# Material Removal Model and Contact Control of Robotic Gasbag Polishing Technique

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***Abstract*—Material removal model of robotic gasbag polishing is established on the basis of Preston equation and distributions of linear velocity and contact stress in the circular contact area between the flexible polishing tool and mould surface. Contact force determines the maximal contact stress and influences the material removal and surface quality. Linear velocity and contact stress fields together with the distribution of material removal are calculated and simulated respectively according to certain internal pressure, downward depth, inclining angle, rotating speed and structural characteristics of rubber gasbag. The material removal mechanism is analyzed for optimizing the polishing process and trajectory planning is proposed for more convenient operation. The theoretical analysis and experimental results indicate that the surface roughness Ra 5nm can be obtained under the situations of effective cushioning action of flexible contact, reliable control accuracy of robotic gasbag polishing system, tiny wiped off process of mould surface, etc.**

***Keywords*—robotic gasbag polishing, material removal model, contact control, surface quality**

## I. INTRODUCTION

Polishing plays a key role in obtaining certain level of form and texture by removing the traces and damaged layer caused by pre-machining or grinding before mould's application in industry. Manual polishing for smooth and well-proportioned surface quality and geometrical accuracy is time-consuming and costly. Conventional rigid rubbing polishing methods make the mould surface relatively rough in local areas because of the harmful mechanical vibration and existence of numerous tiny potholes. With the development of mould industry and its related manufactures, it is very important to develop some automated polishing techniques for more reliable, accurate and stable process control, much higher polishing efficiency and more subtle surface quality. Many useful ideas and efficient methods have been put forward in the aspect of automated polishing as follows. An automated precession process which is based on an innovative pressurized tool of variable effective size with controllable internal pressure for polishing lenses, mirrors and other objects and obtained a superb texture on samples was reported by D. D. Walker et al [1, 2]. A polishing system composed of two subsystems, a three-axis machining center and a two-axis polishing robot, which is able to keep the polishing tool normal to the die surface during operation was developed by M. C. Lee et al [3]. A robotic die polishing station controlled by a PC and a robot controller, which consists of a pneumatic grinding tool, a six-degree-of-freedom industrial robot manipulator, and grinding abrasives, was proposed by B. S. Ryuh et al [4]. An efficient polishing process for precision polishing tasks with a new compliant abrasive tool was implemented by M. J. Tsai et al [5]. An automatic planning and programming system based on a CAD system for robotic polishing and several approaches for realizing automated polishing from different research groups were described and reviewed in detail by J. J. Márquez et al [6].

Robotic gasbag polishing technique based on flexible polishing principle is presented here. This technique provides six degrees of freedom by the Motoman-HP20 articulated robot, controllable internal pressure of rubber gasbag, detachable modules for satisfying different polishing requirements, etc. Rubber gasbag with certain hardness represents the idea of flexible polishing and realizes a relatively large compliant contact. Moreover, certain granularity of abrasive powders adhered to the polishing cloth and distributed in the contact area play an important role in micro-ploughing, micro-cutting micro-fatigue or micro-cracking on the mould surface. In all, the robotic gasbag polishing technique can make the polishing process more stable and subtle because of effective cushioning action of flexible contact, reliable control accuracy of robotic gasbag polishing system, tiny wiped off process of mould surface, and so on.

Research on the contact force, stress field, velocity field, material removal model, material removal mechanism and trajectory planning of robotic gasbag polishing under the situations of certain internal pressure, downward depth, inclining angle, rotating speed and structural characteristics of rubber gasbag is the main purpose here. It will improve the efficiency of material removal, optimization of the polishing process, quality of mould surface, etc.

## II. MATERIAL REMOVAL MODEL AND MECHANISM

Material removal model of robotic gasbag polishing is established on the basis of Preston equation and distributions of linear velocity and contact stress in the assumptive circular contact area. Meanwhile, the material removal mechanism is analyzed for optimizing the polishing process.

