Limitations of the Narrow Fuzzy-Control and the Concept of Generalized Fuzzy-Control

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Abstract—The fuzzy-control usually takes the error and the error derivative between the output and the set point of the system as the input of the controller. But from the pure mathematical point of view, the variable and it’s every rank derivative of the system can be seen as the each state of the system. This paper shows some limitation of the classical fuzzy-control and brings forward the concept of generalized fuzzy-control base of the concept of classical fuzzy-control. The concept of generalized fuzzy-control and its application is explained as an example by state feedback based on IFO-KΔx plus PID in the coordinate control system of power plant.

Keywords—fuzzy-control; state feedback; state observer; IFO-KΔx

I. INTRODUCTION

The rationale of ordinary fuzzy-control takes the error and the error derivative between the output and the set point of the system as the input of the controller. This is equal with the output feedback control of the classical control[1]. But when a system with pure lag and the great inertial is controlled by the ordinary fuzzy-controller, the stability of the system is poor, and there is oscillation phenomenon as the steady-state error or follow-error is smaller. This is mainly caused by pure delay time constant.

The [2] summarily reviewed the research work for improving steady-state performance of the fuzzy-control in the [3-7]. The [3] put forward to adopt two-step control to improve steady-state performance of time-variable object fuzzy-control, but the second control of this method is after the narrow domain, the domain of u is still unable to accurately fixed and the number of rules is become huge. Narrow domain approximation is adopted in [4] to eliminate the residual error of fuzzy-controller. The value of proportion gene ku can only determined by experience after the narrow domain because the model of the system is unknown, so the steady-state error is still great. The [5] is based on three-dimensional fuzzy regulation, introducing feedback fuzzy regulator as a parallel correction with many variable, complex deduction and large rule tables, so it comes true very hard. An adaptive fuzzy-controller is put forward in [6], this controller can automatic adjust the control parameters of the controller according to e, de, and error-integral. But the parameter ku0, kuΔ and ku1 contained in ku can only be determined by steady-state experimental results, this is restricted in the real-time control. A method called polynomial replacement is proposed in the [7]. But this method takes two fuzzy inference processes, involving four control variables and two control rule tables, with cumbersome reasoning. Although from the principle of speaking, these fuzzy-controllers above have improved the steady-state performance to a certain extent, but they don’t eliminate static-error caused by fuzzy-control fundamentally. Even the characteristics such as easily understandable, easy to amend, simple operation and easy to achieve are lost [2].

In addition, in the study on the rationale of ordinary fuzzy-control, the [8] proves that the e and de as input, u as output, the fuzzy-controller of looking-up table can not eliminate static-error. The [9] proves that the linear control rules and nonlinear fuzzy algorithm, the structure of the two-dimensional fuzzy-controller taking du as output is the sum of the overall excessive value relay and partial nonlinear PI controller. When e and de are very small, the typical fuzzy-controller is similar to the linear PI controller. In other words, under certain condition, the fuzzy-controller taking du as output can eliminate static-error. However, results show that, if the domain of du much is too big and there will be an overshoot and fails to reach the purpose of eliminating static-error; otherwise, if the domain of du much is too small, although the static-error is eliminated, the response speed is too slow [2].

From the pure mathematical point of view, the variable and it’s every rank derivative of the system can be seen as the each state of the system. The [10] studies the system which is linear in each sect and whose parameter changes slowly, resolves the control process on the basis of the fuzzy-control’s essence and finds the essential relationship between the fuzzy-control and the state-feedback control, which is the control that variable is the function of the state. And analogy between the state feedback and fuzzy-control has drawn by analyzing the control variable.

