Robust Stabilization of an Unmanned Motorcycle

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***Abstract*— Linearized equations of motion for a motorcycle with small roll angles are derived and used to design a robust cascade control scheme that stabilizes the motorcycle over a range of speeds. Stabilization is achieved by measuring the roll angle and its rate of change, and controlling the steering torque. The approach is validated via simulations and experiments performed with a radio-controlled scooter.**

***Keywords*—robust control, quantitative feedback theory (QFT), cascade control, remote operation**

I. INTRODUCTION

Motorcycles are inherently unstable systems subjected to non-holonomic contact constraints. Due to the non-holonomic constraints at the point of contact between the tire and the ground, in absence of slipping only longitudinal movement is feasible. These make the development of an autonomous or tele-operated motorcycle especially challenging and very few studies have been devoted to this topic. The vast majority of studies related to motorcycles dynamics focus on the stability of uncontrolled motorcycles and the vibration modes that result from external disturbances (e.g. side wind). Two modeling approaches have been used: analytical equations, which are linearized around small roll and steering angles, and multi-body representation. The latter typically results in a large number of state variables and is not suitable for controller design purposes. The former approach was initiated by Whipple [1], who was the first to present a fully general set of equations of motion for a bicycle. Dohring [2] presented a linear set of motion equations for a motorcycle based on the Newton-Euler equations. This approach was further investigated by Weir [3] who assumed that slip was present at the tire-ground point of contact. Weir further assumed that the forces at the point of contact are proportional to the slip angle. An alternative approach based on Lagrange equations was presented by Neimark and Fufaev [4] and further extended by Hand [5]. Other models have been developed over the years, and a thorough critical survey can be found in [5].

As noted before, few studies focused on motorcycle control, and each study used a custom model. Getz and Marsden [6] developed a control algorithm for a very simplified bicycle model, using the steering angle and forward velocity as control variables. The bicycle was modeled as a point mass, wheels inertia was neglected, and geometry was simplified in that steering head angle and trail were both assumed to be zero. Beznos et al. [7] used two fast spinning gyroscopes as actuators to keep a bicycle upright. Iuchi et al. [8] developed a stabilization controller for a bicycle traveling along a straight line at a predetermined constant speed. Finally, Yi et al. [9] presented 2006 a trajectory tracking and balancing control algorithm for an autonomous motorcycle. They used the bicycle model derived in [6] and extend it by including rake angle and trail. A controller based on steering angle and rear wheel torque was derived in order to track an arbitrary trajectory while maintaining stability, and the proposed control system was validated by numerical simulations.

In the present study, the linearized equations of motion for a motorcycle were developed in a thorough manner using a Newtonian approach (Section III), and a stabilizing robust controller was synthesized based on these equations (Section IV). The control scheme was validated through simulations (Section V) and implemented on a scooter operated via remote control (Section VI).

II. EXPERIMENTAL SYSTEM

A 50cc scooter with automatic variable transmission was retrofitted to be operated by remote control. A microchip PIC18F8520 microprocessor was installed on the motorcycle and received throttle, brake and roll angle commands via an R/C receiver. The roll angle was measured by a dynamic gyro-enhanced inclinometer (Microstrain FAS-G) and the roll rate was measured using a gyroscope (Silicon Sensing Systems, CRS02). Both analog signals were acquired by the microcontroller at a sampling rate of 10kHz and averaged at 50Hz. Every 20ms, the microcontroller, which was programmed in C language, issued pulse width modulated (PWM) commands to the throttle, brake and steering motors via appropriate H-bridges. Steering, on which this study focused, was controlled by a 24V DC motor with a 1:45 reduction transmission gear. Fig.1 shows the scooter and its physical properties are detailed in the Appendix.

Figure 1. Picture of the experimental system

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III. DYNAMIC MODEL

The following assumptions were made for deriving the equations of motion of a motorcycle:

- There exists enough friction between tires and road to prevent sliding.
- The change of forward velocity is quasi-static.
- The road is flat and horizontal.
- Only small deviations from straight ahead motion during which the motorcycle is vertical are considered.
- The tires are of negligible width.
- The motorcycle is symmetrical.

