Adaptive Control of an Aerial Robot using Lyapunov Design

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***Abstract*—In this article, based on feedback linearization using Lyapunov design method, an adaptive controller is proposed for an Aerial Robot or unmanned aerial vehicle (UAV). After introducing a nonlinear dynamics model of the system in case of longitudinal equations, comparing controllers are designed to manage the system performance during various maneuvers. Also, stability analysis for the designed adaptation law is studied and discussed. To evaluate the performance of designed controllers for a given system, a comprehensive simulation program is developed. One of the most important results of this study is that tracking errors for the two state variables exponentially converge to zero, even in the presence of parameters uncertainty. Therefore, it is shown that the proposed adaptive controller is able to perform perfect path tracking maneuvers.**

***Keywords*—Aerial robotics, Unmanned Aerial Vehicle, Nonlinear Dynamics, Adaptive Control**

Nomenclature

δ_e	Elevator angle
δ_T	Thrust input
γ_i	Positive constant
λ_i	Positive constant
θ	Pitch angel
Γ	Adaptive gain matrix
Ψ	Filter matrix
g	Gravity acceleration
k_p	Control law constant
p	Pitch rate
q	Roll rate
r	Yaw rate
u	Velocity in x direction of body
u_0	Velocity along x direction of body coord.
v	Velocity in y direction of body
w	Velocity in z direction of body
m	Main Body mass
P	Parameter
x	State variable
X	Force in x direction
Y	Force in y direction
Z	Force in z direction
$\partial x/\partial u$	Stability derivatives

I. 1. Introduction

To increase the mobility of on-orbit robotic systems, Space Free-Flying Robots (SFFR) in which one or more manipulators are mounted on a thruster-equipped base, have been proposed. It is expected that robotic systems play an important role in future space applications, including servicing, construction, and maintenance of space structures on orbit. Therefore, dynamics and motion control of SFFR have been studied extensively, [1-4]. Flying capability opens new opportunities in terrestrial applications as well, to perform field services and tasks like search and rescue, observation and mapping, [5-7]. An Aerial Robot or unmanned aerial vehicle (UAV) may be defined as an aerial vehicle (mostly without on-board manipulators) that uses aerodynamic forces to support its flight in a desired manner, so that a modern UAV is a fully autonomous flying system. Recent technological advancements in airframe materials, guidance systems, propulsion and payloads promise more complex goals to be achievable and yet remain cost-effective. Controlling the motion of a UAV is a challenging task, the interaction of the air flows generated by propeller contribute to complex aerodynamic forces affecting the vehicle's motion. The system's dynamics is not only coupled and nonlinear, but also difficult to be characterized due to the complexity of the system's aerodynamic properties, [8].

An adaptive controller differs from an ordinary controller in that there is a mechanism for online adjustment of the controller parameters based on measured variables. There are two main approaches for constructing adaptive controllers. One is the so-called model reference adaptive control method, and the other is the so-called self-tuning method. Various nonlinear control methods, fuzzy, and adaptive have been applied to UAVs in case of specific longitudinal and lateral maneuvers, [9-12]. Pota has conducted a through research and discussion on speed control problem, [13], while Wise has studied a trajectory problem, [14]. Also adaptive control with a single hidden layer adaptive element has been successfully used on a number of aircraft, [15-17].

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In this article, based on the feedback linearization approach, a non-linear adaptive controller is proposed for an aerial robot. First, nonlinear dynamics model of longitudinal motion is extracted, which will be used to develop the controller. Stability condition for the designed adaptation law is investigated using Lyapunov method to guarantee the stability of controller. To evaluate performance of the designed controller for a real UAV system, a comprehensive simulation program has been prepared. Exploiting this simulation routine, the system is simulated under the proposed controller, and state variables errors and trajectory tracking problem will be discussed.