### A. Contact Analysis

The characteristics on contact area include downward depth, inclining angle, rotating speed, contact size, contact force, stress field, velocity field, etc. The values of contact size, contact force, contact stress, linear velocity, material removal rate can be changed easily by adjusting the inclining angular $\theta$ between the axis of rubber gasbag and the normal of local contact area, the internal pressure of the flexible rubber gasbag $p$, the downward depth $h$ when the rubber gasbag presses against the mould surface, the angular velocity of polishing tool $\omega$ and other factors. Fig. 1 shows the contact area between the hemispherical rubber gasbag and plane mould surface. Fig. 2 shows the coordinate and linear velocity in the assumptive circular contact area.

In Fig. 1 and Fig. 2, it is assumed that the contact area is an approximate circle, $O$ is the centre of hemispherical rubber gasbag, $O_1$ is the point of intersection between the axis of rubber gasbag and mould surface, $O_2$ is the center of the circular contact area, $A$ and $B$ are the two boundary points respectively, $L$ expresses the distance between points $O$ and $O_2$, and $2r$ expresses the distance between points $A$ and $B$, i.e. the diameter of the circular contact area. Moreover, $x$-axis is along the line $AB$ which the point $F$ is on, $H(x,y)$ is a given point on the contact area whose coordinate data is $(x,y)$, the line $HF$ is perpendicular to $x$-axis and the line $FG$ is perpendicular to the line $OO_1$. The distance between points $O$ and $A$ or $B$ is the radius of the hemispherical rubber gasbag named $R$ here. The shape surrounded by the points $O$, $A$ and $O_2$ is a right-angled triangle. It is obvious that $HF \perp OO_2$ and $HF \perp O_1O_2$, thus it can be obtained that $HF \perp OO_1$, then the result of $GH \perp OO_1$ is derived based on the other known information $FG \perp OO_1$. In the robotic gasbag polishing process, the parameter $L$ can be properly adjusted for compensating the form error of mould surface in certain degree.

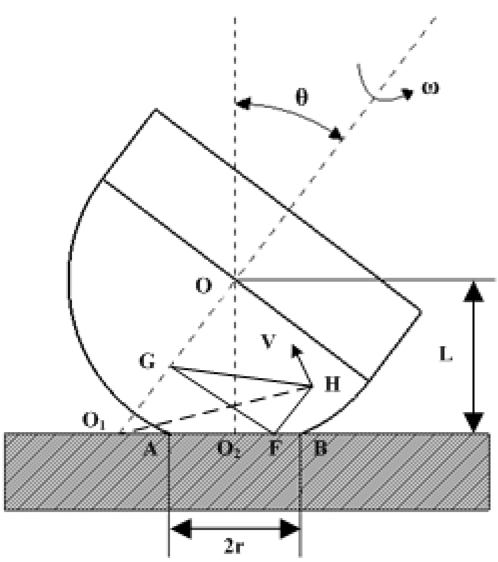



Figure 1. Contact area between the rubber gasbag and mould surface

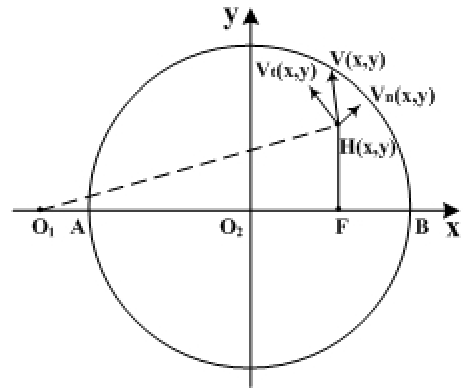



Figure 2. Coordinate and linear velocity in circular contact area

## B. Distribution of Linear Velocity

Based on the discussion and assumption above, it can educe the following equations in turn.