The motorcycle is viewed as consisting of two sub-assemblies (Fig. 2):

- The front sub-assembly which includes the front wheel, the front fork and the steering axis.
- The rear sub-assembly which consists of the rear wheel, the motorcycle's chassis and the engine.

Using the symbols defined in the Appendix, the lateral acceleration of whole motorcycle is:

$$F_{xf} + F_{xr} = m_t h_t \ddot{x} - m_t l_t \ddot{\theta} - m_f d \ddot{\psi} - m_t V \dot{\theta} \qquad (1)$$

In (1), the left-hand terms are the lateral forces exerted on the tires at the contact points by friction. The first right-hand side term is the contribution of roll angular acceleration of the motorcycle, the second term is the contribution of yaw angular acceleration of the motorcycle, the third term is centripetal acceleration of the motorcycle and the fourth term is the lateral acceleration of front assembly due to steering.

The roll equation is:

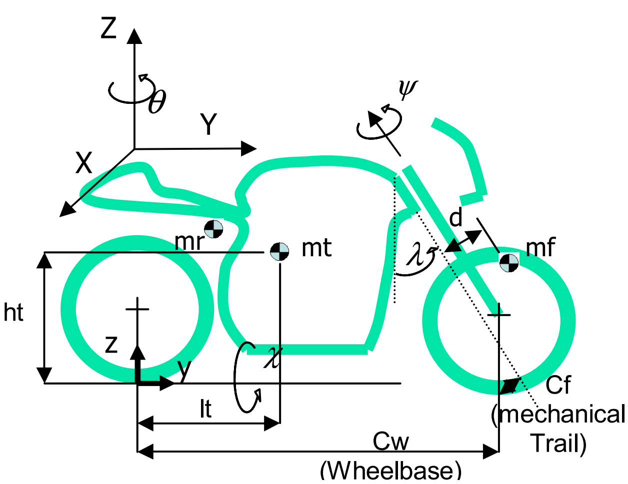

Figure 2. Schematic representation of the motorcycle and definition of the main variables used in the model

$$T_{yy}\ddot{\chi} + T_{yz}\ddot{\theta} + F'_{\lambda y}\ddot{\psi} - \left(I_{xx1}\frac{V}{r_f} + I_{xx2}\frac{V}{r_r}\right)\dot{\theta} - \left(I_{xx1}\frac{V}{r_f}\right)\cos\lambda\,\dot{\chi} - \qquad (2)$$

$$-m_t h_t V\dot{\theta} = m_t g h_t \chi - g m_f d\cdot\psi - c_f\left(\frac{m_t g l_t}{c_w}\right)\psi + M_\chi$$

In (2), the first and second left-hand side terms are the contributions of the time derivatives of the angular momentums of the whole motorcycle about the rear contact point. The third term is the contribution of the time derivative of angular momentum of the steering assembly about the steering axis. The fourth term is the gyroscopic moment due to the yaw angular velocity of the motorcycle and the fifth term is the gyroscopic moment due to the roll angular velocity of the front assembly. The sixth term is the moment required for lateral (centripetal) acceleration of the center of mass. The first right-hand side term is the moment exerted by the weight of the leaning motorcycle, the second term is the moment exerted by weight of the steered front assembly (being out of the symmetry plane of the motorcycle), the third term is the moment exerted by the normal force acting at the front contact point on the steered front assembly and the fourth term denotes external disturbances roll moment, such as for instance side-wind.