II. DYNAMICS EQUATIONS

Considering the airplane as a rigid body, its equations of motion are supposed to be ODEs with constant coefficients. Coefficients in ODE are representations of aerodynamic stability derivatives of mass and inertia properties of the plane. These equations could be stated as first order ODEs. For instance, using equation of motion for a rigid body and considering Euler angles and gravity and lifting forces, dynamics equation along longitudinal axis of plane can be written as:

$$X - mgS_\theta = m(\dot{u} + qw - rv) \tag{1}$$

Each variable in this equation is substituted with its initial value added with a perturbed value as:

$$\begin{aligned} &u = u_1 + \Delta u, v = v_1 + \Delta v, w = w_1 + \Delta w \\ &X = X_1 + \Delta X, q = q_1 + \Delta q, r = r_1 + \Delta r \end{aligned} \tag{2}$$

So, we obtain:

$$\begin{aligned} &X_1 + \Delta X - mgS_{\theta_1+\Delta\theta} = m(\frac{d}{dt}(u_1 + \Delta u) \\ &\quad +(q_1 + \Delta q).(w_1 + \Delta w) - (r_1 + \Delta r).(v_1 + \Delta v)) \end{aligned} \tag{3}$$

The force ΔX indicates a change in thrust and aerodynamic forces along x direction, which can be presented as a Taylor series in terms of perturbed variables. Assuming ΔX as a function of u, w, δ_e, δ_T parameters then ΔX can be written as:

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\Delta\delta_T \tag{4}$$

where $\partial X/\partial u, \partial X/\partial w, \partial X/\partial\delta_e, \partial X/\partial\delta_T$ are known as stability derivatives and their values are defined in reference flight condition. The variables δ_e and δ_T define the elevator angle and fuel gate attitude. Equation (1) for the initial flight condition is written as:

$$X_1 - mgS_{\theta_1} = m(\dot{u}_1 + q_1 w_1 - r_1 v_1) \tag{5}$$

Assuming symmetric flight conditions yields:

$$v_1, w_1, q_1, r_1 \approx 0 \tag{6}$$

By subtracting equation (3) from previous equation and substituting ΔX while reformatting the result, the nonlinear equation of rigid body motion along x direction is obtained as:

$$\begin{aligned} &\left(\frac{d}{dt} - X_u\right)\Delta u - X_w \Delta w + g.S_\theta + \Delta q.\Delta w \\ &\qquad -\Delta r.\Delta v = X_{\delta_e}\Delta\delta_e + X_{\delta_T}\Delta\delta_T \end{aligned} \tag{7}$$

where $X_w = \partial X/\partial w\ /m$ and $X_u = \partial X/\partial u\ /m$. For the two remained equations of longitudinal motion a similar approach is performed and the following equations will be obtained:

$$\begin{aligned} &-Z_u\Delta u + \left((1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right)\Delta w - \left((u_0 + Z_q)\frac{d}{dt} - g\sin\theta_0\right)\Delta\theta \\ &\qquad +\Delta p.\Delta v - \Delta q.\Delta u = Z_{\delta_e}\Delta\delta_e + Z_{\delta_T}\Delta\delta_T \end{aligned} \tag{8}$$

$$\begin{aligned} &-M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta \\ &+\left(\frac{I_x - I_z}{I_y}\right)\Delta r.\Delta p + \left(\frac{I_{xz}}{I_y}\right)\Delta p^2 - \left(\frac{I_{xz}}{I_y}\right)\Delta r^2 = M_{\delta_e}\Delta\delta_e + M_{\delta_T}\Delta\delta_T \end{aligned} \tag{9}$$

These nonlinear equations are used to design the controller. Thus, according to two inputs of the system, feedback linearization controller will be designed:

$$\begin{aligned} \Delta\dot{u} &= X_u.\Delta u + X_w.\Delta w - g.\Delta\theta - \Delta q.\Delta w \\ &\quad + X_{\delta_e}.\Delta\delta_e + X_{\delta_T}.\Delta\delta_T \end{aligned} \tag{10}$$

$$\begin{aligned} \Delta\dot{w} &= Z_u.\Delta u + Z_w.\Delta w + u_0.\Delta q + \Delta q.\Delta u \\ &\quad + Z_{\delta_e}.\Delta\delta_e + Z_{\delta_T}.\Delta\delta_T \end{aligned} \tag{11}$$

III. NONLINEAR CONTROLLER DESIGN

Various approaches have been proposed for nonlinear controller design, which include feedback linearization, robust control, adaptive control and gain scheduling, and each of these are most suitable for a specific kind of control problem. Feedback linearization has attracted a great deal of research interests in recent years. The idea of simplifying the form of a system's dynamics by choosing a different state representation is not entirely unfamiliar. In mechanics, for instance, it is well known that the form and complexity of a system model depends considerably on the choice of reference frames or coordinate systems. Feedback linearization techniques can be viewed as way of transforming original system models into equivalent models of a simpler form. Thus, they can also be used in the development of robust or adaptive nonlinear controllers.

