An Optimization Approach for Intersection Signal Timing Based on Multi-Objective Particle Swarm Optimization

Hao Pang Feng Chen

Department of Automation
University of Science and Technology of China
Hefei, Anhui, 230027, China
shamrock@mail.ustc.edu.cn chenfeng@ustc.edu.cn

***Abstract*— Intersection signal timing is one of the key techniques in intelligent transportation system (ITS). Both the average delay and stop frequency are important indices for evaluating the level of service (LOS) for signalized intersections. Traditional signal timing models either optimize only one of them or deal with them as a single objective using weighted average methods. In this paper, a Multi-Objective Particle Swarm Optimization (MOPSO) method is proposed to optimize the both evaluation indices synchronously. A well-distributed set of Pareto optimal solutions is obtained, and the most satisfied solution is selected by the multi-objective decision-maker module. The experimental results indicate this optimal method is steady and effective.**

***Keywords*—average delay, average stop frequency, multi-objective, Particle Swarm Optimization**

I. INTRODUCTION

With the urban population and cars increasing, traffic congestion has become more and more serious. Intersection is an important component of the urban transportation networks; meanwhile, it is also the major occurring place of traffic congestion. Traffic signal control can provide for the orderly movement of traffic flow, and reduce the frequency and severity of traffic jams at the intersection. The main approach of signal control is to implement the timing model for a signalized intersection. Signal timing model can generate the optimal signal timing plan by calculating appropriate cycle length and green split of each phase, which is crucial for signal control and improves the signalized intersection's level of service (LOS) effectively. Therefore, lots of scholars devoted themselves to designing a reasonable timing model. Reference [1] gave a formula of the optimal cycle length for minimizing vehicle average delay, and the green-light time was determined according to the traffic flow ratio. But stop frequency per vehicle was not considered in this model. In [2], on the basis of Webster's model, the stopping compensation coefficient was added to modify the formula for calculating the optimal cycle length, but it was difficult to be acquired this coefficient precisely. Reference [3] transformed the average delay, average stops and traffic capacity into a single objective by calculating their weighted sum, and adopted Tabu search algorithm to find approximate solution. However, the ratio for the weighted coefficient of average delay and average stop frequency was constant in her model, which meant the two parameters had a linear correlation. That was inconsistent with the actual conditions.

Average control delay and average stop frequency are important evaluation indices of traffic signal plan. They are useful for the determination of the cycle length and the green splits. Reference [4] analyzed the correlation coefficient between the two indices and other traffic parameters, and suggested the relationship between these two indices was neither linear nor absolutely monotone, but a grey correlation. It means when one of the two indices is increased, the other might be increased or decreased. Therefore, it is suitable to establish two objectives optimal model so as to optimize both of them simultaneously. Particle Swarm Optimization (PSO) is a fast optimization algorithm based on swarm intelligence. Its inherent characteristics, such as implicit parallelism, can improve the efficiency of multi-objective optimal problem. In this study, a multi-objective optimization algorithm based on PSO is proposed to optimize average delay and average stop frequency simultaneously. This algorithm calculates the Pareto solution aggregate of the cycle length and green-light time of each phase, the most satisfied solution is provided by the decision-maker module. The experimental results demonstrate this algorithm effective.

II. BASIC THEORY

A. Model of Objective Functions

In signal timing models, average control delay functions are essential for evaluating the traffic conditions of a signalized intersection. These functions directly relate with the LOS of the whole intersection. The basic definition of average control delay is the travel time loss caused by traffic friction resistance and signal control [5]. It is also related to other

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traffic parameters such as cycle length, green splits, and saturation. In the case of an unsaturated traffic situation, the delay formula is expressed as (1):

$$d_i = \frac{C(1-\lambda_i)^2}{2(1-\lambda_i X_i)} + \frac{X_i^2}{2q_i(1-X_i)} - 0.65\left(\frac{c}{q_i^2}\right)^{1/3} X_i^{(2+5\lambda_i)} \quad (1)$$

Where d_i (sec/pcu) is average control delay per vehicle on the particular lane group of the i-th phase; (pcu means the passenger car unit.) C (sec) is the cycle length; q_i (pcu/sec) is the flow rate on the particular lane group of the i-th phase; λ_i is the proportion of effective green respect to cycle length of the i-th phase; (i.e. g_i /C and g_i (sec) is effective green time of the i-th phase.) and X_i is the degree of saturation of the i-th phase of the intersection [6].

The first term of (1) represents the delay when the traffic is assumed to be arriving uniformly. The second term of the equation denotes the experienced delay due to vehicles arriving randomly. The third term of the equation is an empirical correction term to give a closer fit for all values of traffic flow. Normally, the last term is quite small compared to the whole delay and is frequently neglected for actual calculation [6].