$$L = R - h \tag{1}$$

$$r = \sqrt{R^2 - L^2} = \sqrt{2Rh - h^2} \tag{2}$$

$$|O_1O_2| = L \cdot \tan\theta = (R-h)\tan\theta \tag{3}$$

$$|O_2F| = |x|; \quad |HF| = |y| \tag{4}$$

$$|O_1F| = |O_1O_2| \pm |O_2F| = (R-h)\tan\theta + x \tag{5}$$

$$|FG| = |O_1F|\cos\theta = (R-h)\sin\theta + x\cdot\cos\theta \tag{6}$$

$$\begin{aligned}|GH| &= \sqrt{|FG|^2 + |HF|^2} \\ &= \sqrt{[(R-h)\sin\theta + x\cdot\cos\theta]^2 + y^2}\end{aligned} \tag{7}$$

$$V_{O_2} = |OO_2|\sin\theta\cdot\omega = \omega(R-h)\sin\theta \tag{8}$$

$$V(x,y) = |GH|\omega = \omega\sqrt{[(R-h)\sin\theta + x\cdot\cos\theta]^2 + y^2} \tag{9}$$

Where, $x^2 + y^2 \le r^2 = 2Rh - h^2$, $V(x,y)$ is the linear velocity of the given point $H(x,y)$, and $V_{O_2}$ is the linear velocity of point $O_2$, which is a special expression of $V(x,y)$ when $x = y = 0$.

Moreover, the vector direction of $V(x,y)$ is perpendicular to the line $GH$ and in the plane of $GHF$. As the line $OO_1$ is perpendicular to the plane of $GHF$, the vector direction of $V(x,y)$ is perpendicular to the plane of $GHO_1$, thus it is perpendicular to the line $O_1H$. Let the vector direction of $V_t(x,y)$ in the plane of the circular contact area and perpendicular to the line $O_1H$. Similarly, let the vector direction of $V_n(x,y)$ perpendicular to the plane of $O_1FH$. Based on the vectorgraph relationship, it can be obtained that $V(x,y)$ can be separated into a horizontal linear velocity $V_t(x,y)$ and a vertical linear velocity $V_n(x,y)$, which is shown in Fig. 2. Let the angles of $\angle HO_1F$ and $\angle HGF$ as $\beta$ and $\gamma$ respectively, then the following equations can be obtained in turn.

$$\tan\beta = HF/O_1F = |y|/[(R-h)\tan\theta + x] \quad (10)$$

$$\tan\gamma = HF/GF = HF/(O_1F\cdot\cos\theta) = \tan\beta/\cos\theta \quad (11)$$

$$V_t(x,y) = V(x,y)\sin\gamma\cdot\cos\theta/\sin\beta \quad (12)$$

$$V_n(x,y) = V(x,y)\sin\gamma\cdot\sin\theta \quad (13)$$

Apparently, the parameters $R$ and $h$ determine the value of $L$ which is not a constant and will influence the contact size and the distributions of linear velocity and contact stress. Also, the values of $\theta$ and $\omega$ influence the distributions of linear velocity of contact stress. So, adjusting the parameters $L$, $\theta$ and $\omega$ can control the distribution of contact stress, material removal rate and surface quality. The value of $2r$ also validates the large flexible contact area of robotic gasbag polishing technique. In order to view the distribution of linear velocity in the circular contact area, the simulation result is given in Fig. 3 by the software of MATLAB when the $\omega$ is 105rad/s, $h$ is 1.0mm, $R$ is 20mm, and $\theta$ is $\pi/9$.

