Object Manipulation by Multiple Arms of A Wheeled Mobile Robotic System

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***Abstract*— Mobile robotic systems, which include a mobile platform with one or more manipulators, are of great interest in most applications, e. g. planet explorations, rescuing operations, nursing, book keeping, storing and many others. To manipulate an object with two or more cooperating manipulators, the Multiple Impedance control (MIC) as a Model-Based algorithm enforces a desired impedance law on each manipulator, the manipulated object, and the moving base itself. However, to apply model-based control laws, it is needed to extract explicit system dynamic model, which of course for such systems may lead to very complicated nonlinear equations of motions. To this end, non-holonomic constraint of a wheeled system is derived, and the obtained dynamics model is reformatted to become a more concise one using Natural Orthogonal Complement Method. Next, the MIC law is applied to manipulate an object by two 6-dof cooperating manipulators mounted on a wheeled platform while the moving base is driven with two differentially driver wheels. Obtained results reveal a good tracking performance of the system, i.e. a coordinated smooth motion of the object, manipulators and the moving base, even in the presence of impacts due to contact with an obstacle, and system flexibility.**

***Keywords*—Wheeled Mobile robots, Dynamics Model, Non-holonomic Constraint, Impedance Control, Object Manipulation.**

I. Introduction

Due to limitations of fixed-base robots, mobile robotic systems have attracted a lot of attentions. Unlike fixed-base robots, mobile manipulators can do tasks that may be out of reach of their manipulators by exploiting the mobility of the base platforms. Therefore, the interaction between the base and the manipulators results in more complicated dynamics equations, which in turn requires a more sophisticated control. Free-flying robots can move freely in space without any constraint, [1], while free-floating robots and wheeled robotic systems are usually subjected to non-holonomic constraints, [2-4].

To apply model-based control laws, it is needed to extract explicit system dynamic model. Attempts for attaining the dynamical equations of motion for mobile manipulators have presented successful results. A systematic method for the kinematics and dynamics modeling of a two degree-of-freedom (DOF) Automated Guided Vehicle (AGV) has been presented by Saha and Angeles, [5-6]. They have employed the notion of Natural Orthogonal Complement to eliminate the Lagrange multipliers. The idea of direct path method, [7], has been utilized for deriving the dynamics of differentially-driven mobile manipulators equipped with multiple arms. Various dynamics modeling approaches, and control algorithms have been used for motion control of a mobile platform, [8-10], while manipulating objects by multiple arms mounted on a moving base has left almost untouched.

The Multiple Impedance Control (MIC) is an algorithm that has been developed for several cooperating robotic systems manipulating a common object, [11]. The MIC law imposes a reference impedance to all elements of a mobile system, including its base, the manipulator end-points, and the manipulated object itself. This algorithm is used for space free-flyers and is shown to give good manipulation results even in the presence of contact phase, and external disturbances, [12].

The main focus of this paper is on object manipulation by multiple 6-DOF manipulators of a wheeled mobile system while the moving base is driven with two differentially driver wheels. To this end, using Lagrange method the equations of motions are derived. Next, the system non-holonomic constraint will be derived, and using Natural Orthogonal Complement (NOC) Method an independent set of equations of motion is derived for the system. Finally, the MIC law is applied to manipulate an object with two cooperating manipulators tracking a given path. The obtained results reveal a coordinated motion of the object, manipulators and the base vehicle.

II. System Dynamics

A. *Basic Definitions and Calculations:*

A wheeled robotic system is considered on a flat surface as depicted in Fig. 1. Direct path method (DPM), [7], is used to express the base and the links center of mass position and orientation, and linear and angular velocities of the base and each link. To derive equations of motion, using Lagrange approach, it can be written:

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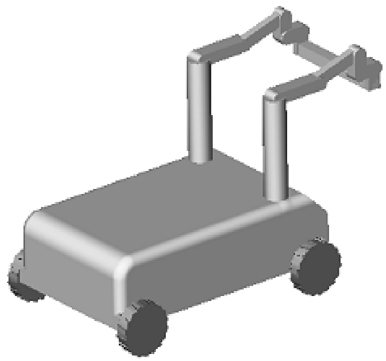

