Modified Dislocated Feedback Synchronization of Unified Chaotic System

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***Abstract*—An modified dislocated feedback method is proposed to acquire the synchronization of the whole unified chaotic system. This method is based on the Lyapunov stability theory, and overcome the limitation of the original dislocated feedback method. Numerical simulations are also provided to show the effectiveness of the method.**

***Index Terms*—unified chaotic system, Lyapunov stability theory, dislocated feedback, synchronization**

I. Introduction

Since the pioneering work of Pecora et al. in 1990[1], chaos synchronization has recently attracted great attentions, and a great deal of efforts have been devoted to its study. Previously, many different techniques and methods have been proposed for achieving chaos synchronization, such as, feedback synchronization[2,3], adaptive synchronization[4,5,6], generalized synchronization[7,8], impulsive synchronization[9,10], etc.

In 1963, Lorenz found the first classical chaotic attractor[11]. In 1999, Chen and Ueta found another chaotic attractor[12], which is similar, but not topologically equivalent to the Lorenz attractor. In 2002, L$\ddot{u}$ and Chen also found the critical attractor between the Lorenz and Chen attractor[13]. To bridge the gap between the Lorenz attractor and Chen attractor, L$\ddot{u}$ et al. presented a unified chaotic system[14]. It includes the Lorenz and Chen systems as two extremes, respectively, and L$\ddot{u}$ system as a transition system[15].

Recently, Tao[16] investigated the dislocated feedback method, which has been used to control the Lorenz system[17,18] and the unified system[19], and has also realized the synchronization of the Lorenz system[16]. However, we found that it can't realize the synchronization of the whole unified chaotic system according to the original dislocated feedback method, can only do the part when $\alpha \leq 1/29$. In this paper, according to modify the control input $u_i, i = 1, 2$, an modified dislocated feedback method is proposed to acquire the synchronization of the whole unified chaotic system. This method is based on the Lyapunov stability theory, and overcome the limitation of the original dislocated feedback method.

The rest of this paper is organized as follows. Section II describes the unified chaotic system. A modified dislocated feedback synchronization is proposed in Section III. Numerical simulations are provided in Section IV to show the effectiveness of the proposed method. Section IV gives some conclusions and discussions.

II. The unified chaotic system

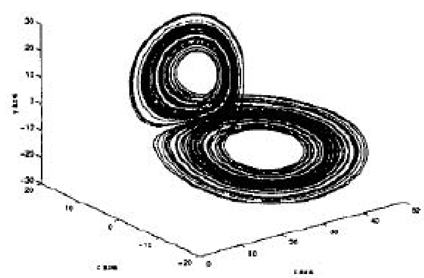

Fig. 1. Lorenz chaotic attractor

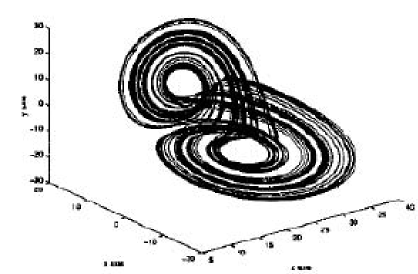

Fig. 2. Lü chaotic attractor

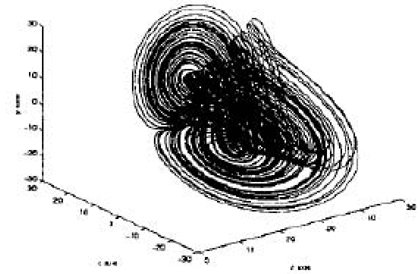

Fig. 3. Chen chaotic attractor

The unified chaotic system can be described by the following differential equations:

$$\left\{\begin{array}{l} \dot{x}=(25\alpha+10)(y-x)\\ \dot{y}=(28-35\alpha)x-xz+(29\alpha-1)y\\ \dot{z}=xy-\frac{8+\alpha}{3}z, \end{array}\right. \quad (1)$$

where $\alpha\in[0,1]$, which contains the canonical Lorenz system and Chen system as two extremes and Lü system as a special case. What is interesting is that, as the parameter α changes continuously from 0 to 1, the resulting system remains continuously to be chaotic. Obviously, when $\alpha=0$, $\alpha=0.8$, and $\alpha=1$, it is the Lorenz chaotic attractor, Lü chaotic attractor and Chen chaotic attractor, as shown in Figs. 1, 2, 3, respectively.