A. *Canonical form of feedback linearization*

In this form of controller, feedback linearization yields cancellation of nonlinearities in so that the closed-loop dynamics matches a linear form. The idea of feedback linearization, i.e. canceling the nonlinearities and imposing a desired linear dynamics, can be simply applied to a class of nonlinear systems described by the so-called companion or controllability canonical form, [18]. In this research, we focus on displacement state variables ($\dot{x}$) among velocity variables

(V) and we deal with the control problem based on obtained equations of motion. So:

$$\dot{x} = V \rightarrow \ddot{x} = A\dot{x} + Bu \tag{12}$$

where u is input of the system, B is input matrix and A is the state matrix which contains nonlinear terms. Now assuming a control law as:

$$\upsilon = \ddot{x}_d + 2k_p\dot{\tilde{x}} + k_p^2\tilde{x} \tag{13}$$

A multi integral form is obtained:

$$\ddot{x} = \nu \tag{14}$$

So control input of the system is obtained:

$$\upsilon = A\dot{x} + Bu \Rightarrow u = B^{-1}(\upsilon - A\dot{x}) \tag{15}$$

Assuming control laws:

$$\begin{cases} \upsilon = \ddot{x}_d - 2.\lambda_1.(\dot{x} - \dot{x}_d) - \lambda_1^2.(x - x_d) \\ \mu = \ddot{z}_d - 2.\lambda_2.(\dot{z} - \dot{z}_d) - \lambda_2^2.(z - z_d) \end{cases} \tag{16}$$

Below multi integral form is achieved:

$$\begin{cases} \Delta\ddot{x} = \Delta\dot{u} = \upsilon \\ \Delta\ddot{z} = \Delta\dot{w} = \mu \end{cases} \tag{17}$$

which satisfies an exponential convergence criterion:

$$\begin{cases} \Delta\ddot{x} - \upsilon = 0 \\ \Delta\ddot{z} - \mu = 0 \end{cases} \Rightarrow \begin{cases} \ddot{\tilde{x}} + 2.\lambda_1\dot{\tilde{x}} + \lambda_1^2.\tilde{x} = 0 \\ \ddot{\tilde{z}} + 2.\lambda_2.\dot{\tilde{z}} + \lambda_2^2.\tilde{z} = 0 \end{cases} \tag{18}$$

By substituting these variables in nonlinear dynamic equations for calculating control input we have:

$$\begin{cases} X_{\delta_e}.\Delta\delta_e + X_{\delta_T}.\Delta\delta_T = \upsilon - X_u.\Delta u - X_w.\Delta w + g.\Delta\theta + \Delta q.\Delta w = \Upsilon \\ Z_{\delta_e}.\Delta\delta_e + Z_{\delta_T}.\Delta\delta_T = \mu - Z_u.\Delta u - Z_w.\Delta w - u_0.\Delta q - \Delta q.\Delta u = \Lambda \end{cases} \tag{19}$$

Solving these two equations results in:

$$\begin{cases} \Delta\delta_e = \dfrac{Z_{\delta_T}.\Upsilon - X_{\delta_T}.\Lambda}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}} = \dfrac{Z_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Upsilon - \dfrac{X_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Lambda \\ \Delta\delta_T = \dfrac{X_{\delta_e}.\Lambda - Z_{\delta_e}.\Upsilon}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}} = \dfrac{X_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Lambda - \dfrac{Z_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Upsilon \end{cases} \tag{20}$$

and by defining the following parameters which are combinations of input matrix elements:

$$P_1 = \frac{Z_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}, \quad P_2 = -\frac{X_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}$$
$$P_3 = -\frac{Z_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}, \quad P_4 = \frac{X_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}} \tag{21}$$

a simplified model for input control is obtained:

$$\begin{Bmatrix} \Delta\delta_e \\ \Delta\delta_T \end{Bmatrix} = KP \tag{22}$$

where $P = [P_1 \quad P_2 \quad P_3 \quad P_4]^T$ and K matrix is defined as:

$$K = \begin{bmatrix} \Upsilon & \Lambda & 0 & 0 \\ 0 & 0 & \Upsilon & \Lambda \end{bmatrix} \tag{23}$$