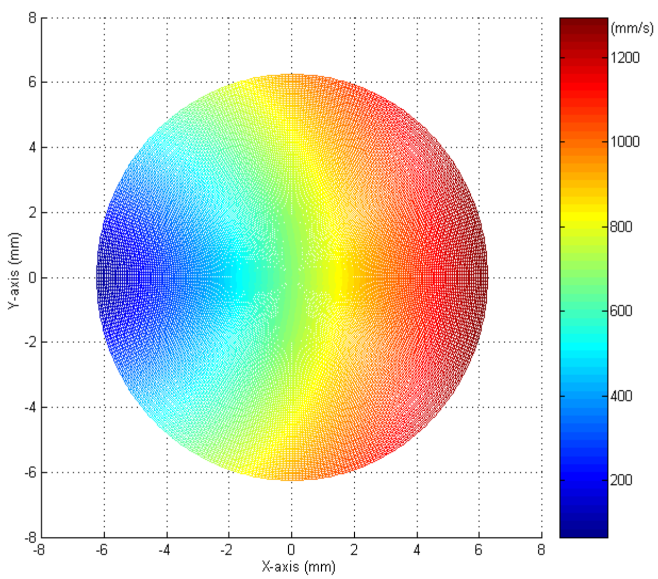



Figure 3. The distribution of linear velocity in circular contact area

*C. Distribution of Contact Stress*

Hertz deduced that an ellipsoidal distribution of pressure would satisfy the boundary conditions of the problem and found that, for the case of a sphere, the distribution of normal pressure satisfy certain equation [7]. Here, we adopt the general expression tentatively to analyze the distribution of contact stress, which is as follows.

$$P(x,y)/P_m = 3\sqrt{1-(x^2+y^2)/r^2}\,/2,\; x^2+y^2 \le r^2 \quad (14)$$

Where, the distribution of contact stress $P(x,y)$ reaches a maximum value which is 1.5 times the mean contact stress $P_m$ at the center of contact area, i.e. $x = y = 0$, and falls to zero at the edge of the circular contact area in theory, i.e. $x^2 + y^2 = r^2$. When the contact force $F$ which can be obtained from the result of finite element analysis under different contact parameters combination or experimental measurement is given, thus $P_m = F/\pi r^2$.

*D. Material Removal Model*

The classical Hertz contacting theory and material removal Preston equation provide basis for calculating the distribution of contact stress in the contact area and constructing the material removal model of robotic gasbag polishing [7-9]. Some effective contributions to the study of material removal model and corresponding experiment in different application fields of precision engineering have been done [10-12]. Here, the material removal model of robotic gasbag polishing is established on the basis of Preston equation and distributions of linear velocity and contact stress in the circular contact area between the flexible polishing tool and plane mould surface.

In the polishing process, local material removal rate $dh/dt$ on the certain polishing area has been expressed as follows by Preston equation.

$$dh/dt = K \times P \times V \quad (15)$$

Here, $dh/dt$ is determined by the Preston coefficient $K$, the polishing pressure also called contact stress $P$ and the relative velocity $V$ between polishing tool and mould surface. The Preston coefficient $K$ is related to the material properties of rubber gasbag and mould, granularity of abrasive powders, and other factors such as the influence of different material removal mechanisms, chemical action, and so on. Moreover, Preston coefficient $K$ depends mostly upon abrasives and material properties and only the tangential component of relative velocity on the polishing area affects the local removal rate [10]. So, the (15) can be written as follows.

$$dh(x,y)/dt = KP(x,y)V_t(x,y) \quad (16)$$

The Preston coefficient $K$ can be obtained by orthogonal experiment using "fixed point polishing" method on the basis of different combinations of separate parameters such as internal pressure, angular velocity, downward depth, polishing time and inclining angular of polishing tool, and certain granularity of abrasive powders, etc. When $x = y = 0$, $dh(0,0)/dt = KP(0,0)V_t(0,0)$. So, measuring the central removal depth in contact area accurately after experiments is the significant task for obtaining the Preston coefficient. The distribution feature of material removal in circular contact area under certain polishing parameters, some of which have been listed in the part of simulation of distribution of linear velocity, is shown in Fig. 4.