Figure.1:The considered mobile robotic system manipulating an object

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}=Q_i \qquad i=1..........N \tag{1}$$

where T is the total system kinetic energy, U is the total system potential energy, N is the system degrees-of-freedom (DOF), q_i , $\dot{q}_i$, and Q_i are the i-th element of the vector of generalized coordinates, generalized speeds, and generalized forces, respectively, as defined below:

$$q=\left\{R_b^T \ ,\phi \ ,q_1^T ,\ q_2^T\right\}^T \tag{2}$$

where R_b and ϕ are position vector and the yaw angle of the base, and q_1 and q_2 are the first and second manipulator vectors of joint angles respectively , as below:

$$R_b=(x_G, y_G)^T \tag{3a}$$

$$q_1=(\theta_1^{(1)},...,\theta_6^{(1)})^T \tag{3b}$$

$$q_2=(\theta_1^{(2)},...,\theta_6^{(2)})^T \tag{3c}$$

The terms of the total system kinetic energy (T) are explicitly detailed in [4] for a general unconstrained mobile robotic system.

The system gravitational potential energy contains terms including both the base gravitational potential energy and those of links, as obtained below:

$$U=m\,\vec{g}\,.\,\vec{R}_b+\sum_{m=1}^{n}\sum_{i=1}^{N_m}\left(m_i^{(m)}\,\vec{g}\,.\,(\vec{R}_b+\vec{r}_{c\,i}^{\,(m)})\right) \tag{4}$$

where g is the gravity acceleration vector, $m_i^{(m)}$ and $r_{c\,i}^{(m)}$ are mass and the position vector of the center of mass (CM) for the i-th link of the m-th manipulator with respect to the base CM, respectively.

B. Extracting Dynamics Equations:

Exploiting Lagrange equations, Eq. (1), the system dynamics equation is extracted as:

$$H(q)\ddot{q}+C(q,\dot{q})+G(q)=Q \tag{5}$$

The mass matrix (H), non-linear velocity vector (C) and gravity vector (G), are obtained in the following form:

$$\begin{aligned}H_{ij}&=M\frac{\partial\vec{R}_b}{\partial q_i}.\frac{\partial\vec{R}_b}{\partial q_j}+\frac{\partial\vec{\omega}_b}{\partial\dot{q}_i}.I_b.\frac{\partial\vec{\omega}_b}{\partial\dot{q}_j}\\&+\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial\vec{r}_{ck}^{\,(m)}}{\partial q_i}.\frac{\partial\vec{r}_{ck}^{\,(m)}}{\partial q_j}+\frac{\partial\vec{\omega}_k^{(m)}}{\partial\dot{q}_i}.I_k^{(m)}.\frac{\partial\vec{\omega}_k^{(m)}}{\partial\dot{q}_j}\right)\\&+\left(\sum_{m=1}^{n}\sum_{k=1}^{N_m}m_k^{(m)}\frac{\partial\vec{r}_{ck}^{\,(m)}}{\partial q_i}\right).\frac{\partial\vec{R}_b}{\partial q_j}\\&+\left(\sum_{m=1}^{n}\sum_{k=1}^{N_m}m_k^{(m)}\frac{\partial\vec{r}_{ck}^{\,(m)}}{\partial q_j}\right).\frac{\partial\vec{R}_b}{\partial q_i}\end{aligned} \tag{6}$$

Vector C could be written as:

$$C=C_1\,\dot{\mathbf{q}}+C_2 \tag{7a}$$

where:

$$\begin{aligned}C_{1ij}&=M\frac{\partial\vec{R}_b}{\partial q_i}.\left(\sum_{s=1}^{N}\frac{\partial^2\vec{R}_b}{\partial q_s\partial q_j}\right)+\frac{\partial\vec{\omega}_b}{\partial\dot{q}_i}.I_b.\frac{\partial\vec{\omega}_b}{\partial q_j}+\vec{\omega}_b.I_b.\frac{\partial^2\vec{\omega}_b}{\partial\dot{q}_i\,\partial q_j}\\&+\frac{\partial\vec{R}_b}{\partial q_i}.\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\sum_{s=1}^{N}\frac{\partial^2\vec{r}_{c\,k}^{\,(m)}}{\partial q_s\partial q_j}\dot{q}_s\right)+\\&+\left(\sum_{s=1}^{N}\frac{\partial^2\vec{R}_b^{(m)}}{\partial q_s\partial q_j}\dot{q}_s\right).\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial\vec{r}_{c\,k}^{\,(m)}}{\partial q_i}\right)\\&+\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(\begin{aligned}&m_k^{(m)}\frac{\partial\vec{r}_{c\,k}^{\,(m)}}{\partial q_i}.\left(\sum_{s=1}^{N}\frac{\partial^2\vec{r}_{c\,k}^{\,(m)}}{\partial q_s\partial q_j}\dot{q}_s\right)\\&+\frac{\partial\vec{\omega}_k^{(m)}}{\partial\dot{q}_i}.I_k^{(m)}.\frac{\partial\vec{\omega}_k^{(m)}}{\partial q_j}+\vec{\omega}_k^{(m)}.I_k^{(m)}.\frac{\partial^2\vec{\omega}_k^{(m)}}{\partial\dot{q}_i\partial q_j}\end{aligned}\right)\end{aligned} \tag{7b}$$

