Kinematics Control of Wheeled Robot Based on Angular Rate Sensors

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Abstract—To improve the navigation precision of the ground wheeled robot, implement the autonomous kinematics motion control, and overcome the yawing problem when only using an optical encoder, a small-sized, low cost and reusable independent angular rate sensor (gyroscope) module based on iMEMS technique is used. The kinematics model of the robot is explained, and the principle that a robot can move straight based on a gyroscope is proposed. The angular rate is sampled to get the yawing angle of the robot and then it is inputted into a digital PID controller with a dead zone. The incremental speeds of each of the two driving wheels are calculated to attenuate the deviation. Methods for the robot to turn a fixed angle or move along a polygon are also presented. Experiments show that these algorithms have effectively improved the performance the robot moves rather than using time-consuming algorithms or installing bulky and power cost computers.

Keywords—wheeled robot, iMEMS, angular rate sensor (gyroscope), kinematics control, digital PID control

I. INTRODUCTION

The mobile robot is one of most popular topics in robot field these years. We can classify them into several types according to how robots walk, mainly including wheeled robots, the track mounted robots, the legged robots and the snake-shaped robots, and so on [1]. The wheeled robot can move smoothly on the flat surface, but it can not move through complex landscape. The track-mounted robot may cross barriers, stairs and so on, but friction between the caterpillar band and the ground is big, which increases the energy consumption. The design of legged robot and the snake-shaped robot adopts the bionics method, imitating the multiped or snake to move. These two robots have strong obstacle crossing ability. However, it is a bit complex to control. Among these robots, the wheeled robots are widely applied, because of their low energy consumption and simple mechanism. Therefore, we chose it as our research object.

On the wheeled robot, several kinds of sensors are usually used to obtain the orientation or position, or called the localization. These localization approaches mainly relate to indoor, outside, dynamic and static environments [2]. The commonly used sensors include the inertial ones, the magnetism ones, the laser ones and the visual ones [3]. A magnetic sensor can get the direction the robot moves; a gyroscope can detect the angular speed or displacement of a robot; a camera can track a featured object; a GPS (Global Position System) receiver can get the longitude and latitude in every certain time. Nevertheless, using only one type of these sensors may have the defects as follows: in inertial localization system, information coming from gyroscope will drift along when time passes and the error is accumulated. Magnetic sensors are likely to be influenced by the magnetic field and they will mislead the robot towards the wrong direction. The long time used to process image data limits the using of the visual sensors because of the transmission rate and execution time of a certain algorithm, so that the real time performance is in a degree difficult to assure. The GPS signal cannot be received in an indoor environment or will be affected by trees or buildings.

Thereby, it is of great help to integrate the merits of the sensors when multiple sensors are combined. Reference [4] introduces a method to keep the direction using a gyroscope and a CCD camera. Reference [5] proposes combining the inertial navigation as the foundation and the magnetic sensor to make a revision. Reference [6] presented method to combine state estimation via extended Kalman filter from gyro and acceleration sensors and wheel angular encoders’ measurements.

But it is necessary to apply a fast-computing processor to run the relatively complex and time-consuming algorithms or to process images. In [4], the author uses an industrial computer to process the images and a DSP (Digital Signal Processor) to control the motor. In [5], Kalman filtering calculation is also required on an industrial computer. Reference [6] uses a 600 MHz DSP to execute predictive control algorithm and navigation system. All of them need high cost and heavy energy consumption. And it is inconvenient to lessen the size of the robot further when the industrial computer is mounted.

In our wheeled robot, namely Voyager II, we adopt iMEMS (Integrated Micro Electromechanical System) technique based gyroscopes, and incorporate an NXP ARM7 microcontroller, an A/D converter and a serial interface to form a motion control module. Hence, both the size and power consumption are greatly decreased. If any part of the module is broken, it could be substituted with another one immediately rather than discard the entire control board. This module thus can be also applied to other mobile robots.
We install a brushless DC motor controller to drive the wheels, and equip optical encoders to get the speed and direction. During the period it runs, the attitude variation is sampled and inputted into a PID controller. Thus, speeds of the wheels are corrected, and the precision of the locomotion is therefore enhanced.

Similarly, in [7], a sensor fusion method is also used on a wheeled robot compared with an encoder-only way, but the error is still distinct. In our work, we lessen the relative error to 0.6%.