III. Modified dislocated feedback synchronization of the unified chaotic system

In Ref.[16], the dislocated feedback method was given to synchronize the Lorenz chaotic system. However, as this method is carried on the unified chaotic system, we found that it can not synchronize the whole unified system, can only do the part when $\alpha\le 1/29$. In this section, we will synchronize the whole unified chaotic system through a modified dislocated feedback method.

A. the synchronization of the unified chaotic system according to the original dislocated feedback method

Let the system (1) is the drive system, then the response system is described by

$$\left\{\begin{array}{l} \dot{x}_1=(25\alpha+10)(y_1-x_1)+u_1\\ \dot{y}_1=(28-35\alpha)x_1-x_1z_1+(29\alpha-1)y_1+u_2\\ \dot{z}_1=x_1y_1-\frac{8+\alpha}{3}z_1, \end{array}\right. \quad (2)$$

where u_1, u_2 are the dislocated feedback gains.

According to the original method of dislocated feedback synchronization which was given in Ref.[16], let the dislocated feedback gains $u_1=0$, $u_2=-k_2(x_1-x)$, respectively, then the response system can be described by:

$$\left\{\begin{array}{l} \dot{x}_1=(25\alpha+10)(y_1-x_1)\\ \dot{y}_1=(28-35\alpha)x_1-x_1z_1+(29\alpha-1)y_1-k_2(x_1-x)\\ \dot{z}_1=x_1y_1-\frac{8+\alpha}{3}z_1. \end{array}\right. \quad (3)$$

Let the system errors be $e_1=x_1-x$, $e_2=y_1-y$, $e_3=z_1-z$. Thus, the error system is given by:

$$\left\{\begin{array}{l} \dot{e}_1=(25\alpha+10)(e_2-e_1)\\ \dot{e}_2=(28-35\alpha)e_1-xe_3-ze_1-e_1e_3\\ \qquad +(29\alpha-1)e_2-k_2e_1\\ \dot{e}_3=xe_2+ye_1-\frac{8+\alpha}{3}e_3+e_1e_2. \end{array}\right. \quad (4)$$

Consider the Lyapunov candidate function

$$V=\frac{1}{2}(\frac{1}{\beta}\dot{e_1}^2+\dot{e_2}^2+\dot{e_3}^2),$$

where $\beta>0$. Then we have

$$\begin{aligned}\dot{V}&=\tfrac{1}{\beta}e_1\dot{e_1}+e_2\dot{e_2}+e_3\dot{e_3}\\ &=-\tfrac{25\alpha+10}{\beta}e_1^2+[\tfrac{1}{\beta}(25\alpha+10)-z+(28-35\alpha)\\ &\quad -k_2]e_1e_2+(29\alpha-1)e_2^2+ye_1e_3-\tfrac{8+\alpha}{3}e_3^2\\ &\le-\tfrac{25\alpha+10}{\beta}e_1^2+[\tfrac{1}{\beta}(25\alpha+10)+M_3+(28-35\alpha)\\ &\quad -k_2]e_1e_2+(29\alpha-1)e_2^2+ye_1e_3-\tfrac{8+\alpha}{3}e_3^2\\ &=-\tfrac{25\alpha+10}{\beta}e_1^2+(\sigma-k_2)e_1e_2+(29\alpha-1)e_2^2\\ &\quad +ye_1e_3-\tfrac{8+\alpha}{3}e_3^2,\end{aligned}$$

where $\sigma=\frac{25\alpha+10}{\beta}+M_3+28-35\alpha$,
when $k_2\le\sigma$, we have,