and:

$$C_{2\,i}=-\left(\vec{\omega}_b.I_b.\frac{\partial\vec{\omega}_b}{\partial q_i}+\sum_{m=1}^{n}\sum_{k=1}^{N_m}\vec{\omega}_k^{(m)}.I_k^{(m)}.\frac{\partial\vec{\omega}_k^{(m)}}{\partial q_i}\right) \tag{7c}$$

where ω_b and I_b are the base angular velocity and inertia matrix and $\omega_k^{(m)}$ and $I_k^{(m)}$ angular velocity and inertia matrix for each link respectively and M is the total mass of the robotic system. Finally, vector G is obtained as follows:

$$G_i=M\,\vec{g}\,.\frac{\partial\vec{R}_b}{\partial q_i}+\vec{g}\,.\sum_{m=1}^{n}\sum_{k=1}^{N_m}m_k^{(m)}\frac{\partial\vec{r}_{ck}^{\,(m)}}{\partial q_i} \tag{8}$$

It should be noted that extracting the explicit dynamics model of Eq. (5), all its terms, including mass matrix (H), non-linear velocity vector (C) and gravity vector (G), can be symbolically obtained via Maple.

III. Non-holonomic Constraint

Fig. 2 illustrates a wheeled mobile robot base that moves by its two rear independent wheels .The motion of the wheels is restricted to be slipless, therefore the wheels will move along x-axis of base-attached coordinates. So, one can write:

$$tan(\phi) = \frac{\dot{y}_o}{\dot{x}_o} \tag{9}$$

where point O is in the middle of the rear axle, and ϕ describes the base orientation (yaw angle) as shown in Fig. 2. The velocity of the base CM, i.e. point G, can be written as:

$$\vec{V}_G = \vec{V}_O \ + \ \dot{\phi}\vec{k} \times l_G \vec{i}' \tag{10}$$

Substituting Eq. (9) into (10) yields a non-holonomic constraint as:

$$\dot{x}_G \, sin(\phi) - \dot{y}_G \, cos(\phi) + l \, \dot{\phi} = 0 \tag{11}$$

where $\dot{x}_G$ and $\dot{y}_G$ are base linear velocity along x and y axes respectively. Considering Eq. (11) the DOF of the base at the speed level is reduced to two, as discussed in [10]. One can choose the angular velocities of the right and left wheels, i.e. ω_r and ω_l, as new general speeds of the base. Then, it can be written:

$$\dot{q} = S\,(q)\,v \tag{12a}$$

where $S(q)$ is a Jacobian matrix which relates the new general speeds to the prior ones. For the base we have:

$$\begin{Bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\phi} \end{Bmatrix} = Jac_{32} \begin{Bmatrix} \omega_l \\ \omega_r \end{Bmatrix} \tag{12b}$$

where Jac_{32} is a non-square Jacobian matrix which relates the new general speeds of the base to the prior ones as follows:

$$Jac_{32} = \begin{bmatrix} \frac{r}{2}\cos(\phi) + \frac{l_G \ r}{b}\sin(\phi) & \frac{r}{2}\cos(\phi) - \frac{l_G \ r}{b}\sin(\phi) \\ \frac{r}{2}\sin(\phi) - \frac{l_G \ r}{b}\cos(\phi) & \frac{r}{2}\cos(\phi) + \frac{l_G \ r}{b}\cos(\phi) \\ \frac{-\ r}{b} & \frac{r}{b} \end{bmatrix} \tag{13}$$