Based on the work above, this paper further discuss the inherent nature of fuzzy-control, points out that the existing classic fuzzy-control is a narrow intelligent control method. Corresponding to this, the concept and definition of the generalized fuzzy-control are given. And thought method of generalized intelligent control is expatiated taking a practical case as example. The contact and interaction relations between the two methods of intelligent-control and classical control are revealed.
II. THE LIMITATIONS OF CLASSIC FUZZY-CONTROL AND NARROW INTELLIGENT CONTROL

Classical fuzzy-control usually has the following process [1]: 1. Fuzzying input variables, that is multiplying the input is: changing rate; 2. Fuzzying reasoning, that is fuzzying decision-making from E, EC and fuzzy-control rule R, R, according to synthesis rule of reasoning, then get the fuzzy-control variables. If the error E, the error changing rate EC and control variable u are admitted for the same domain, then fuzzy decision-making can be written as expression [12]:

\[ u = -\frac{1}{2}(E + EC) \]

3. non-fuzzying, fuzzy-control variable u is converted to precise variable u, that is multiplied by the proportion factor \( K_u \), then there is \( u = K_u \cdot u \), so

\[ u = -\frac{1}{2}K_u K_e e + \frac{1}{2}K_u K_e \dot{e} = k_1 e + k_2 \dot{e} \quad (1) \]

If the controlled object can be charged with piecewise linearization, the state space expression of a two-order with single input and single output system at certain linearization subsection can be expressed as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} u,
\]

\[ y = [c_1 \ c_2]
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} \quad (2) \]

The error e between output value and the set point, and the changing rate \( \dot{e} \) can be expressed as (r is the set value):

\[ e = y - r = c_1 x_1 + c_2 x_2 - r \quad (3) \]

\[ \dot{e} = \dot{y} = c_1 \dot{x}_1 + c_2 \dot{x}_2 = c_1 (a_1 x_1 + a_{12} x_2 + b_1 u) + c_2 (a_{21} x_1 + a_{22} x_2 + b_2 u) \quad (4) \]

Put formulas (3),(4) into formula (1)

\[ u = f(e, \dot{e}) = \frac{k_1 e + k_2 \sum_{j=1}^{3} (c_j a_j)}{1 - k_2 \sum_{i=1}^{3} c_i b_i} \quad (5) \]

If state feedback control is taken, then the control variable is:

\[ u = kx - vr = k_1 x_1 + k_2 x_2 - vr \quad (6) \]

Where \( k_1, k_2 \) are coefficients of feedback, \( v \) is coefficient of set point.

When the controlled object is three-order with single input and single output:

\[
A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}, B = \begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix}, C = [c_1 \ c_2 \ c_3] \quad (7)
\]

The error e between output value and the set point e, the changing rate \( \dot{e} \) and the changing accelerate \( \ddot{e} \) can be expressed as: (r is set point)

\[ e = y - r = \sum_{i=1}^{3} c_i x_i - r \quad (8) \]

\[ \dot{e} = \dot{y} = \sum_{i=1}^{3} c_i a_i x_i + \sum_{i=1}^{3} c_i b_i u \quad (9) \]

\[ \ddot{e} = \ddot{y} = \sum_{i=1}^{3} \sum_{k=1}^{3} c_i a_i a_i x_i + \sum_{i=1}^{3} \sum_{j=1}^{3} c_i a_i b_j u \quad (10) \]

If the error e and the changing rate \( \dot{e} \) are taken for fuzzy-control, then the simplified method of two-order system can be obtained:

\[ u = \frac{\sum_{i=1}^{3} k_1 c_i + k_2 \sum_{j=1}^{3} (c_j a_j)}{1 - k_2 \sum_{i=1}^{3} c_i b_i} \frac{k_1 r}{1 - k_2 \sum_{i=1}^{3} c_i b_i} \quad (11) \]

Where \( k_1, k_2 \) are the same meanings with formula (1).

If the error e, the changing rate \( \dot{e} \) and the changing accelerate \( \ddot{e} \) are taken for fuzzy-control, then their controlling variables can be fixed by the differential equations as follow:

\[ u = \sum_{j=1}^{3} k_1 c_j + k_2 \sum_{j=1}^{3} (c_j a_j) \frac{x_j}{1 - k_2 \sum_{j=1}^{3} c_j b_j} + k_2 \sum_{j=1}^{3} c_j b_j u - k_1 r \quad (12) \]

Where \( k_1 = \frac{1}{3} K_e K_{e}, k_2 = -\frac{1}{3} K_e K_{e}, k_3 = -\frac{1}{3} K_e K_{e} \) (\( K_e \) is proportion factor, \( K_e, K_{e}, K_{e} \) are quantify factors of e, \( \dot{e}, \ddot{e} \) respectively).