$$\begin{aligned}\dot{V}&\le-\tfrac{25\alpha+10}{\beta}e_1^2+(\sigma-k_2)|e_1e_2|+(29\alpha-1)e_2^2\\ &\quad +M_2|e_1e_3|-\tfrac{8+\alpha}{3}e_3^2\\ &=-|e|^TP|e|,\end{aligned} \quad (5)$$

where M_2 and M_3 are the boundaries satisfying $|y|,|y_1|\le M_2$, $|z|,|z_1|\le M_3$, $|e|=(|e_1|,|e_2|,|e_3|)^T$ and

$$P=\left(\begin{array}{ccc} \frac{25\alpha+10}{\beta} & \frac{k_2-\sigma}{2} & -\frac{M_2}{2}\\ \frac{k_2-\sigma}{2} & 1-29\alpha & 0\\ -\frac{M_2}{2} & 0 & \frac{8+\alpha}{3} \end{array}\right).$$

To make the origin of error system(4) be asymptotically stable, the symmetrical matrix P should be positive definite. That is,

$$\left\{\begin{array}{l} \frac{25\alpha+10}{\beta}>0\\ \frac{25\alpha+10}{\beta}(1-29\alpha)>\frac{(k_2-\sigma)^2}{4}\\ (\frac{(8+\alpha)(25\alpha+10)}{3\beta}-\frac{M_2^2}{4})(1-29\alpha)-\frac{(8+\alpha)(k_2-\sigma)^2}{12}>0. \end{array}\right. \quad (6)$$

From the second inequality of (6), we can notice that, only when $\alpha\le\frac{1}{29}$, this formula is reasonable. So it is limited to synchronize the whole unified system with original dislocated feedback method. In the next section, we will modify the dislocated feedback method to synchronize the whole unified chaotic system.

B. Modified dislocated feedback synchronization of the unified chaotic system

In this part, according to modify the control inputs $u_i, i=1,2$, the modified dislocated feedback method is proposed to realize the synchronization of the whole unified chaotic system.

Theorem 1. Let $u_1=0$, $u_2=-k_2(x_1-x)-29(y_1-y)$. Then the new response system (7) and the drive system (1) realize synchronization when $\sigma-\sqrt{(\frac{4(25\alpha+10)}{\beta}-\frac{3}{8+\alpha}M_2^2)(30-29\alpha)}<k_2\le\sigma$.

Proof. Let $u_1=0$, $u_2=-k_2(x_1-x)-29(y_1-y)$, then we have the new response system as follows:

$$\left\{\begin{array}{l} \dot{x}_1=(25\alpha+10)(y_1-x_1)\\ \dot{y}_1=(28-35\alpha)x_1-x_1z_1+(29\alpha-1)y_1\\ \qquad -k_2(x_1-x)-29(y_1-y)\\ \dot{z}_1=x_1y_1-\frac{8+\alpha}{3}z_1. \end{array}\right. \quad (7)$$

Let the system errors be $e_1 = x_1 - x$, $e_2 = y_1 - y$, $e_3 = z_1 - z$. Thus, the error system is given by

$$\begin{cases} \dot{e}_1 = (25\alpha + 10)(e_2 - e_1) \\ \dot{e}_2 = (28 - 35\alpha)e_1 - xe_3 - ze_1 - e_1e_3 \\ \qquad +(29\alpha - 30)e_2 - k_2e_1 \\ \dot{e}_3 = xe_2 + ye_1 - \frac{8+\alpha}{3}e_3 + e_1e_2. \end{cases} \tag{8}$$

Consider the Lyapunov candidate function

$$V = \frac{1}{2}(\frac{1}{\beta}\dot{e_1}^2 + \dot{e_2}^2 + \dot{e_3}^2),$$

where $\beta > 0$.Then we have

$$\begin{aligned} \dot{V} &= \tfrac{1}{\beta}e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} \\ &= -\tfrac{25\alpha+10}{\beta}e_1^2 + [\tfrac{1}{\beta}(25\alpha + 10) - z + (28 - 35\alpha) \\ &\quad -k_2]e_1e_2 + (29\alpha - 30)e_2^2 + ye_1e_3 - \tfrac{8+\alpha}{3}e_3^2 \\ &\le -\tfrac{25\alpha+10}{\beta}e_1^2 + (\sigma - k_2)e_1e_2 + (29\alpha - 30)e_2^2 \\ &\quad +ye_1e_3 - \tfrac{8+\alpha}{3}e_3^2, \end{aligned}$$