Next , the constraint Eq. (11) can be expressed as:

$$A(q).\dot{q} = 0 \tag{14}$$

where $A(q)$ is

$$A\,(q) = \begin{bmatrix} \sin(\phi) & -\cos(\phi) & l_G & 0_{1\times\sum_{m=1}^{n} N_m} \end{bmatrix} \tag{15}$$

Considering (14), and defining corresponding Lagrangian multiplier as λ, [13], Eq. (5) can be written as:

$$H(q)\ddot{q} + C(q,\dot{q}) + G(q) + A(q)^T.\lambda = Q \tag{16}$$

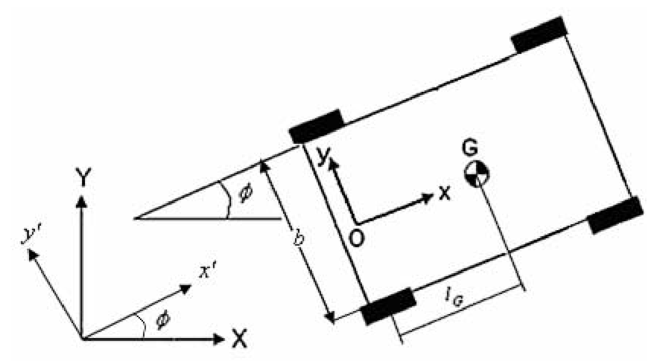

Figure .2: A wheeled base with no-slippage constraint

Then, using *Natural Orthogonal Complement Method*, [5], the equations of motion can be derived as a set of unconstrained equations which is detailed next.

The relationship between constrained general forces and unconstrained ones can be written as

$$Q_{N\times 1} = E\,(q) \quad \tau_{(N-1)\times 1} \tag{17a}$$

in which $E\,(q)$ is an $N \times (N-1)$ matrix, and

$$\tau = \begin{bmatrix} \tau_l & \tau_r & \tau_1^1 & \tau_2^1 & \ldots\ldots\ldots\ldots\ldots \end{bmatrix}^T \tag{17b}$$

To obtain $E\,(q)$, based on principle of virtual work, one can write

$$Q^T \ . \ dq \ = \tau^T . \ dv \tag{17c}$$

Substituting dq from Eq. (12a), it is obtained:

$$\tau = S^T \ Q \tag{17d}$$

Finally, substituting Q from Eq. (17a), we will have

$$S^T \, . E = 1_{(N-1)\times(N-1)} \tag{18}$$

Now, substituting (17a) into (16), and multiplying by S^T, we will have

$$S^T\left(H\,(q) \ \ddot{q} \ + C_1\dot{q} + C_2 + \ G\,(q) - E \ \tau\right) + S^T A\,(q)^T \ \lambda = 0 \tag{19a}$$

Noting the fact that $A(q)$ is in the null space of $S(q)$, the second part will vanish, and (19a) reduces to :

$$S^T H\,(q) \ \ddot{q} \ + S^T C_1\dot{q} + S^T C_2 + S^T G\,(q) = \tau \tag{19b}$$

Now, based on Eq. (12) we have:

$$\ddot{q} = S\,\dot{v} + \dot{S}\,v \tag{20a}$$

where

$$\dot{S}(q,\dot{q}) = \begin{bmatrix} \frac{\partial Jac_{32}}{\partial t} & 0_{3\times(N-3)} \\ 0_{(N-3)\times 2} & 0_{(N-3)\times(N-3)} \end{bmatrix}_{N\times(N-1)} \tag{20b}$$

Finally, substituting $\dot{q}$ and $\ddot{q}$ in (19b), the equations of motion for the constrained system will reduce into an independent set in the following form:

$$\tilde{H}\ \ \dot{v}+\tilde{C}_1 v+\tilde{C}_2+\tilde{G}=\tau \tag{21a}$$

in which

$$\tilde{H}=S^T.H.S \tag{21b}$$

$$\tilde{C}_1=S^T.(C_1 S+H\dot{S}) \tag{21c}$$

$$\tilde{C}_2=S^T.C_2 \tag{21d}$$

$$\tilde{G}=S^T.G \tag{21e}$$

These matrices are symbolically calculated in Maple and the concise dynamics model is obtained.