The yaw equation is:

$$T_{yz}\ddot{\chi} + T_{zz}\ddot{\theta} + F''_{\lambda z}\ddot{\psi} + \left(I_{xx1}\frac{V}{r_f} + I_{xx2}\frac{V}{r_r}\right)\dot{\chi} - \left(I_{xx1}\frac{V}{r_f}\right)\sin\lambda\,\dot{\psi} + \qquad (3)$$

$$+m_t l_t V\dot{\theta} = -C_w F_{xf}$$

In (3), the first and second left-hand side terms are the contributions of the time derivative of the angular momentums of the whole motorcycle about the rear contact point. The third term is the contribution of the time derivative of the angular momentum of the steering assembly about the steering axis. The fourth term is the gyroscopic moment due to the roll angular velocity of motorcycle and the fifth term is the gyroscopic moment due to the steering angular velocity of the front assembly. The sixth term is the moment required for the lateral (centripetal) acceleration of the center of mass. The right-hand side of (3) is the moment exerted by the lateral force at the front wheel contact point about the Z-axis.

The equation for the steering torque (front assembly) is:

$$F'_{\lambda y}\ddot{\chi} + F''_{\lambda z}\ddot{\theta} + F'_{\lambda\lambda}\ddot{\psi} + \left(I_{xx1}\frac{V}{r_f}\right)\cos\lambda\,\dot{\chi} + \left(I_{xx1}\frac{V}{r_f}\right)\sin\lambda\,\dot{\theta} + \qquad (4)$$

$$+m_f d\cdot V\dot{\theta} = M_\psi + C_f F_{xf} - g\upsilon\chi + g\sin\lambda\upsilon\psi$$

In (4), the first, second and third left-hand side terms are the contributions of the time derivatives of the angular momentum of the front assembly about the steering axis. The fourth and fifth terms are the gyroscopic moments due to the roll and yaw angular velocity of the front assembly, respectively. The sixth term is the moment required for lateral (centripetal) acceleration of the front center of mass. The first right-hand side term is the steering torque, the second term is the moment exerted by the front lateral force at the front wheel contact point, and the third and fourth terms are the moments exerted by the normal force and by the weight of the front assembly when out of the symmetry plane of the motorcycle.

Assuming no slip between the tires and the road yields a geometric constraint that can be used to eliminate θ as follows. From Figs. 3 and 4,

$$\theta = \left(C_f / C_w \right) \psi \tag{5}$$

and therefore:

$$\dot{\theta} = \left(C_f / C_w \right) \dot{\psi} . \tag{6}$$

Also,

$$\beta = \psi \cos(\lambda) \tag{7}$$

and

$$\tan(\beta) = C_w / R \tag{8}$$

where β is the effective steering angle. For small angles:

$$\beta = C_w / R \tag{9}$$

where R is the turning radius, and

$$V / R = \dot{\theta} \tag{10}$$

Combining (7), (9) and (10) yields:

$$\dot{\theta} = V \left(\cos\lambda / C_w \right) \psi . \tag{11}$$

Summing (6) and (11) yields:

$$\ddot{\theta} = \left(C_f / C_w \right) \ddot{\psi} + V \left(\cos\lambda / C_w \right) \dot{\psi} \tag{12}$$

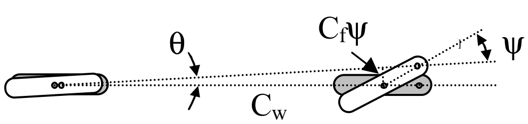

Figure 3. Yaw of rear assembly due to steering of front

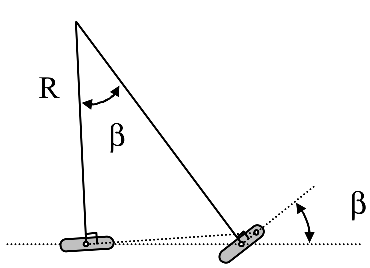

Figure 4. Turning motorcycle viewed from above

Finally, substituting (12) into (2) and (4) yields the linearized equations of motion that are used to design the controller:

Roll equation:

$$\begin{aligned} & T_{yy}\ddot{x} - m_t g h_t x + \left[F'_{\lambda y} + T_{yz}\frac{C_f}{C_w} \right]\ddot{\psi} + V\left[T_{yz}\frac{Cos\lambda}{C_w} - \frac{C_f}{C_w}\left(\frac{I_{xx1}}{r_f} + \frac{I_{xx2}}{r_r} \right) - \right. \\ & \left. -\frac{I_{xx1}}{r_f}Cos\lambda - m_t h_t \frac{C_f}{C_w} \right]\dot{\psi} + V^2\left[-\frac{Cos\lambda}{C_w}\left(\frac{I_{xx1}}{r_f} + \frac{I_{xx2}}{r_r} \right) - \right. \\ & \left. -m_t h_t \frac{Cos\lambda}{C_w} + g\nu \right]\psi = M_\chi \end{aligned} \tag{13}$$

Steering equation:

$$\begin{aligned} & \left[F'_{\lambda\lambda} + 2F''_z\frac{C_f}{C_w} + T_{zz}\frac{C_f^{\,2}}{C_w^{\,2}} \right]\ddot{\psi} + V\left[F''_{\lambda z}\frac{Cos\lambda}{C_w} + m_f d\cdot\frac{C_f}{C_w} + T_{zz}\frac{C_f}{C_w^{\,2}}Cos\lambda + m_t l_t\frac{C_f^{\,2}}{C_w^{\,2}} \right]\dot{\psi} + \\ & + V^2\left[\frac{I_{xx1}}{r_f}\frac{Cos\lambda}{C_w}Sin\lambda + m_f d\frac{Cos\lambda}{C_w} + m_t l_t\frac{C_f}{C_w^{\,2}}Cos\lambda \right]\psi - [g\nu Sin\lambda]\psi + \\ & + \left[F'_{\lambda y} + T_{yz}\frac{C_f}{C_w} \right]\ddot{x} + V\left[\frac{I_{xx1}}{r_f}Cos\lambda + \left(\frac{I_{xx1}}{r_f} + \frac{I_{xx2}}{r_r} \right)\frac{C_f}{C_w} \right]\dot{x} + g\nu x = M_\psi \end{aligned} \tag{14}$$

IV. CONTROLLER DESIGN

A feedback control loop was designed to stabilize the motorcycle for speeds ranging from 2.5 [m/s] to 6.5 [m/s]. All the physical properties of the motorcycle, such as masses, moments of inertia and locations of centers of masses were measured or determined experimentally (see Appendix). A 5-15% uncertainty was considered in eight of these parameters (see Appendix) and the quantitative feedback theory (QFT, e.g. [10]) approach was used to design a cascade controller as depicted in Fig. 5, using the software developed in [11]. Following the standard QFT procedure, design specifications for the outer loop were defined in time-domain and translated into frequency-domain specifications (Fig. 6). In addition, it was required that the sensitivity of both loops be smaller than 8 dB for all frequencies, which ensured stability. The feedback controller of the outer loop (G_1) was designed assuming that the inner loop would be sufficiently regulated so that it could be approximated temporarily as 1. A simple proportional gain that brought the bandwidth within the desired range was used (Fig. 7):

$$G_1 = 15 \tag{15}$$

After designing G_1, the tolerance and sensitivity bounds for the inner loops were computed (Fig. 8) and the following controller was found to meet the specifications:

$$G_2(s) = 35\frac{\left(1 + \dfrac{s}{3}\right)}{\left(1 + \dfrac{s}{20}\right)} \tag{16}$$

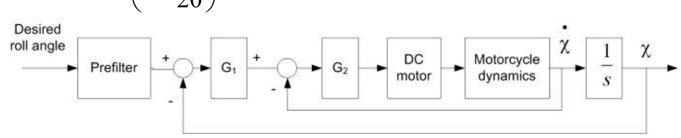

Figure 5. Cascade control loop

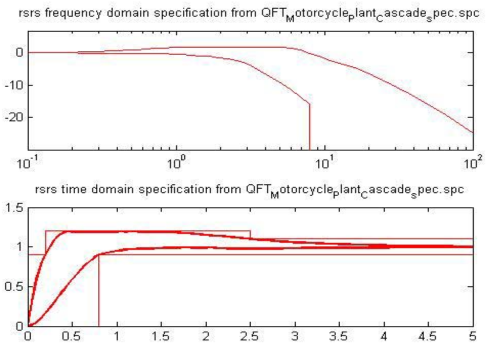

Figure 6. Design specifications in the time domain (bottom frame) and frequency domain (top frame)