The structure of this article is as follows: in the second part, we introduce the kinematics model of wheeled robot based on techniques used to modulate speed differences between two wheels; in the third part, we propose an algorithm to revise the robot direction and let it move along a straight line or a polygon; in the fourth part, we compare the results of robot moving along a straight line between the robot with angular rate sensor and without; finally, we sum up the whole article, and put forward the future work.

II. KINEMATICS MODELLING

Fig. 1 is the sketch of mobile robot. It has four wheels. The two front wheels are driving wheels, which are equipped with two brushless DC motors; two back wheels are versatile wheels, which are just used to support weight.

Suppose the robot body is rigid, the position of any point on the robot axis of symmetry is represented by \( p(x, y) \). We choose a point on this line, which is denoted as “S”, away from the axis of two front wheels. Velocity of this point can be described as:

\[
\begin{bmatrix}
    v \\
    \omega
\end{bmatrix} = \begin{bmatrix}
    \cos \theta & -S \sin \theta \\
    \sin \theta & S \cos \theta
\end{bmatrix} \begin{bmatrix}
    v \\
    \omega
\end{bmatrix}
\]

(1)

Where, \( v \) represents the linear velocity along the direction of symmetry axis, \( \omega \) is the rotating velocity of the vehicle in the world coordinate system.

The relationship between the linear velocity and rotating velocity can be described as:

\[
\begin{bmatrix}
    v \\
    \omega
\end{bmatrix} = \begin{bmatrix}
    1/2 & 1/2 \\
    1/S & -1/S
\end{bmatrix} \begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\]

(2)

Where, \( u_1 \) and \( u_2 \) are respectively linear velocity of left and right wheel.

From (1) and (2), we can construct the equation of moving state and linear velocities of two wheels. Then we can work out the position of point “P” in the world coordinate system by integrating \( q \) [8].

When the robot moves along a line, \( \omega \) equals to zero, so we can just insure \( u_1 = u_2 \). If the speed of left wheel is greater than the right wheel, the robot will turn right; and vice versa. So this method is also called the differential speed mode drive.

The motors applied to driving wheels adopt closed loop control. Theoretically, the speed of the motor is a constant value. While, in fact, the diameter of two wheels are not perfectly equal. Besides, the loads upon two wheels are not the same, so the distortion is different. Moreover, the friction between wheels and ground leads to slight fluctuation. Lastly, the wheels may meet obstacles or slips. All these causes are sorted as systematic errors or non-systematic errors in [9].

Based on the four reasons above, speeds of the two wheels cannot keep the same if we want the robot move straightly.

III. KINEMATICS CONTROL ALGORITHM

In this part, we will detailed discuss the algorithm to let the robot move straight. And this approach can be also applied when it is ordered to turn a fixed angle or move around a polygon with some modifications.

A. Linear Move

According to (2), results can be also obtained as below:

\[
\begin{bmatrix}
    v \\
    \omega
\end{bmatrix} = \begin{bmatrix}
    1/2 & 1/2 \\
    1/S & -1/S
\end{bmatrix} \begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\]

(3)

\[
\omega = \frac{u_1 - u_2}{S}
\]

(4)

Assume \( \Delta u = u_1 - v = v - u_2 \)

(5)

From (3) to (5), we have:

\[
\Delta u = \frac{\alpha S}{2}
\]

(6)

\[
\begin{cases}
    u_1 = v + \Delta u \\
    u_2 = v - \Delta u
\end{cases}
\]

(7)

If we now get the angle difference between the original direction and that in the fact, we can figure out the real velocities of two wheels by the actual linear speed \( v \) and (6) and (7).

According to the formula below:

\[
\begin{cases}
    u_1 = v + \Delta u \\
    u_2 = v - \Delta u
\end{cases}
\]

(8)

We can control the speed of two wheels and make up the offset, then the robot will move as we ordered.

In order to gain \( \omega \), we put an angular rate sensor (gyroscope) on the axle of the robot.

A gyroscope is one kind of sensor used to measure rotational speed and angle. It can collect the rotational speed,
work out the change of attitude and then by the integral calculation it can figure out the current direction angle of robot. Hereby, the algorithm to compensate the heading angle of robot is proposed.