IV. KINEMATICS CONSIDERATIONS

The jacobian matrix for the considered robotic system in case of no constraint will be in the following form

$$\dot{X}_{task}=Jac\ \dot{q} \tag{22}$$

where $\dot{X}_{task}$ is the vector of task space speeds that can be in the following form

$$\dot{X}_{task}=\begin{bmatrix}\dot{x}_G & \dot{y}_G & \dot{\phi} & (\dot{x}_e^1)^T & (\dot{x}_e^2)^T\end{bmatrix}^T \tag{23}$$

where x_e^i describes the i-th end effector linear position in the inertial frame, and its orientation using Euler angles. Therefore, the jacobian matrix can be written as:

$$Jac=\begin{bmatrix}1_{3\times3} & 0_{3\times(N-3)}\\ Jac^{21}_{(N-3)\times3} & Jac^{22}_{(N-3)\times(N-3)}\end{bmatrix} \tag{24}$$

As described before, if we use v instead of $\dot{q}$, it is obtained

$$\dot{X}_{task}=\tilde{J}ac\ \dot{v} \tag{25}$$

where

$$\tilde{J}ac=Jac\bullet S(q)=\begin{bmatrix}1_{2\times2} & 0_{2\times(N-3)}\\ Jac^{21}\cdot Jac_{3\times2} & Jac^{22}\end{bmatrix} \tag{26}$$

Finally, the time derivative of jacobian matrix is obtained as

$$\tilde{J}_{dot}=\begin{bmatrix}0_{2\times2} & 0_{2\times(N-3)}\\ Jac^{21}_{dot}\cdot Jac_{32}+Jac^{21}\cdot Jac^{32}_{dot} & Jac^{22}_{dot}\end{bmatrix} \tag{27}$$

V. THE MIC LAW

The MIC law enforces an impedance relationship at the object level, as well as the manipulators-base level, and yields proper results even in the presence of object flexibility, and impacts due to contact with the environment. This strategy allows coordinated motion and force control of wheeled mobile robots to perform a desirable manipulation task. To apply the MIC law, a desired impedance relationship at the object level is written as:

$$M_{des}\ddot{e}+k_d\dot{e}+k_p e+F_c=0 \tag{28}$$

where M_{des}, k_d, k_p are the desired mass, damping and stiffness matrices, and F_c is the contact force (in contact phase), and $e=x_{des}-x$ is the object tracking error. On the other hand, as described in [11], the object equation of motion can be obtained as:

$$\mathrm{M}\ \ddot{\mathrm{x}}+\mathrm{F_w}=\mathrm{F_c}+\mathrm{F_o}+\mathrm{G}\ \mathrm{F_e} \tag{29}$$

where M, $\mathrm{F_w}$, $\mathrm{F_o}$, G, and $\mathrm{F_e}$ are mass matrix, vector of nonlinear velocity terms, external forces/torques, grasp matrix and finally forces/torques exerted by the manipulators end effectors, respectively. As mentioned before, the MIC law enforces the same impedance on various parts of the system. Therefore, we can write the same impedance law for the system as:

$$\tilde{M}_{des}\ddot{\tilde{e}}+\tilde{k}_d\dot{\tilde{e}}+\tilde{k}_p\tilde{e}+F_c=0 \tag{30}$$

where $\tilde{e}=\tilde{x}_{des}-\tilde{x}$ is the tracking error in the system controlled variables.

According to the MIC law the applying forces/torques could be divided in two parts as follows:

$$\tau_{app}=\tau_f+\tau_m \tag{31}$$

where τ_m is the required actuator forces/torques for the motion of the system, and τ_f is the required forces to be applied on the object by end effectors.

The controlling forces/torques τ_m could be calculated based on a feedback linearization approach as follows, [11]:

$$\tau_m=\hat{H}\tilde{M}^{-1}_{des}\left(\tilde{M}_{des}\ddot{\tilde{x}}+\tilde{k}_d\dot{\tilde{e}}+k_p\tilde{e}+\tilde{U}_{fc}F_c\right)+\hat{C} \tag{32a}$$

where

$$\hat{H}=\tilde{J}ac\,\tilde{H}\,\tilde{J}ac^{-1} \tag{32b}$$

$$\hat{C}=\tilde{J}ac^{-1}\,\tilde{C}-\hat{H}\,\tilde{J}_{dot}\,\dot{v} \tag{32c}$$

$$\tilde{U}_{fc}=\begin{bmatrix}Jac^T_{32} & 1_{3\times3} & \ldots\ldots & 1_{3\times3}\end{bmatrix}^T \tag{32d}$$