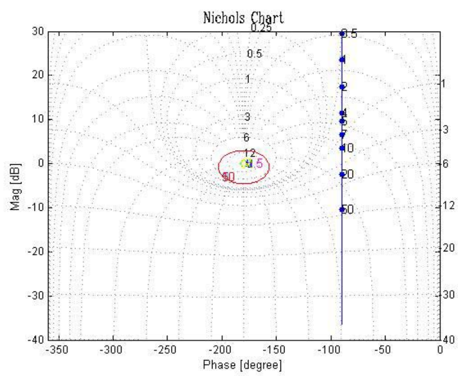

Figure 7. Outer loop sensitivity bounds and system controlled with G_1 under the temporary assumption that the inner controlled plant = 1.

Figure 9 shows the actual outer loop recalculated with the controller G_2. It can be seen that although the actual inner loop differed significantly from unity (as assumed at the first design stage, Fig. 7), the outer loop remained stable and met the specifications.

Finally, the following prefilter was added to adjust the system bandwidth according to the specifications (Fig. 10):

$$F(s) = 1.1\frac{1}{\left(1+\frac{s}{5}\right)^2} \tag{17}$$

Since the actual controllers were implemented on a digital microcontroller with a sampling rate of 50 Hz, G_2 and F were translated into their respective discrete-time forms using the matched zero-pole translation:

$$G_2(z) = 198.1\frac{(\text{z-0.94194})}{(\text{z - 0.67030})} \tag{18}$$

$$F(z) = \frac{0.009962}{(z^2\text{-1.81 z +0.8187})} \tag{19}$$

The loop-shaping design was checked with these discrete controllers and the loops were found to meet the design requirements (not shown).

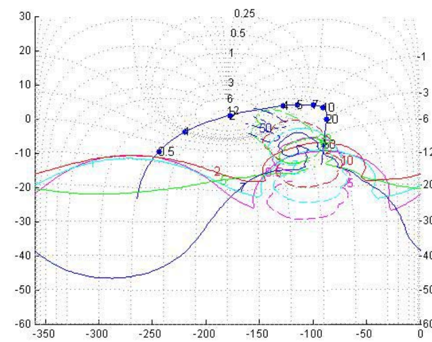

Figure 8. Inner loop sensitivity bounds and system controlled with G_1 and G_2

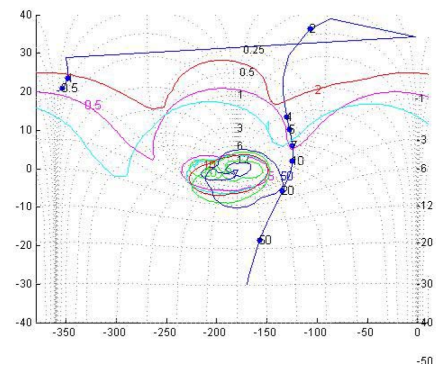

Figure 9. Outer loop sensitivity bounds and system controlled with G_1 and G_2

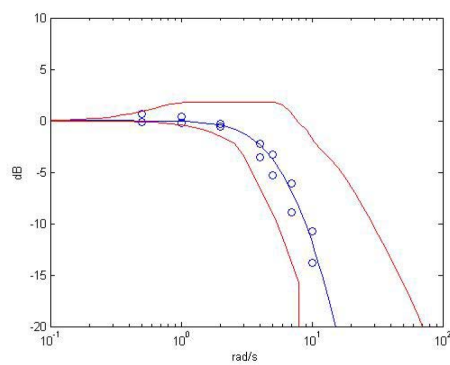

Figure 10. Specifications and closed loop with prefilter.