First, when the robot is immobile, in other words, the rotating speed is zero, we calibrate the state of gyroscope and record its value. Then we let the robot run towards an appointed orientation. At the same time, the gyroscope collects the angular speed $\omega$ every 100 microsecond (uSec.). The current angular rate value $\omega$ can be worked out when calculating the arithmetic mean value divided by 100. The sum of rotational speed, $\theta \Delta$, is the yawing offset which is also the input of PID controller. The output of the controller, $\Delta u$, is the speeds increments between the two wheels and the speed of their middle point. Finally, the result will be figure out by (8) and sent to the motor driver by a serial port. The algorithm flow chart is as Fig.2 [10].

$$\omega = \sum_{i=1}^{\infty} \alpha_i / 100$$

$$\Delta \theta = \sum \omega$$

To avoid modulating too frequently, the control algorithm uses the incremental digital PID controller with a dead zone $\varepsilon$, range of which should be determined by the results of repetitious experiments [11]. The formula of incremental digital PID controller to output $\Delta u(k)$, which represents the increment of the differential speeds, is as follows:

$$\Delta u_k = u_k - u_{k-1} = K_p \left( e_k - e_{k-1} \right) + \frac{T_i}{T} e_k + \frac{T_D}{T} \left( e_k - 2e_{k-1} + e_{k-2} \right)$$

$$= K_p \left( 1 + \frac{T_i}{T} + \frac{T_D}{T} \right) e_k - K_p \left( 2 - \frac{T_i}{T} \right) e_{k-1} + K_p \frac{T_D}{T} e_{k-2}$$

Let:

$$A = K_p \left( 1 + \frac{T_i}{T} + \frac{T_D}{T} \right)$$

$$B = K_p \left( 2 - \frac{T_i}{T} \right)$$

$$C = K_p \frac{T_D}{T}$$

Such that

$$\Delta u_k = Ae_k - Be_{k-1} + Ce_{k-2}$$

The flow chart is as Fig.3.

On initialization, we assume

$$e(k-1) = e(k-2) = 0$$

Figure 2. Flow chart of kinematics control algorithm

$$e(k) = \Delta \theta - 0 = \Delta \theta$$

(13)

The increment output is determined by (11) when the error is beyond the dead zone (i.e. $|e(k)| \geq \varepsilon$), of which, $A$, $B$ and $C$ are determined using the trial and error method [12].

The differential speed is got through

$$\Delta u = u(k) + \Delta u(k)$$

(14)

Where, $u(k)$ is the result calculated in the last servo period. While the error is within the dead zone, the PID control is not necessary.

Finally, update the variables as follows:

$$e(k-1) \rightarrow e(k-2)$$

$$e(k) \rightarrow e(k-1)$$

(15)

(16)

This is for the next servo period.

B. Fixed-Angle Move and Polygon Move

When we appoint a fixed angle, similar calculations are performed as above, while the reference value of each PID servo period is modified. Thus the robot will gradually tune its speeds of both wheels so that the appointed angle is achieved.

The polygon move is consisted with several linear moves and fixed-angle moves as described above.

However, the robot cannot move along a polygon using the same method since there is the accumulated error during each basic move. And the error is gradually increasing when it moves longer. When it runs to the end point, the polygon is not closed with a few gaps and distortions.

In order to close the polygon, we use a global variable to record the error between the appointed angle and the actual angle in each linear move.

Here, we suppose that robot is ordered to move along a rectangle. At every corner, the robot is supposed to turn 90 degree. But in fact there is a slight error. If the former move has an error angle of $\Delta \theta$, the appointed angle for the next move is modified by $90^\circ - \Delta \theta$. 

![Figure 3. Flow chart of incremental digital PID control algorithm with a dead zone](image-url)
And the angle of final move can be calculated using the formula below:

$$\theta = -\sum_{i=1}^{n} \Delta \theta_i$$  \hspace{1cm} (17)

Where $\Delta \theta_i$ is the angle error in the move of side i.

IV. EXPERIMENTAL RESULTS

A. Experiments

We select iMEMS technique based Inertial Measurement Unit (IMU) ADXRS150 of Analog Devices Inc. as the angular rate sensors of this robot, the dynamic range of which is $\pm 150^\circ/s$ and the 3dB bandwidth (frequency response) is 40Hz. It also has 2000g powered shock survivability [13]. The output voltage swings from zero volt to 5 volt, and can be sampled by ADS1256 (a 24-bit analog-to-digital converter of Texas Instruments Inc.). The main chip is LPC2294 (ARM7) of NXP. We incorporated all of the above with a serial interface and a power supply on a single module, size of which is only 100mm by 50mm as Fig. 4 and Fig. 5.