If state feedback control is taken, then the control variable of three-order system with single input and single output is:

\[ u = kx - vr = k_1 x_1 + k_2 x_2 + k_3 x_3 - vr \quad (13) \]

Where \( k_1, k_2, k_3 \) are coefficients of state feedback, \( v \) is coefficient of set point.

Comparing formula (5) with formula (6), formula (11) with formula (13) respectively, it can be seen that each of the two formulas have the same form formally. They are all state functions. From the point of fuzzy-control, the control variable changes following the error e and the derivative of error \( \dot{e} \), where e and \( \dot{e} \) are caused by the change of state substantively.
This point can be seen from formulas (3-4) or formulas (8-9), so the essential of fuzzy-control is function of state.

Although the essential of fuzzy-control is function of state, fuzzy-control cannot configure pole arbitrarily like state feedback for the following reasons:

1. For more than three-order (including three-order) system, if fuzzy-control only takes error $e$ and the changing rate $\dot{e}$, it can be seen by comparing formula (11) with formula (13) that both the formulas are the same in form, moreover:

$$k_1^i c_p + k_2^i \sum_{i=1}^{p} (c_i a_p) = \left(1 - k_2^i \sum_{i=1}^{p} c_i b_i \right) = v$$

Choosing $k_1^i$ and $k_2^i$, the coefficients $k_1, k_2, k_3$ of state feedback can be calculated by formula (14). On the contrary, choosing $k_1, k_2, k_3$, to get $k_1^i$ and $k_2^i$ by formula (14) is turned into that making the two variables meet the three dual linear equations in formula (14) at the same time. There is no solution usually except special circumstances in algebra. That is to say for more than three-order (including three-order) system, if fuzzy-control only takes error $e$ and the changing rate $\dot{e}$, it cannot configure pole arbitrarily like state feedback.

2. For three-order system, if fuzzy-control only takes error $e$ and the changing rate $\dot{e}$, it can be seen from formula (12) that the process needs to solve a differential equation and then compare the result with formula (13), this is very complex. The rest may be deduced by analogy, for a $n$-order ($n > 3$) system, if fuzzy-control takes $n$ input variables, it can be known from formula (12) that the control variable $u$ is expressed by a differential equation included $(n-2)$-order derivative of $u$, this is very complex.

Fuzzy-control itself is unable to overcome uncertainties, for two-order system, if fuzzy-control takes error $e$ and the changing rate $\dot{e}$, it can be seen from formula (5) and formula (6) that both the two formulas are the same in form, and that:

$$k_1^i c_1 + k_2^i \left( c_1 a_1 + c_2 a_2 \right) = \left(1 - k_2^i \left( c_1 b_1 + c_2 b_2 \right) \right) = k_1$$

$$k_1^i c_2 + k_2^i \left( c_1 a_2 + c_2 a_2 \right) = \left(1 - k_2^i \left( c_1 b_1 + c_2 b_2 \right) \right) = k_2$$

$$k_1^i \left(1 - k_2^i \sum_{i=1}^{p} c_i b_i \right) = v$$

For the two variables $k_1^i, k_2^i$ meeting the dual linear equations in formula (16) at the same time can be solved in the algebra. After $k_1^i, k_2^i$ are obtained, $v$ is confirmed by (17). When the parameters above are matched, fuzzy-control and state feedback control seem to be equivalent, but there is an important difference virtually. The coefficients of state feedback gain $k_1^i, k_2^i$ are fixed according to the theorem that state feedback pole collocation, then $k_1^i, k_2^i$ are not related to the controlled object. While the equivalent state feedback gain $k_1^i c_1 + k_2^i \left( c_1 a_1 + c_2 a_2 \right)$ and $k_1^i \left( c_1 b_1 + c_2 b_2 \right)$ in formula (5) is related to the characteristic of controlled objects such as $b_1, b_2, c_1, c_2, a_1, a_2$. If the controlled object is the parameters slow-time-variable or is uncertain, then the coefficients of state feedback gain in (5) is also time-variable or uncertain. It is difficult to overcome time-variable or uncertain.