V. SIMULATION RESULTS

Before carrying out actual tests with the motorcycle, simulations based on (13) and (14) were conducted to check the ability of the controllers to stabilize the motorcycle over the desired range of speeds. The Matlab Simulink model is shown in Fig.11, and it can be seen that quantizers were included to account for the resolutions of the sensors and actuator. Typical results are presented in Figs 12 and 13 that show the roll angle and the steering angle in response to

- a 10 degrees step command at time 0
- a 10 Nm steering torque disturbance between t = 3 and t=3.1 seconds

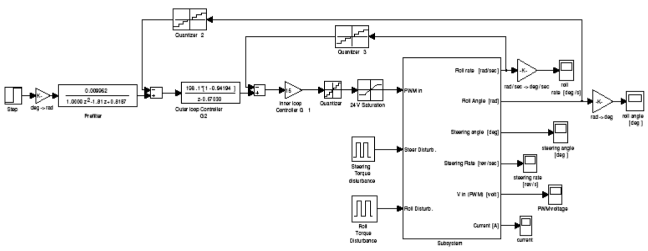

Figure 11. Simulilnk block diagram model

- a 100 Nm roll torque disturbance between t = 4 and t=4.1 seconds.

In Fig. 12 the various curves correspond to different parameter values and velocities. It can be seen that for all parameter values and velocities the motorcycle remains stable despite the strong disturbances.

Figure 13, which presents the steering angle for forward velocity of 4 [m/s], shows that the initial direction of steering is opposite to its steady state value, which is due to the non-minimum phase nature of the steering angle response to a desired roll angle and is known by motorcyclists as "counter steering".

It must be emphasized that although much faster controllers with higher bandwidths could be synthesized to yield better simulated performances, in practice it was observed that such controllers resulted in awakening un-modeled vibration modes of the motorcycle (due to chassis flexibility) and resonances due to nonlinearities in the loop (such as the backlash of the steering motor gear, minor as it may be). In addition, the controllers were intentionally kept as simple as possible since this allowed operation of the microcontroller in fixed-point mode, which ensured fast calculations. Attempts to implement more complex controllers that required much slower calculations performed in floating-point mode showed that the disadvantages of the slower computations outweighed the theoretical benefits of such complex controllers.

VI. Experiments

Experiments using the motorcycle described in Section II were conducted, and short video clips of these tests can be found at http://www.technion.ac.il/~linkerr/unmanned_motorcycle . These tests validated the ability of the controller to stabilize the system. However, due to the limited memory space available in the microcontroller, continuous recording of the roll angle and roll rate was not possible. Therefore, a data-logging device is being added for this purpose and detailed results will be presented in a future paper.

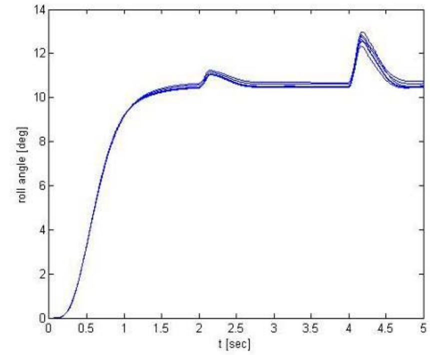

Figure 12. Simulated roll angle for various speeds and parameter values

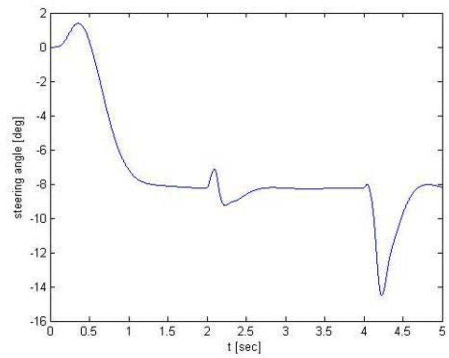

Figure 13. Simulated steering angle for speed of 4 [m/s]

VII. Conclusion

Linearized equations of motion for a motorcycle were derived, and the various parameters included in the model were estimated by simulation and/or CAD (Computer Aided Design) modeling. This model was used to design a robust cascade feedback controller that stabilizes the motorcycle for velocities ranging from 2 [m/s] to 6.5 [m/s]. After validation of the controller through simulation, the controller was implemented on an experimental radio-controlled scooter and the tests demonstrated the ability of the controller to stabilize the system.