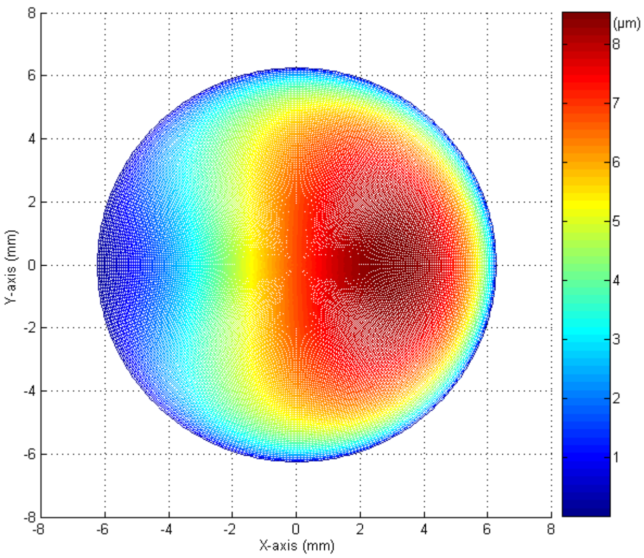
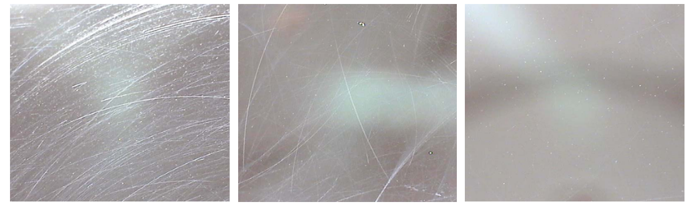



Figure 4. The distribution of material removal in circular contact area

### E. Material Removal Mechanism

Material removal mechanism plays an important role in optimizing the polishing process, improving the efficiency of material removal and quality of mould surface, and so on. It has been reported that since the abrasives are suspended in the fluid they are, in principal, able to roll freely between work-piece and counterface or to penetrate into the counterface losing the freedom of rotation. These two kinds of abrasive motion result in different modes of material removal mechanisms. While in case of freely rolling abrasives material removal is caused by micro-fatigue and micro-cracking of the surface, embedded abrasives lead to micro-ploughing or micro-cutting [12]. So, in the robotic gasbag polishing process, the contact stress, relative velocity, granularity of abrasive powders, and downward depth influence the material removal rate mentioned above, also the material removal mechanism and surface topographies. Analysis of material removal mechanism of robotic gasbag polishing can help us to hold the optimal polishing technics, such as when the internal pressure of rubber gasbag and rotating speed of polishing tool should be adjusted, when the granularity values and kinds of abrasive powders should be changed, how the polishing stages and times should be arranged, and so on. Fig. 5 shows the polishing quality and surface topography in different stages under different internal pressures, rotating speeds, feeding rates, and downward depths of polishing tool, granularity values, and so on, which will be listed in the part of experimental research.



a. polishing stage 1 b. polishing stage 2 c. polishing stage 3

Figure 5. Surface topographies in different polishing stages

## III. CONTACT CONTROL

The robotic gasbag polishing technique based on flexile polishing principle can enhance the effect of contact compliance and reduces the variation of contact force. The internal pressure of rubber gasbag influences the contact stress, local material removal and error compensation. The suitable radius of rubber gasbag is adopted based on the requirement of variation extent of curvature on the polished surface and material removal rate. The changes of curvatures in different contact points, the inclining angular, the downward depth, and the rotating speed of the polishing tool alters the contact stress, contact size, distribution of linear velocity and local material removal rate. The feed rate can influence the dwell-time. The granularity value and kinds of abrasive powder also take a vital function in local material removal rate and surface quality, etc.

Trajectory planning, control of contact force and position, and polishing process have been paid more attention in order to make automated polishing technique more stable and effectual. A CAM system which is capable of dealing with kinematic modeling of the polishing robot, check of the robots work-range, linear trajectory, and polishing patterns was developed by M. C. Lee et al [3]. A constant contact force control method and an automatic planning and programming system based on date from a CAD system and stages of the polishing process were described by J. J. Márquez et al [6]. Control of robotic mold polishing including a CAD/CAM-based position/force controller and a complete non-taught operation of the position and contact direction were considered by F. Nagata et al [13]. A trajectory generator needed for generating the robot polishing path, which is taken from the cutter location data was introduced L. Feng-yun et al [14]. A shape adaptive motion control system which consists of surface measurement, surface reconstruction, tool trajectory planning, and axis motion control was presented by Z. W. Yang et al [15].