B. *Adaptive control*

Adaptive control is an approach to dealing with uncertain systems or time-varying systems, [19]. Although the term adaptive can have broad meanings, current adaptive control designs apply mainly to systems with known dynamic structure, but unknown constants or slowly-varying parameters. Adaptive controllers, whether developed for linear systems or for nonlinear systems, are inherently nonlinear.

C. *Using Lyapanov design*

The controller design procedure is stated by using obtained input control equations using feedback linearization method in previous section and four unknown parameters $Z_{\delta_T}, X_{\delta_T}, Z_{\delta_e}, X_{\delta_e}$.

Using four unknown parameters, four new unknown parameters are defined as $\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4$ and as a result control input will become:

$$\begin{cases} \Delta\delta_e = \hat{P}_1.\Upsilon + \hat{P}_2.\Lambda \\ \Delta\delta_T = \hat{P}_3.\Upsilon + \hat{P}_4.\Lambda \end{cases} \tag{24}$$

The control law which completes first stage of design procedure is similar to feedback linearization method. It is assumed that $\hat{P}$ is indicator of unknown parameter and $\tilde{P}$ is indicator of estimated parameter error and the relationship between these two parameters is defined as:

$$P - \hat{P} = \tilde{P} \Rightarrow P = \hat{P} + \tilde{P} \tag{25}$$

Lyapanov design method for the control law satisfies two needs of selecting an adaptation rule for regulating parameters, and analyzing convergence characteristics of the controller. For these purposes and considering uncertainty in parameters, calculated control input will be substituted in nonlinear dynamic equations of the system. Using above definitions, dynamic equations are simplified as:

$$\begin{cases} P_1.(\Delta\ddot{x} - M_1) + P_2.(\Delta\ddot{z} - M_2) = \hat{P}_1.\Upsilon + \hat{P}_2.\Lambda \\ P_3.(\Delta\ddot{x} - M_1) + P_4.(\Delta\ddot{z} - M_2) = \hat{P}_3.\Upsilon + \hat{P}_4.\Lambda \end{cases} \tag{26}$$

where:

$$\begin{cases} M_1 = X_u.\Delta u + X_w.\Delta w - g.\Delta\theta - \Delta q.\Delta w \\ M_2 = Z_u.\Delta u + Z_w.\Delta w + u_0.\Delta q + \Delta q.\Delta u \end{cases} \tag{27}$$

and using the definition:

$$\begin{cases} (\hat{P}_1 + \tilde{P}_1).(\Delta\ddot{x} - M_1) + (\hat{P}_2 + \tilde{P}_2).(\Delta\ddot{z} - M_2) = \hat{P}_1.\Upsilon + \hat{P}_2.\Lambda \\ (\hat{P}_3 + \tilde{P}_3).(\Delta\ddot{x} - M_1) + (\hat{P}_4 + \tilde{P}_4).(\Delta\ddot{z} - M_2) = \hat{P}_3.\Upsilon + \hat{P}_4.\Lambda \end{cases} \tag{28}$$

and after simplifications we achieve:

$$\begin{bmatrix} \Delta\ddot{x}-\upsilon \\ \Delta\ddot{z}-\mu \end{bmatrix} = \begin{bmatrix} \hat{P}_1 & \hat{P}_2 \\ \hat{P}_3 & \hat{P}_4 \end{bmatrix}^{-1} \begin{bmatrix} -\Delta\ddot{x} & -\Delta\ddot{z} & 0 & 0 \\ 0 & 0 & -\Delta\ddot{x} & -\Delta\ddot{z} \end{bmatrix} \begin{bmatrix} \tilde{P}_1 \\ \tilde{P}_2 \\ \tilde{P}_3 \\ \tilde{P}_4 \end{bmatrix} \quad (29)$$