Acknowledgment

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APPENDIX

Model parameters

Parameter	Description	Measured Value
C_w	Wheel base	1.08 [m]
C_f	Mechanical trail	86.53 [mm]
λ	Steering head angle	27°
r_r	Rear wheel radius	0.205 [m]
I_{xx1}	Front wheel inertia, about wheel axis	0.0431 [kgm^2]
l_r	Horizontal distance from rear contact point to rear COG	0.4 [m] ±15%
h_r	Vertical distance from rear contact point to rear center of mass	0.4096 [m] ±15%
m_r	Mass of rear assembly	105.3572 [m] ±15%
I_{yyr}	yy moment of inertia of rear assembly, about front COG, measured in rear coordinate system	4.7235 [kgm^2] ±15%
I_{zzr}	zz moment of inertia of rear assembly, about front COG, measured in rear coordinate system	11.2341 [kgm^2] ±15%
I_{yzr}	yz product of inertia of rear assembly, about front COG, measured in rear coordinate system	0.2204[kgm^2]
m_f	Mass of front assembly	13.9242 [kg] ±5%
l_f	Horizontal distance from front COG to front tire contact point	-0.1073 [m]
h_f	Height of front COG	0.4110 [m]
I_{xxf}	xx moment of inertia of front assembly, about front COG, measured in front coordinate system	1.1421 [kgm^2]
I_{yyf}	yy moment of inertia of front assembly, about front COG, measured in front coordinate system	1.1467 [kgm^2] ±15%
I_{zzf}	zz polar moment of inertia of front assembly, about front COG, measured in front coordinate system	0.0530 [kgm^2] ±15%
I_{yzf}	yz product of inertia of front assembly, about front COG, measured in Front coordinate sys.	-0.0120 [kgm^2]
R_f	Front wheel radius	0.205 [m]
I_{xx2}	Polar mass moment of inertia of rear wheel, about wheel axis	0.0431 [kgm^2]
ψ	Steering angle	
χ	Roll angle	
θ	Heading angle of motorcycle	
d	Distance of front COG from steering axis	4.5 [mm]

Constitutional relationships

$$F'_{yy} = I_{yy_f}\cos^2\lambda + I_{yz_f}\sin 2\lambda + I_{zz_f}\sin^2\lambda$$

$$F'_{yz} = I_{yy_f}\sin\lambda\cos\lambda - I_{yz_f}\cos 2\lambda - I_{zz_f}\sin\lambda\cos\lambda$$

$$F'_{zz} = I_{yy_f}\sin^2\lambda - I_{yz_f}\sin 2\lambda + I_{zz_f}\cos^2\lambda$$

$$m_t = m_r + m_f$$

$$h_t = \frac{m_r h_r + m_f h_f}{m_t}$$

$$l_t = \frac{m_r l_r + m_f l_f}{m_t}$$

$$T_{yy} = I_{yy_r} + m_r h_r^2 + F'_{yy} + m_f h_f^2$$

$$T_{zz} = I_{zz_r} + m_r l_r^2 + F'_{zz} + m_f\left(C_w + l_f\right)^2$$

$$T_{zz} = I_{zz_r} + m_r l_r^2 + F'_{zz} + m_f\left(C_w + l_f\right)^2$$

$$F'_{\lambda y} = -I_{yz_f}\cos\lambda - I_{zz_f}\sin\lambda - m_f h_f d$$

$$F''_{\lambda z} = -I_{yz_f}\sin\lambda + I_{zz_f}\cos\lambda + m_f\left(C_w + l_f\right)d$$

$$F'_{\lambda\lambda} = I_{zz_f} + m_f d^2$$

$$\nu = m_f d + m_t\frac{l_t}{C_w}C_f$$