The average control delay in the whole cycle length should equal to the value of weighted average of each phase and be expressed as (2):

$$d = \sum_1^n d_i q_i \Big/ \sum_1^n q_i \quad (2)$$

Where n is the number of phase in one cycle [7].

The average stop frequency per vehicle is also an important parameter for judging the LOS of a signalized intersection. It represents the number of stops when one vehicle passes through the signalized intersection. The average stop frequency per vehicle of the i-th phase is expressed as (3):

$$h_i = 0.9\frac{1-\lambda_i}{1-y_i} \quad (3)$$

Where h_i means average stop frequency per vehicle of the i-th phase; and y_i is the flow ratio of the particular lane group of the i-th phase. It represents the ratio of actual flow rate to saturation flow rate [7].

Similar to the average delay, the average stop frequency in the whole cycle length should equal the value of weighted average of each phase and be expressed as (4):

$$h = \sum_1^n h_i q_i \Big/ \sum_1^n q_i \quad (4)$$

Where n is also the number of phase in one cycle [7].

As a result, the optimal timing model taken into consideration both average control delay and average stop frequency is described as follows：

$$\min \ y = \{d(C,\lambda_i), h(C,\lambda_i)\} \quad (5)$$

B. Multi-Objective Optimization Problem

If there are two and more objectives to be optimized simultaneously, there is no longer a single optimal solution but a whole set of possible solutions of equivalent quality [8]. Generally, the definition of minimum multi-objective problem with n decision variables x and m objectives y is expressed as follows:

$$\min \ y = f(x) = \{f_1(x), f_2(x), \cdots, f_m(x)\} \quad (6)$$

Where $x = (x_1, x_2, \cdots, x_n)$;

$$y = (y_1, y_2, \cdots, y_n);$$

$$x \in S = \{x \mid g_j(x) \le 0, j = 1, 2, \cdots, p\};$$

And where x is called decision vector; y is called objective vector; S is the feasible solution region, and g_j represents the j-th constraint of this problem [8].

1) DEFINITION 1:

A decision vector $u \in S$ is referred to dominate a decision vector $v \in S$ only if:

$$\forall i \in \{1, \ldots, n\} \quad f_i(u) \le f_i(v)$$

$$\text{And } \exists j \in \{1, \ldots, n\} \quad f_i(u) < f_i(v)$$

Usually, u dominates v can be written as $u \succ v$ [8].

Based on this definition, Pareto optimal solutions can be defined as follows:

2) DEFINITION 2:

Let $u \in S$ be an arbitrary decision vector.

(a) The decision vector u is referred to be non-dominated regarding a set $S' \subseteq S$ only if there is no vector in S' which dominates u;

(b) The decision vector u is called Pareto optimal solution only if u is non-dominated regarding the whole feasible solution space S.

The key to solving multi-objective problem is to find the Pareto optimal solutions from the feasible solution region, and decision makers can choose the most satisfied solution or a set of optimal solutions from them [8].

C. Particle Swarm Optimization

The Particle Swarm Optimization (PSO) is a parallel stochastic search algorithm first proposed by Kennedy and Eberhart. It is a population-based algorithm and its procedure is simpler than Genetic Algorithm's. This algorithm simulates social behavior such as flying bird flock in searching of food. The behavior of each particle is affected by the behaviors of neighborhoods and the whole swarm [9].

This algorithm is initialized with a swarm of random solution at the feasible solution region. The individual called particle flows through the solution space by following the current best one. At each iteration, the position of each particle is updated by a new velocity calculated through (7) and (8) which is based on its precious velocity, the position of the best solution so far has been achieved by the particle itself (pbest), and the position at which the best solution so far has been achieved by the global swarm (gbest):

$$V(t+1)=w^*V(t)+c_1u\left(pBest(t)-p(t)\right)+c_2v\left(gBest(t)-p(t)\right) \quad (7)$$

$$p(t+1)=p(t)+V(t+1) \quad (8)$$

Where w is a weight determining the proportion of the particle's previous preserved; c_1 and c_2 are two positive acceleration constants, u and v are two uniform random sequences produced from U (0, 1) [9].

Fitness values obtained from the objective functions drive the particles to "fly" through the solution space and are attracted to both their personal best solution and the best position found by the global swarm. Finally, they converge to the optimal solution [9].