Fuzzy-control itself has the factors inherently which lead to system unstable. It can be seen from (5) and (11) that there are such coefficient as $\frac{1}{1 - k_2(c_1 b_1 + c_2 b_2)}$ or $\frac{1}{1 - k_2(c_1 b_1 + c_2 b_2)}$ of equivalent state feedback gain in (5) is also time-variable or uncertain. It is difficult to overcome time-variable or uncertain.

In the designing and debugging stage, this situation can be avoided by selecting $k_1$ appropriately. When $k_1$ is fixed, in the process of fuzzy-control system operation, if the controlled object has uncertainty of some parameter, then the probability of $k_2(c_1 b_1 + c_2 b_2)$ or $k_2(c_1 b_1 + c_2 b_2)$ approximate to 1, he control variable $u$ will change greatly, this will cause the system surge easily.

From the pure mathematical point of view, the variable and it’s every rank derivative of the system can be seen as the each state of the system. But from the point of view of control theory, the state $x_i$ and error $e$ of system and its every rank derivative $\dot{e}, \ddot{e}$ and so on, they are different. It can be seen from formula (2) that when $[c_1 \ c_2]$ changes, $y$ will change, too. But $[c_1 \ c_2]$ does not affect the state $x_i$. This will give different consequences to two control methods. If the state-feedback control is used, the control variable $u$ does not change; If the fuzzy-control is used, the control variable $u$ will change.

The mechanism of fuzzy-control is to sum up the people’s control action and to describe people’s manual decision-making using language. Finally all above is summed up into a series of conditional sentences (control rules). Fuzzy-control system needs the person’s experience and decision-making action model.

Experience is very important in fuzzy-control, so whether or not get the experience which reflects the essence of objective things and how to get the method of the experience is important. This relates to whether fuzzy-control is successful.

Experience obtained by the isolated observation results or the behaviour of individual in not enough long time is not enough, especially there are some uncertainties in system.
III. THE CONCEPT OF GENERALIZED FUZZY-CONTROL AND ITS MATHEMATICAL ANALYSIS

This paper presents the classic fuzzy-control as narrow fuzzy-control, and correspondingly, the concept of generalized fuzzy control defined as follows:

Definition: No matter what method in what way to obtain or to measured full states of a controlled object, or any part or function of the states, the state feedback control and PID regulator integrated control system based on these states, or function of the states, can be called generalized fuzzy-control.

Proposition 1: The state feedback control based on the incremental function observer \((IFO-K\Delta x)\) can be called as generalized fuzzy-control.

The characteristic of controlled object: it has non-linear, parameters slow-time-variable, the time lag and the great inertia, which lead to the accurate mathematical model of controlled object difficult to obtain, or although the model can be given according to working mechanism or its operating experience, the parameters of the model can not track the actual controlled object due to time-variable. Thereby many difficulties are brought to design and debug of control system.

Background of engineering: generator unit coordinated control system, boiler overheat or reheat steam temperature control system of 300MW, 600MW large-capacity thermal power unit

Taking coordinated control of power plant as an example, the controlled object can be described by controllable linear system and a group of nonlinear functions as follow:

\[
\begin{align*}
\dot{x} &= Ax + Bu, \\
y &= Cx, \\
u &= f(v)
\end{align*}
\]  

Where: \(x \in \mathbb{R}^n, A, B \text{ and } C\) are real matrices with proper order, and \(v, y, u\) are input and output vectors of proper dimension, \(f\) is the relation of nonlinear function with proper dimension.