To obtain τ_f, based on the object dynamics as described in Eq. (29), the required controlling force $GF_{e\,req}$ could be written:

$$GF_{e\,req}=MM^{-1}_{des}\left(M_{des}\ddot{x}_{des}+k_d\dot{e}+k_p e+F_c\right)-F_\omega-F_o \tag{33}$$

The required desired force obtained in (33) could be used to determine τ_f for both manipulators, [11]:

$$\tau_f=\begin{Bmatrix}0_{2\times1}\\ F^1_{e\,req}\\ F^2_{e\,req}\end{Bmatrix} \tag{34}$$

It can be shown that by application of the MIC law, all participating manipulators, the moving base and the manipulated object behave with the same desired impedance behavior, [14].

VI. Simulation Results and Discussions

The simulated system consists of two 6-DOF manipulators mounted on a wheeled mobile platform as shown in Fig. 1, while the moving base is driven with two differentially driver wheels. All geometric and mass properties of the mobile base, and each of the two identical manipulators are given in Tables (1)-(2). The first manipulator is equipped with a remote center compliance (RCC) which is initially free of tension or compression, and its stiffness and damping properties are chosen as, [15]:

$$k_e = 2.4\times10^4\ kg/s^2 \text{ and } b_e = 5.5\times10^2\ kg/s$$

and the object parameters are

$$m_{obj} = 3\ kg\quad,\ r_e^{(1)} = -r_e^{(2)} = (-0.45,0,0)$$

where $r_e^{(i)}$ is the position i-th end effector with respect to the object center of mass. The obstacle is at $x_w = 4.65m$, and it is taken as a spring with $k_w = 1e5\,N/m$. The MIC algorithm is used to successfully move an object in a mixed circular-straight path as depicted in Fig. 3a. Therefore, to examine the capabilities of the MIC law the object will deliberately face a contact on its path (along straight part), where a smooth stop at the obstacle will be desired.

Fig. 3b shows the object real path in comparison with desired path, and it is revealed that they have very good correspondence. As it is seen, an accordant motion of both end-effectors results in smooth motion of the object on a 5-meter-radius circular path which continues through a straight part at the end of its maneuver. The existence of flexibility in the system due to RCC does not have any undesired effect on the object control which reveals the high capabilities of the MIC law in the presence of flexibility in the system. Fig. 4 shows the change of robot base yaw angle ϕ which varies with constant slope on the circular part of its path (until it reaches the straight part of its path), then the base orientation undergoes no change .

TABLE (1): PROPERTIES OF THE BASE

mass	*I* $(kg.m^2)$	l_G	*b*	$r_w\ (m)$	$\tau_{limit}^{left/right}$ $N.m$
50	20	1.25	1.5	0.25	200

TABLE (2): PROPERTIES OF EACH MANIPULATOR LINKS

i-th body	*length (m)*	$m_i^{(m)}$ kg	$I_i^{(m)}$ $kg.m^2$	$\tau_i^{(m)}\ N.m$
1	1	3	0.25	75
2	0.5	3	0.2	75
3	0.5	3	0.2	75
4	0.2	1	0.1	75
5	0.2	1	0.1	75
6	0.1	1	0.1	75

Fig. 5 shows variation of the contact force, which occurs at the time about $t = 19\,sec$. As it is seen its amount is zero during no contact phase, but during contact phase gradually increases. Based on the MIC law this amounts finally converges to a constant value that is governed by the defined system parameters. Fig. 6 and Fig. 7 show the torques variation of the left and right wheels, and the 1st end effector torques. As it is seen, the wheels react correspondingly, also at the time about $t = 6.7\,sec$ sudden changes appear, which corresponds to the time of switching from the circular part to the straight part of the path.

Fig. 8 shows the position error of the object, and the first and second end effector. As it is seen these figures show two points of disruption; which correspond to the time of switching from the circular part to the straight part of the path, and the impact due to contact with the obstacle, respectively. After the contact, the object tracking error will undergo a steady error which implies that the MIC law will not allow the object to experience excessive force. In other words, the cause of the error lies in the fact that the MIC law concentrates on the object dynamical behavior by applying defined impedances both at the object level and the robotic system level, rather than explicit force/position tracking control. So as it is seen the object will come into smooth stop at the obstacle, which shows the success of applying the desired impedance.