III. TIMING MODEL BASED ON MOPSO

A. MOPSO Algorithm Process

Multi-objective particle swarm optimization (MOPSO) uses PSO algorithm to optimize each objective in the feasible solution region. This optimization approach is effective to solve multi-objective optimization problem and easy to be implemented. Strength Pareto Evolutionary Algorithm (SPEA) is a classical method among all major multi-objective EAs which is based on the Pareto-optimality and dominance [8]. This study proposes the MOPSO algorithm based on SPEA to optimize average delay and average stop frequency simultaneously. The detailed procedure is described as follows:

Step 1: Initialize the particle swarm P. In this model, each particle represents a signal timing plan in the same traffic condition. The dimension of the particle equals to the number of phases on this plan, and its position represents the vector of the green-light time of each phase. The initialized position of each particle is generated randomly in the feasible solution region and the initialized speed is set to zero. The initialization of pbest and gbest of each particle is set to itself. The value range of the position $(X_{\min}, X_{\max})$ and the maximum speed $V_{\max}$ should be initialized to avoid the individuals flowing out of the solution space. An empty external set of non-dominated individuals P' is also created.

Step 2: Update the position and speed of each particle by using (7) and (8).

Step 3: Calculate the objective function values of each particle. In this model, average delay and average stops are calculated with (2) and (4).

Step 4: Update the pbest of each particle. For each individual in the swarm, the pbest is replaced by the current position if the current position dominates the pbest of this particle. Otherwise, the pbest is not updated.

Step 5: Find all non-dominated members of P at this iteration, and copy them to P'.

Step 6: Remove solutions within P' which are dominated by any other member of P'.

Step 7: If the number of stored non-dominated solutions exceeds a given maximum N, reduce the population size of P' to N by means of clustering procedure. This iterative procedure clusters particles which have the minimum distance in P'.

Step 8: For each particle, select a solution randomly from P' and compare to the gbest of this particle. If the solution dominates gbest, then replaces it; else do not update the gbest of this particle. In order to accelerate the convergence speed of this algorithm, the value of gbest should not vary frequently. That makes the particle search on a steady approach. A variation coefficient of the particle's gbest is also introduced to avoid the algorithm dropping into local optimum [10].

Step 9: If maximum number of generations is reached, then stop; else go to Step 2.

B. Design of Multi-Objective Decision-Maker

Multi-Objective Decision-maker is a mechanism that makes sure the most suitable solution for the practical situation be selected from the set of non-dominated solutions. By using MOPSO algorithm, a set of Pareto optimal solutions are generated. However, single intersection usually uses only one signal timing plan. The multi-objective decision-maker helps to choose the most suitable timing plan according to the environment and dynamic traffic conditions. When the saturation degree of the intersection is closed to 1, the decision-maker may choose the solution which optimizes the average delay. When the intersection closes to residential areas, it may select the solution which optimizes the average stop frequency to avoid pollution increasing due to vehicles start and brake. The multi-objective decision-maker is designed as follows:

Step 1: Design a fuzzy matrix about the proportion of each index considered in the decision. The dimension of this matrix is usually set to 3, and its elements can be changed frequently.

Step 2: According to the actual situation about this intersection, determine which is more important between average delay and average stop frequency. Then select an element form the fuzzy matrix to represent the ratio of weight between control delay and stop frequency.

Step 3: Modify the value of objectives by using the element selected at Step 2. Then sort the members within the set of non-dominated solutions by the value of the objective which is more important.

Step 4: Offer the most fitting solution. Because the set of non-dominated solutions has been modified by considering the effect of actual situation, and also been sorted by the value of the key objective. So the first solution will be the best signal timing plan that meets the optimization of the both indices and the practical traffic condition about this intersection.

IV. EXPERIMENTS AND DISCUSSION

In order to verify the property of this traffic timing model based on MOPSO, we select two intersections in different areas for our experiments.

1) *INTERSECTION 1:*

The first intersection called Baihuajin is in Hefei, Anhui province, China. It is a crossroad and locates near the people's living region. The flow rate of each approach is usually not high, and the number of phase is set to 4. The traffic data from 13:30 to 14:30 at May 23, 2003 are listed in Table 1:

TABLE I. TRAFFIC DATA ON BAIHUAJIN

Lane Direction	Actual Flow Rate (pcu/h)	Saturation Flow (pcu/h)
East straight	290	1650
East left-turn	118	1550
South straight	219	1550
South left-turn	108	1450
West straight	275	1650
West left-turn	78	1550
North straight	331	1550
North left-turn	176	1450

2) INTERSECTION 2:

The second intersection called Longshan Road locates in Anqing, Anhui province, China. This intersection is also a crossroad, and its location is on a slope top. The capacity of each lane is lower than the first intersection because of the narrow width of lanes and the existence of the slope. However, the flow rate of each lane is often high because this intersection is near the city center. So the number of phase is usually set to 2. The traffic data from 8:00 to 12:00 are listed in Table 2:

TABLE II. TRAFFIC DATA ON LONGSHAN ROAD

Lane Direction	Actual Flow Rate (pcu/h)	Saturation Flow (pcu/h)
East straight	245	850
East left-turn	79	850
South straight	356	900
South left-turn	117	900
West straight	265	850
West left-turn	45	850
North straight	386	900
North left-turn	312	900

According to the actual situation, some constraints have to be described before the experiments:

$$0.5 \le X_i \le 0.95;$$
$$60 \le C \le 180;$$
$$10 \le g_i \le 45;$$
$$L = 3 \text{ (sec)};$$

Where L is the lost time of each phase in a cycle length.