The procedure of trajectory planning and control of internal pressure of rubber gasbag of robotic gasbag polishing are analyzed as follows. If the information of free-form surface mould is known, the CAD model can be established directly. If not, just utilize surface measurement by contact probe or non-contact probe to digitize the surface, and then reconstruct the surface. Based on the selected tool path and considering the cutting tolerance and other factors, the cutting contact curve over the established surface is obtained. Then, generate the cutter location (CL) data which consists of a position vector and its normal vector from the cutting contact curve by the offset. The parameters of normal vector under the environment of CAD/CAM are different from the rotary angles of robot. So, the transformation matrix between the CL data and robot is needed. Also, transforming the polishing data in the work coordinate system to that in the robot coordinate system is necessary during the polishing trajectory planning. Moreover, a low-cost pressure control system adopting the control method of PWM is adopted for obtaining required control accuracy and stable contact characteristics. In order to get the path, position, orientation, feed rate and other parameters on a plane mould surface without the need of conventional teaching-operation mode, a trajectory generator is proposed as shown in Fig. 6.

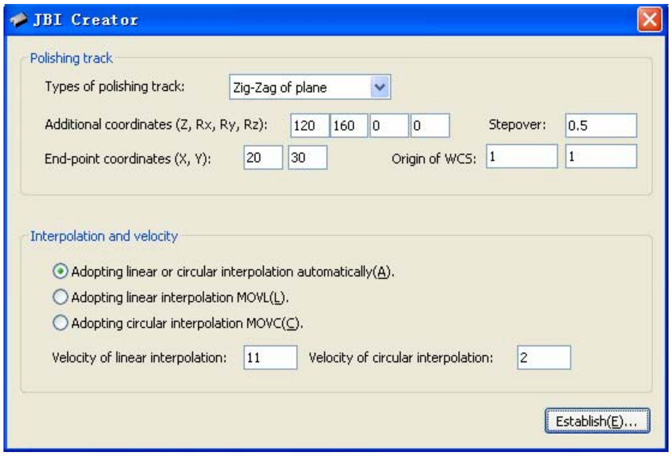



Figure 6. Track generator for robotic gasbag polishing

## IV. Experimental Research

The experimental research on concave mould is done based on robotic gasbag polishing system. The experiment conditions are listed as follows. The material of mould is die steel whose original surface roughness is Ra 0.425μm. The radius of gasbag is 10 and 20mm, the internal pressure of gasbag is 0.12 and 0.10 MPa, the rotating speed of polishing tool changes between 500–1500r/min, the feed rate is 120 and 60mm/min, the granularity value of abrasive powders varies from W3, W1.5 to W0.5, the downward depth alters from 0.3 to 1.0mm, and so on. The comparison of surface quality of unpolished and polished concave moulds is shown in Fig. 6. The surface qualities in three different stages shown in Fig. 5 are 41nm, 20nm and 5nm respectively, which are operated according to the analysis of material removal mechanism and optimal polishing process. So, it can be said that robotic gasbag polishing can achieve stable polishing process and surface roughness Ra 5nm.

a. unpolished concave mould    b. polished concave mould

Figure 7. Comparison of unpolishied and polished concave moulds

## V. Conclusion

The linear velocity and contact stress fields together with the distribution of material removal of robotic gasbag polishing in circular contact area between the flexible polishing tool and mould surface are calculated and simulated respectively according to certain internal pressure, downward depth, inclining angle, rotating speed and structural characteristics of rubber gasbag. Material removal model of robotic gasbag polishing is established on the basis of Preston equation and distributions of linear velocity and contact stress. The analysis of material removal mechanism is useful to optimize the polishing process, improve the efficiency of material removal and quality of mould surface, and so on. The procedure of trajectory planning and control of internal pressure of rubber gasbag of robotic gasbag polishing are analyzed in order to make the polishing process more stable and effectual. The theoretical analysis and experimental results indicate that the surface roughness Ra 5nm can be obtained.

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