when $k_2 \le \sigma$, we have,

$$\begin{aligned} \dot{V} &\le -\tfrac{25\alpha+10}{\beta}e_1^2 + (\sigma - k_2)|e_1e_2| + (29\alpha - 30)e_2^2 \\ &\quad +M_2|e_1e_3| - \tfrac{8+\alpha}{3}e_3^2 \\ &= -|e|^TP|e|, \end{aligned}$$

where $|e| = (|e_1|, |e_2|, |e_3|)^T$ and

$$P = \begin{pmatrix} \frac{25\alpha+10}{\beta} & \frac{k_2-\sigma}{2} & -\frac{M_2}{2} \\ \frac{k_2-\sigma}{2} & 30 - 29\alpha & 0 \\ -\frac{M_2}{2} & 0 & \frac{8+\alpha}{3} \end{pmatrix}.$$

To make the origin of error system (8) be asymptotically stable, the symmetrical matrix P should be positive-definite. That is,

$$\begin{cases} \frac{25\alpha+10}{\beta} > 0 \\ \frac{25\alpha+10}{\beta}(30 - 29\alpha) > \frac{(k_2-\sigma)^2}{4} \\ (\frac{(8+\alpha)(25\alpha+10)}{3\beta} - \frac{M_2^2}{4})(30 - 29\alpha) - \frac{(8+\alpha)(k_2-\sigma)^2}{12} > 0. \end{cases} \tag{9}$$

It can also be written by,

$$\begin{cases} \frac{25\alpha+10}{\beta} > 0 \\ \frac{(k_2-\sigma)^2}{4} < \frac{25\alpha+10}{\beta}(30 - 29\alpha) \\ \frac{(k_2-\sigma)^2}{4} < (\frac{25\alpha+10}{\beta} - \frac{3M_2^2}{4(8+\alpha)})(30 - 29\alpha). \end{cases} \tag{10}$$

Then, we have

$$\begin{aligned} &\tfrac{25\alpha+10}{\beta}(30 - 29\alpha) - (\tfrac{25\alpha+10}{\beta} - \tfrac{3M_2^2}{4(8+\alpha)})(30 - 29\alpha) \\ &= (30 - 29\alpha)(\tfrac{25\alpha+10}{\beta} - \tfrac{25\alpha+10}{\beta} + \tfrac{3M_2^2}{4(8+\alpha)}) \\ &= (30 - 29\alpha)\tfrac{3M_2^2}{4(8+\alpha)}, \end{aligned}$$

from $0 < \alpha < 1$, we have

$$(30 - 29\alpha)\frac{3M_2^2}{4(8 + \alpha)} > 0,$$

and then

$$\frac{25\alpha + 10}{\beta}(30-29\alpha)-(\frac{25\alpha + 10}{\beta}-\frac{3M_2^2}{4(8 + \alpha)})(30-29\alpha) > 0.$$

Thus,

$$\frac{25\alpha + 10}{\beta}(30 - 29\alpha) > (\frac{25\alpha + 10}{\beta} - \frac{3M_2^2}{4(8 + \alpha)})(30 - 29\alpha).$$

Then, the second inequality of (10) can be deleted, we can only consider the third one of it:

$$\frac{(k_2 - \sigma)^2}{4} < (\frac{25\alpha + 10}{\beta} - \frac{3M_2^2}{4(8 + \alpha)})(30 - 29\alpha).$$

Obviously, we can get

$$|k_2 - \sigma| < \sqrt{(\frac{4(25\alpha + 10)}{\beta} - \frac{3}{8 + \alpha}M_2^2)(30 - 29\alpha)},$$

again

$$k_2 \le \sigma,$$

in the end, we have

$$\sigma - \sqrt{(\frac{4(25\alpha + 10)}{\beta} - \frac{3}{8 + \alpha}M_2^2)(30 - 29\alpha)} < k_2 \le \sigma.$$

Hence, the matrix P is positive-definite as $\sigma - \sqrt{(\frac{4(25\alpha+10)}{\beta} - \frac{3}{8+\alpha}M_2^2)(30 - 29\alpha)} < k_2 \le \sigma$, $0 < \beta < \frac{4(25\alpha+10)(8+\alpha)}{3M_2^2}$, M_2 and M_3 are the boundaries satisfying $|y|, |y_1| \le M_2$, $|Z|, |Z_1| \le M_3$.