Then, applying state-space method to stability analysis, it can be described by state-equations as follow:

\[
\begin{align*}
\dot{z} &= Fz + G\dot{y} + H\dot{u}, \\
\dot{w} &= Mz + N\dot{u}, \\
w_1 &= f_{w_1}(\dot{y}_1, A\dot{y}_1, R_{\text{MDM}}) \\
w_2 &= -w + A\dot{y}_{2,\text{PID}} + w_1 \\
\dot{\mu} &= v - f_{\mu}(R_{\text{MDM}}) \\
\dot{\mu} &= y - f_{\mu}(R_{\text{MDM}}) \\
-\dot{y}_{1,\text{PID}} &= I_1, y_1 + I_2, z_1 + I_3, \cdot \dot{y}_1 \\
\dot{z}_1 &= (k+1) \cdot A\dot{y}_1, \\
z_1(t_0) &= z_{10} \\
\dot{y}_2 &= A\dot{y}_2, \\
z_2(t_0) &= z_{20}
\end{align*}
\]  

The relation between active power (MW) and total fuel flow (%) of a 600MW unit is shown as figure 2. It can be seen that the relation between unit and total fuel flow is not one by one because of some changes.
Where: $z \in R^F, F, H, G, N, M$ are real matrices with proper order, $w, w_1, w_2, \Delta u, \Delta y, \Delta y_1, \Delta y_2, \Delta y_{PID}, \Delta y_{PID}$ are vectors of intermediate results with proper dimension, $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ are parameter vectors of regulator PID with proper dimension, $I$ is an identity matrix, $z_1, z_2$ are state-vectors with proper dimension, $\bar{y}_1, \bar{y}_2$ are vectors of process variable to be controlled with proper dimension and belong to $y$ that is output vector. The variable $R_{DM}$ is measurable and $f_i, f, f_1, f_2$ represent relation of nonlinear function with proper dimension. $[F M]$ is observable.

The united formulas (23) to (25) form a boiler-turbine coordinated control system of generator unit that nonlinear system (21)-(26) is asymptotically stable. When the sufficient conditions as follow are satisfied:

1. The matrix $F$ is nonsingular and $\Re \lambda(F) < 0$

2. $\lim_{t \to \infty} |\Delta u| = \lim_{t \to \infty} |\Delta y| = |\xi_1| \leq \omega_1$

3. $\lim_{t \to \infty} |\xi_2| \leq |\omega_2|$

4. $\lim_{t \to \infty} |\Delta y| = |\xi_2| \leq |\omega_2|$

5. $\lim_{t \to \infty} |\Delta y| = |\xi_2| \leq |\omega_2|$

6. $\lim_{t \to \infty} |\Delta y| = |\xi_2| \leq |\omega_2|$

7. $\lim_{t \to \infty} |\Delta y| = |\xi_2| \leq |\omega_2|$

and existing a real matrix $T$ with proper order making:

1. $TA - FT = GC$

2. $MT-NC = K$

Moreover

$\lim_{t \to \infty} |\Delta y| = |\xi_2| \leq |\omega_2|$

Then the closed-loop control system shown as united system (21)-(26) is asymptotically stable.

Proof: Setting

$z_3 = z + F^{-1} HI \xi_1 + F^{-1} GI \xi_2$

$w_3 = w + MF^{-1} HI \xi_1 + MF^{-1} GI \xi_2 = Mz_3 + N\Delta \hat{y}$

$e = z_3 - T \Delta x, e = w - NI \xi_2 - K \Delta x$

It can be achieved by the condition in the theorem 1 comes into existence, meanwhile according to median theorem, then:

$\lim_{t \to \infty} e = Fe + [GC + FT - TA] \Delta x$

$+ H \Delta u - TB \Delta u - HI \xi_1 - GI \xi_2$ (30)

If define

$\Delta \hat{y} = C \Delta x + I \delta_1, C \Delta x$

Where $\delta_1$ is vector of the unknown perturbation elements, then

$\dot{e} = Fe + [GC + FT - TA] \Delta x + GI \delta_1, C \Delta x$

$+ H \Delta u - TB \Delta u - HI \xi_1 - GI \xi_2$

Moreover, noticing that formulas (29), (28), (23), (21) and (31), Then

$e = Me + [N(C + MT) - K] \Delta x - NI \xi_2 - K \Delta x$ (33)

When the condition in the theorem 1 comes into existence, meanwhile according to median theorem, then:

$\lim_{t \to \infty} e = Fe + [GC + FT - TA] \Delta x$

$+ H \Delta u - TBf(\xi) \Delta x - HI \xi_1$ (34)

$e = Me + [N(C + MT - K) \Delta x$ (35)

When the conditions and in the theorem 1 are existent, then the formulas (34) and (35) can be shown as:

$\lim_{t \to \infty} e = Fe + H \Delta u - TBf(\xi) \Delta x - HI \xi_1$ (36)

$e = Me$ (37)

It can be known from the conditions in the theorem 1 that:

$\lim_{t \to \infty} e(t) = \lim_{t \to \infty} |z_1(t) - T \Delta x(t)| = 0$ (38)

Furthermore, there will be:

$\lim_{t \to \infty} e(t) = \lim_{t \to \infty} |w_3(t) - NI \xi_2 - K \Delta x(t)| = 0$ (39)

So the stability of the observer can be proved.

Now, the stability of the entire system can be proved. It can be known from the condition in the theorem 1 that:

$\lim_{t \to \infty} w_2 = \lim_{t \to \infty} y_1 = \lim_{t \to \infty} y = \gamma$ (40)

From formula (40) , (23) and the condition in the theorem 1, then:

$\lim_{t \to \infty} v_{PID} = \lim_{t \to \infty} (w_2 - w_1 + w)$

$\lim_{t \to \infty} v - \lim_{t \to \infty} f, (R_{DM}) \xi_3 + \lim_{t \to \infty} w$ (41)

Uniting the condition and in the theorem 1 with (41), It can be obtained that:
\[-M\left(F^{-1}HI\xi_1 + F^{-1}GI\xi_2\right) = \lim_{t \to \infty} Mz\]  
(42)

\[\lim_{t \to \infty} z = \left(F^{-1}HI\xi_1 + F^{-1}GI\xi_2\right)\]  
(43)

Notice that formulas (27),(29),(38),(39), then:

\[\lim_{t \to \infty} \xi = \lim_{t \to \infty} \left(\xi + F^{-1}HI\xi_1 + F^{-1}GI\xi_2\right) = 0\]  
(44)

\[\lim_{t \to \infty} T\Delta x = \lim_{t \to \infty} (\xi - \xi) = 0\]  
(45)

Because \(T\) is not equal to \(\theta\), if formula (45) coming into existence constantly, there will only be:

\[\lim_{t \to \infty} \Delta x = 0\]  
(46)

The theorem 1 has been proved.

IV. APPLICATION

The generalized fuzzy-control base of state feedback based on IFO-K\(\Delta x\) plus PID is well applied in power plant of 660MW turbine-generator-unit with super-critical-press boiler. Figure 3 shows the trend of the active power changed from 525MW to 495MW to 445MW, with the rate 12MW/min, and the press changed from 24.2MPa to 22.9MPa, the whole process control is well. It decreases the work load of operator and increases the security and stability of the unit.

Figure 4 shows the trend of reheated steam temperature control of 300MW unit generator in power plant system.

Reheated steam temperature is adjusted by changing the open of recirculation smoke damper, so as to change the smoke recycle flow. The control scheme is base of generalized fuzzy control state feedback based on IFO-K\(\Delta x\) plus PID.

It can be seen from the figure 4 that the state feedback is relate to the changed rate of reheated steam temperature, the PID output is relate to the error of the reheated steam temperature, the final control output is constituted by above two parts.

Figure 3. The trend of boiler turbine coordinated control system of state feedback based on IFO-K\(\Delta x\) plus PID

Figure 4. The trend of reheated steam temperate control of state feedback based on IFO--K\(\Delta x\) plus PID

V. CONCLUSION

By analyzing the classical fuzzy control and control rule, it can be deduced that controlling quantity is state function in essence and indicate the limitation of classical fuzzy control. The concept of generalized fuzzy-control is presented based on the concept of classical fuzzy control. The control system made of state feedback based on IFO-K\(\Delta x\) plus PID as the example of generalized fuzzy-control is analyzed. The practice shows that it is typical generalized fuzzy-control system and very good in application of power plant.

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