This module can be also used in other mobile robots. Once any part of the module is broken, all we need to do is to change the broken part. So we don’t have to discard the entire board as before.

![Top layer of gyroscope module](image1)

![Bottom layer of gyroscope module](image2)

Figure 4. Top layer of gyroscope module

Figure 5. Bottom layer of gyroscope module

The platform is Voyager II type ground wheeled mobile robot. And the experiments are carried out on a ceramic tile floor (as is shown in Fig. 6).

Linear move is fundamental to fixed-angle move and polygon move. So, in order to evaluate its performance, two groups of experiments on robot’s walking along a straight line are performed to contrast the capability differences between with the angular rate sensor and without.

We let the robot move a 10-meter distance and record the deviation displacement every time it passes a one-meter mark. Before the robot starts, we align the edge of the right driving wheel with a white line which indicates the target moving direction. After initialization such as calibration, the robot walks along the white line. At the meantime, we follow the robot to draw a sign where the edge of the right wheel passes when it moves by every one-meter mark. When the robot reaches the end point, the deviation displacement from the mark to the white line is measured and recorded.

In each group, experiments at different speed are also performed. Repetitious experiments are conducted at the same velocity, the arithmetical mean values of them as the results of each group.

After the linear move experiments, we also operate the robot to turn a fixed angle of round. The angles are from 30 degree to 360 degree at the interval of 30 degree and the speeds are all at 0.32m/s. The actual angles are measured by a protractor.

Finally, the robot is ordered to run a polygon round including a triangle and a rectangle. Its linear speed is also at 0.32m/s.

B. Results

The experimental results of linear moves are shown in Fig. 7 and Fig. 8. The abscissa denotes the displacement (in meter) that the robot moves along the white line, while the ordinate represents the error that shows how far the robot deviates from the white line (in millimeter).

When there is no angular rate sensor, as is illustrated in Fig. 7, the robot deviates from the specific white line for a long distance.

When the angular rate sensor is utilized, the displacement between the designated line and the robot has sharply reduced. The curves only fluctuate in a small degree. And we note that the smaller the speed is, the greater the error is.
Compared with the similar works in [7], the error between the actual measured and estimated method was 46mm when it runs no more than 3m at the velocity of 1m/s. The relative error was about 1.5%. While, in our experiment, the error is 55mm at the speed of 0.48m/s when it runs along a 10m straight line where the relative error is about 0.6%.

![Figure 7](image1.png) The robot moves along the straight line without angular rate sensor

![Figure 8](image2.png) The robot moves along the straight line with angular rate sensor

In the turning experiments, there is also great improvement when using the angular rate sensor. And the polygon tracks are finally closed using the method introduced in part III-B.

V. CONCLUSION

This paper firstly analyzes the problems in mobile robot navigation when we only use one sensor or fuse several types of sensors. Complex and time-consuming algorithms must be implemented on expensive processors and the size of robot cannot be decreased further. Secondly, kinematics modeling of the two-wheel differential speed mode driven robot is built. Then the principle for the robot to move along a straight is brought forward. In order to overcome the defects that the robot deviates away for a long distance when only using the optical encoder, a navigational algorithm with an angular rate sensor to improve the performance is proposed. This algorithm uses an incremental digital PID controller with a dead zone, and samples the yawing angle of robot at every servo period. Incremental speeds for the two driving wheels are calculated to control the motors. In this part, the fixed-angle move and polygon move algorithms are also presented. Eventually, experiments show that these methods effectively attenuate the deviation. Meanwhile, the small-sized independent and reusable gyroscope module broadens its application when the bulk and cost are restricted.

VI. FUTURE WORK

Based on the experiments presented above, further efforts could be taken to improve the A/D conversion precision and lower the temperature drift extent of the angular rate sensor so that the robot can run within a smaller error range after running a long time.

Moreover, we find that when the robot starts moving at a relevantly higher speed, a noticeable slippage and obvious fluctuating adjustment occurs. This makes the robot leave the original orientation the moment it starts. We will work on it considering the friction and slippages.

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