According to the Lyapunov theorem, $e_1, e_2, e_3 \to 0$ for $t \to \infty$. Therefore, the response system (7) and the drive system (1) will realize synchronization.

IV. A NUMERICAL EXAMPLE

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation results for Lorenz, L$\ddot{u}$ and chen system. In simulation experiments, values of the parameter k is chosen for 10.

Case I: Lorenz system. When $\alpha = 0$, Eqs.(1) is Lorenz system. The simulation results are shown in Fig.4-5. It can be seen that the synchronization errors converge to zero rapidly.

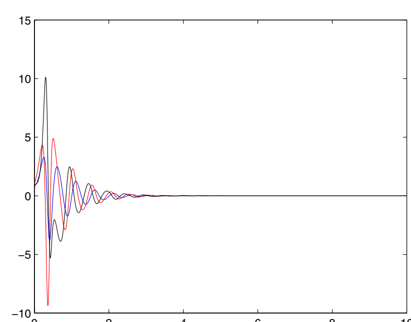

Fig. 4. the synchronous error between systems (1) and (7) with $k_2 = 10$ and $\alpha = 0$

Case II: L$\ddot{u}$ system. When $\alpha = 0.8$, Eqs.(1) is L$\ddot{u}$ system. The simulation results are shown in Fig.6-7. It can be seen that the synchronization errors converge to zero rapidly too.

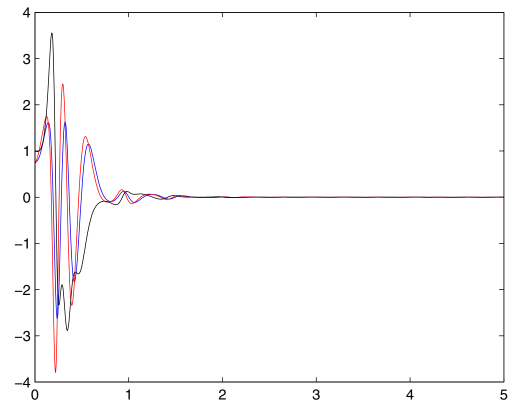

Fig. 5. the synchronous error between systems (1) and (7) with $k_2 = 10$ and $\alpha = 0.8$

Case III: Chen system. When $\alpha = 1$, Eqs.(1) is Chen system. Under the same simulation condition of Case I and Case II, the simulation results are given in Fig.8-9. It can also be seen that the synchronization errors converge to zero rapidly.

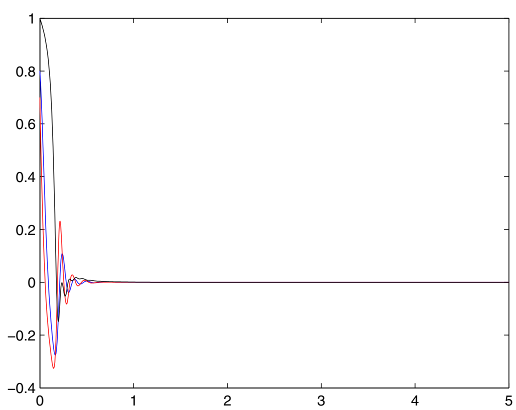

Fig. 6. the synchronous error between systems (1) and (7) with $k_2 = 10$ and $\alpha = 1$

V. Conclusion

This letter modified the dislocated feedback method to synchronize the unified system. And the fundamental synchronous criteria is given for the modified dislocated feedback synchronization. Moreover, the numerical simulations are given to show the effectiveness of the criteria.

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