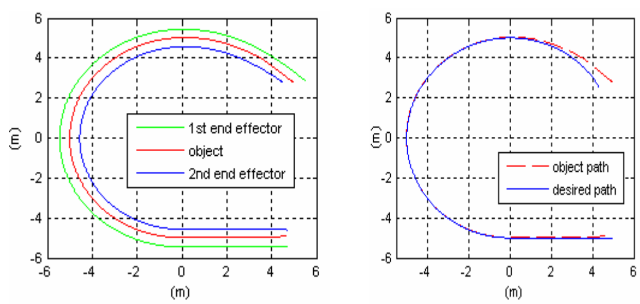

Figure 3. (a) The object and end-effectors real paths; (b) Comparison between the object CM real and desired paths

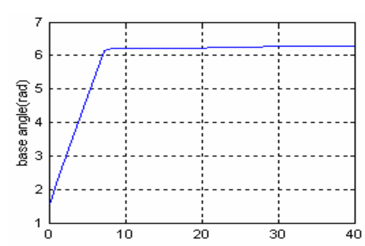

Figure 4. Variation of the base yaw angle with time

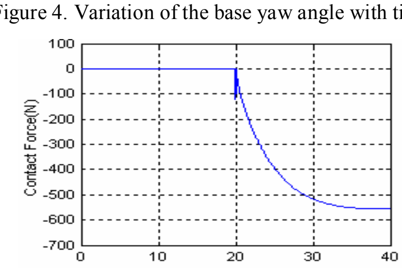

Figure 5. The contact Force

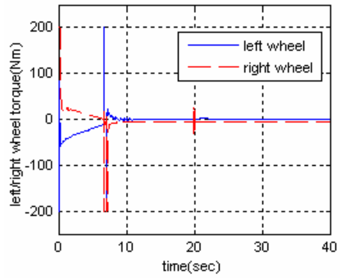

Figure 6. Variation of the left & right wheel torques

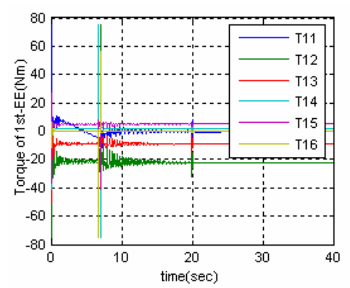

Figure 7. Variation of the 1st end effector torques

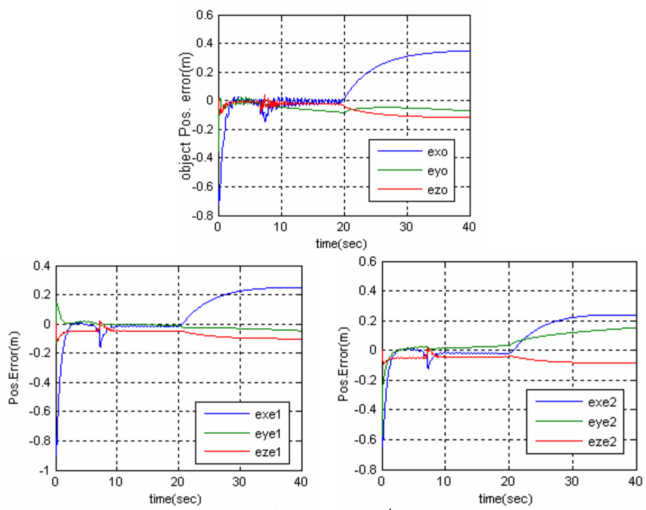

Figure 8. The object (top), 1st (left) and 2nd (right) end effector position tracking errors

VII. Conclusions

In this paper, using Lagrange method the equations of motions were derived based on direct path method (DPM). Next, the system non-holonomic constraint was derived, and using Natural Orthogonal Complement Method the independent set of equations of motion for the system was derived. Finally, the MIC law was applied to manipulate an object by two 6 DOF cooperating manipulators, one of them equipped with a remote center compliance (RCC), mounted on a wheeled platform while the moving base is driven with two differentially driver wheels. The obtained results reveal a coordinated smooth motion of the object, manipulators and the moving base, even in the presence of system flexibility, and impacts due to contact with an obstacle.

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