If the right hand side tends to be zero, an exponential convergence condition is guaranteed. This situation is exactly equal to the situation which estimation error of parameters tend to be zero using an adaptive law. So, we can write:

$$\begin{bmatrix} \ddot{\tilde{x}} + 2.\lambda_1.\dot{\tilde{x}} + \lambda_1^2.\tilde{x} \\ \ddot{\tilde{z}} + 2.\lambda_2.\dot{\tilde{z}} + \lambda_2^2.\tilde{z} \end{bmatrix} = \hat{B}.K.\tilde{P} \quad (30)$$

By substituting controller input in dynamic equations of the system, we go through designing adaptation rule and stability guarantee of the system, with uncertain parameters. So:

$$\begin{aligned} \ddot{X} &= A\dot{X} + BU = A\dot{X} + B\left(\hat{B}.K.\tilde{P}\right) \\ Y &= \dot{\tilde{X}} + \Psi\tilde{X} = CX \end{aligned} \quad (31)$$

where $\Psi = diag\,[\Psi_1 \quad \Psi_2 \quad \dots \quad \Psi_r]$ is filter matrix, $\tilde{X} = [\tilde{x} \quad \tilde{z}]^T$ is tracking error and Y is filtered output. Assuming that the system is stable and by means of stability theorems, it could be stated that if the defined system is stable, there exist symmetric and positive definite ρ and Q matrices which satisfy the following equation:

$$A^T\rho + \rho A = -Q \quad (32)$$

$$\rho B = C \rightarrow B^T\rho^T = C \rightarrow B^T\rho = C \quad (33)$$

Now, we define a positive definite Lyapanov function candidate as:

$$V(X,\tilde{P}) = X^T\rho X + \tilde{P}^T\Gamma^{-1}\tilde{P} \quad (34)$$

where $\Gamma = diag\,[\gamma_1 \quad \gamma_2 \quad \dots \quad \gamma_r]\,, \gamma_i \succ 0$ is the gain matrix. Now, the derivative of Lyapanov function is obtained as:

$$\dot{V} = \dot{X}^T\rho X + X^T\rho\dot{X} + 2\tilde{P}^T\Gamma^{-1}\dot{\tilde{P}} \quad (35)$$

By substituting from dynamic equations and reformating and simplifications we achieve:

$$\dot{V} = -X^TQX + 2\tilde{P}^T\left[\Gamma^{-1}\dot{\tilde{P}} + K^T\hat{B}^TY\right] \quad (36)$$

As the first term in right hand side is always negative, if the second term become equal to zero, derivative of Lyapanov function will be negative definite, i.e. $\dot{V} \prec 0$, and thus the system will be stable. So:

$$\dot{\tilde{P}} = -\Gamma K^T\hat{B}^TY \quad (37)$$

where according to the definition of estimated parameters we have $\dot{\tilde{P}} = -\dot{\hat{P}}$ and then adaptation law obtained from Lyapanov method for unknown parameters of the system is:

$$\dot{\hat{P}} = \Gamma K^T\hat{B}^TY \quad (38)$$

IV. SIMULATION RESULTS AND DISCUSSIONS

To study the performance of the proposed adaptive nonlinear controller for a UAV, a comprehensive simulation routine has been conducted. This program after taking the time of simulation calculates reference input value x_r, ideal assumed angle, time and distance for performing defined maneuver, and then according to the entered coefficients by the operator, illustrates the results. First, this program calculates stability derivatives for the given UAV based on geometrical characteristics and stability coefficients. It is noticeable that designing controller using feedback linearization method and followed discussions to complete design and simulation of adaptive controller are based on two horizontal and vertical velocity state variables.

Figures 1 to 6 illustrate take off maneuver for the considered system, applying the proposed adaptive controller, in the presence of uncertainty in parameters. It is observed that using this controller tracking errors of two selected state variables go to zero with an exponential rate (Figure 4) which can be realized by comparing each variable with its reference value (Figure 5). Only fourth state variable is not able to reach its expected value and shows an offset. Considering the fact that designed adaptive controller using Lyapanov method is based on feedback linearization, this offset value could be explained by nonlinear terms in pitch angle rate and its effects on pitch angle. Also, considering a two input UAV system in terms of longitudinal equations (Figure 2 and 3) and controller design which was used for perfect tracking of first and second state variables in case of feedback linearization, this error occurs in fourth state variables.