In these two experiments, we compare current traffic signal plan, Webster traffic model [1], and timing model based on PSO for single objective with MOPSO so as to verify the efficiency of our algorithm. Each method has operated for 50 times. Each method of SPSO and MOPSO iterates 20000 times and the population size is set to 50. The results are shown in Table 3 and Table 4:

TABLE III. COMPARISON OF ALGORITHM RESULTS ABOUT BAIHUAJIN INTERSECTION

Optimization Methods	Delay (sec/pcu)	Stops (num/pcu)
Current signal plan	53.24	0.8063
Webster	43.61	0.8448
SPSO (average delay)	38.83	0.8311
SPSO (average stops)	115.20	0.7819
MOPSO (decision 1)	40.27	0.8301
MOPSO (decision 2)	88.56	0.7827
MOPSO (decision 3)	56.54	0.7901

TABLE IV. COMPARISON OF ALGORITHM RESULTS ABOUT LONGSHAN ROAD INTERSECTION

Optimization Methods	Delay (sec/pcu)	Stops (num/pcu)
Current signal plan	34.56	0.7503
Webster	30.40	0.7782
SPSO (average delay)	28.85	0.7591
SPSO (average stops)	66.64	0.7249
MOPSO (decision 1)	29.02	0.7540
MOPSO (decision 2)	65.75	0.7269
MOPSO (decision 3)	33.68	0.7381

In Table 3 and Table 4, SPSO (average delay) tries to optimize the average control delay while SPSO (average stops) tries to optimize the average stop frequency. The three experiments of MOPSO use the same optimization algorithm, but their decision-makers choose the different ratio of weight between average delay and average stop frequency. MOPSO (decision 1) thinks optimizing average delay is much more important than optimizing average stop frequency, so it chooses the biggest ratio of weight between average delay and stop frequency. On the contrary, MOPSO (decision 2) selects the smallest one. MOPSO (decision 3) considers both of the indices and the intersection's practical environment, and obtains the suitable solution by selecting the appropriate element of that matrix. The Maximum number of members within the non-dominated set equals to 50 for all three MOPSO.

According to the experiments focused on minimizing the average delay, it is obvious that SPSO (average delay) and MOPSO (decision 1) are better than the current timing plan and Webster method, especially when the average delay is high. That the average delay from MOPSO (decision 1) is close to the one from SPSO (average delay) indicates that the Pareto optimal solutions have a reasonable distribution. The comparison between SPSO (average stops) and MOPSO (decision 2) shows the same conclusion. Although average delay from MOPSO (decision 3) is inferior to the result from MOPSO (decision 1) and SPSO (average delay), the other objective is superior to the one from MOPSO (decision 1) and SPSO (average delay).

In Table 3, the solution obtained from MOPSO (decision 3) is closer to the one from MOPSO (decision 2) than MOPSO (decision 1). However, the solution obtained from MOPSO (decision 3) has the opposite situation in Table 4. Because the environment and the traffic conditions of Baihuajin intersection are different from Longshan Road intersection. Baihuajin intersection is near the living areas and the flow rate is not high; in this situation, the decision-maker of MOPSO (decision 3) suggests optimizing average stop frequency is more important than optimizing average delay. On the contrary, Longshan Road intersection locates in the city center and capacity of its lanes is low. To avoid traffic congestion and decrease the degree of saturation, the decision-maker of MOPSO (decision 3) suggests optimizing average control delay is more important than optimizing average stops. The solution of MOPSO (decision 3) is also selected from the Pareto optimal solutions, so the results from this solution will be the most fitting timing plan for fixed-cycle signal control.

V. CONCLUSIONS

In this paper, a multi-objective optimization algorithm based on PSO (MOPSO) is proposed to optimize the average delay and average stop frequency simultaneously. The Pareto solutions of the cycle length and green light time of each phase are obtained as well. Finally, the most satisfied solution is provided by the multi-objective decision-maker module. The experiment results show that this algorithm not only obtains better solution than Webster's optimal model when minimizing average delay, buy also provides other best solutions which minimize average stop frequency or optimize them both. The decision-maker is also designed to choose the suitable solution according to the traffic situation.

The further study will focus on improving the convergence speed of MOPSO algorithm by adding new operators, and researching the interaction between the multi-objective optimization algorithm and the multi-objective decision- maker.

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