it providing the hypotheses are fulfilled. We have to underline the major drawback for these systems : the pixel arrangement is set for only one position of the photosensor relatively to the mirror. If the camera's pose relatively to the mirror is shifted because of chocs or some non stable mechanism, the imaging property is lost.

Finally, simulation results are presently scarce because such systems do not exist. We got only the method to place and shape the pixels, measures through synthetic datas should be produced in order to confirm the efficiency of the proposed approach in futur work.

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