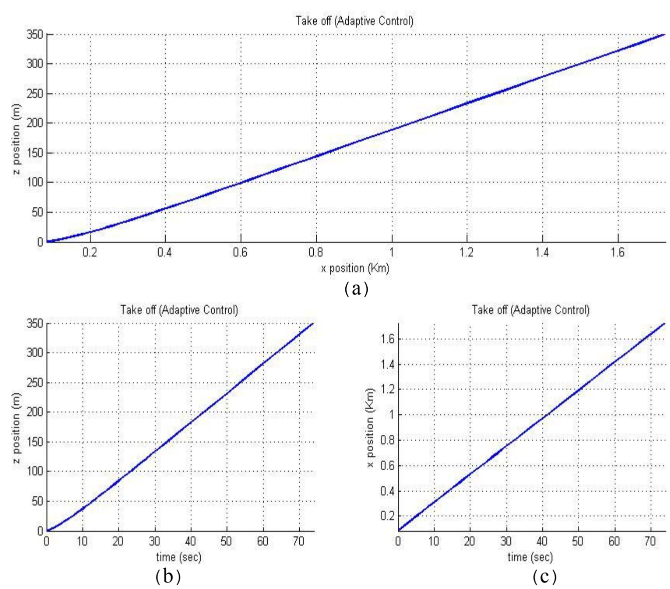

Figure 1 : Flight path in take off maneuver for adaptive controller, a) path in vertical plane, b) vertical displacement versus time, c) horizontal displacement versus time

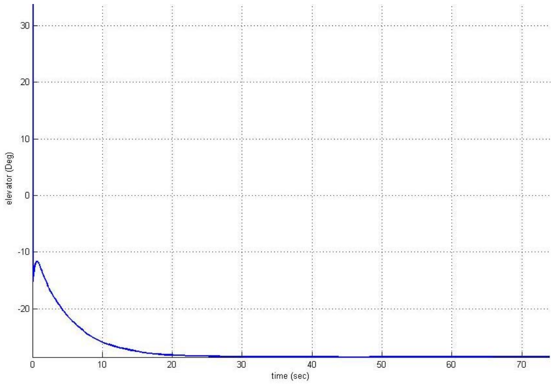

Figure 2: Illustration of elevator input in take off maneuver using adaptive controller

It is illustrated clearly in (Figure 6) that the designed adaptation law using Lyapanov method had been able to estimate unknown parameters according to initial values of parameters and gain of adaptation law which are defined by operator, such that the UAV would be able to perform a satisfactory tracking and guarantee stability of system, in addition to guaranteeing convergence of parameters. Variations in adaptation rule gains change the convergence speed and convergence rate while these parameters are convergent all the time. Observing the fact that changes in coefficients of error polynomial and adaptation law gains which are defined by operator can affect the error convergence rate of state variables, it should be noted that although there is an offset for tracking forth variable, the UAV has been successful in performing a take off maneuver with uncertainty in parameters.

V. CONCLUSIONS

In this research, after giving the definition of a nonlinear feedback linearization controller, a canonical feedback linearization controller for an aerial robot or unmanned aerial vehicle (UAV) was designed. According to nonlinear terms effects in first and second state variables for longitudinal equations, namely vertical and horizontal components, the controllers were designed. Then, adaptation law was designed to encounter uncertainty in system based on feedback linearization controller using Lyapanov design method and obtained controller was implemented on a UAV system. Comprehensive simulation program, after taking error polynomial coefficients and adaptation law gains which are set by the operator, was used for studying take off maneuver.

Applying the proposed adaptive controller, tracking error for two state variables with uncertainty in parameters tends to reach zero with an exponential rate. It is seen that just the fourth state variable can not reach to its expected value and has offset. This could be explained by considering nonlinear terms in pitch angle rate which affect the pitch angle. Also designed adaptation law by means of Lyapanov design method is able to estimate the unknown parameters such that performing a satisfactory tracking is obtained. In addition to guaranteeing convergence of parameters, the proposed controller guarantees stability of the system.

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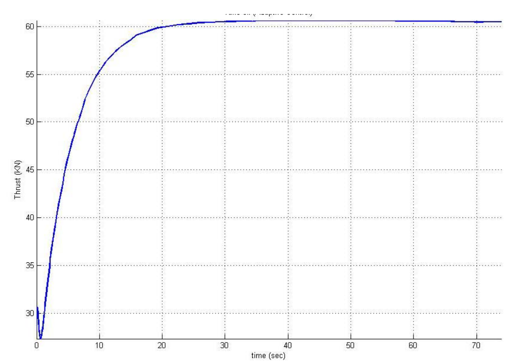

Figure 3: Illustration of thrust input in take off maneuver for adaptive controller

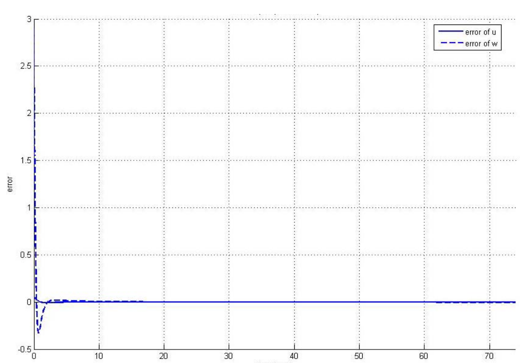

Figure 4 : Illustration of exponential convergence of adaptive controller in take off maneuver

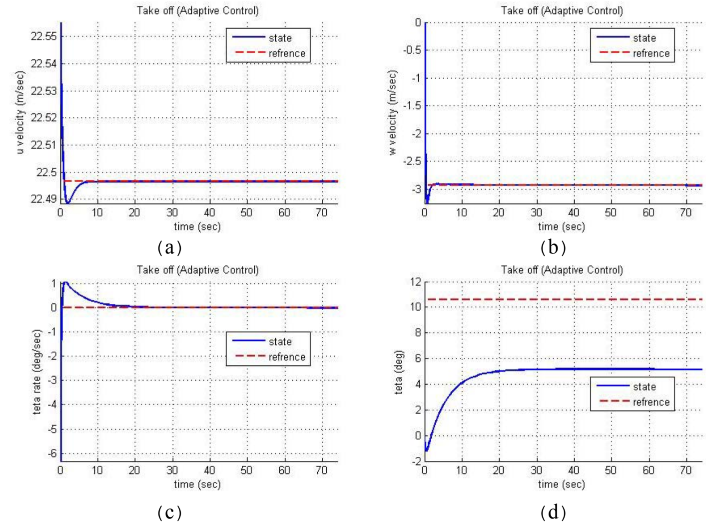

Figure 5 : Illustration of state variables in take off maneuver for adaptive controller, a) horizontal component of velocity, b) vertical component of velocity, c) pitch angle rate, d) pitch angle

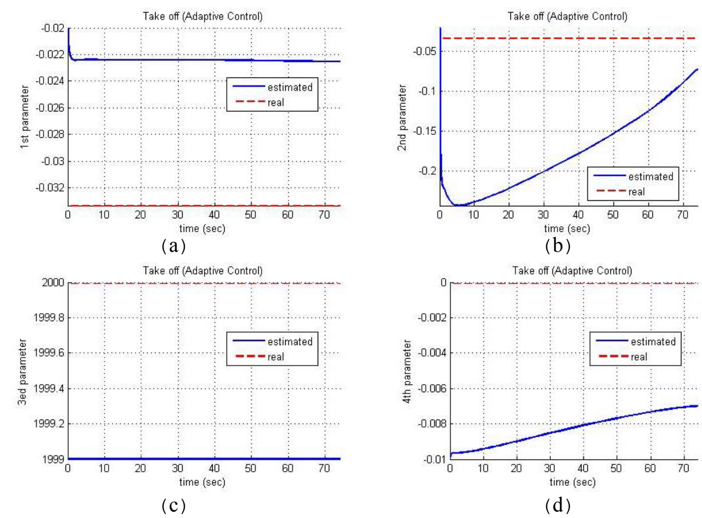

Figure 6 : Illustration of parameter estimation values in take off maneuver using adaptive controller, a) first unknown parameter, b) second unknown parameter, c) third unknown parameter, d) fourth unknown parameter