Dissimilar Redundancy Measurement of Swashplate Angle of Variable Pump Based on the Exact Linearization Model of EHCA

Yan Lang[1], Yunhua Li[2]
School of Automation Science and Electrical Engineering
Beijing University of Aeronautics and Astronautics
Beijing 100083, China
1: yan.lang@126.com, 2: yhli@buaa.edu.cn

***Abstract*—This paper mainly focuses on the swashplate angle dissimilar redundancy measurement of the variable displacement pump in the electrohydraulic compound integrated actuator (EHCA). Dissimilar redundancy means that the practical system can acquire more than one measurement values to describe a certain signal through hardware or software manners. Aiming to the need of software sensing, the signal needed to be rebuilt is computed from the system model. So it proposes the exact linearization model of EHCA to handle the nonlinear problem of load flow in this paper. Comparing to the linearization model which nearing the working point and the general nonlinear model, the exact linearization method which based on nonlinear transformation in EHCA modeling, is not only truly reflects system characters with simple linear control theory, but also provides a reliable model for computing swashplate angle by using other signals measurement value from sensors. This method provides a valuable reference model in the practical dissimilar redundancy applications.**

***Keywords*—electrohydraulic, actuator, exact linearization model, dissimilar redundancy, H_∞ control, swashplate angle**

I. INTRODUCTION

Aircraft itself has the demand of high security, and this goal is based on the high reliability of its components. Many research show that above 60 percent of system faults are occurred in the relative parts of controllers. And most of the controller faults are sensors and actuations failures, especially the sensor malfunctions are more common. It is a very important problem that how to rapidly detect the operating states of sensing system and exactly rebuild the failure signals to keep aircraft in the normal operation. Comparing to the way of backuping several similar channels, dissimilar redundancy based detection is a more effective method, because the latter can acquire more than one measure values of a certain signal through several different hardware or software manners. Hardware sensing is directly obtaining signal value from the corresponding sensor. Usually, it is convenient to measure signals directly by sensor, but considering the potential faults of sensing system, two or three different measurement channels are needed to backup as reference values for monitoring whether the sensing signal is in normal. Once the hardware signal fails, the other backup software signals also can rebuild and replace the fault measures and maintain the whole system operating normally. The following will take the swashplate angle measure of variable displacement pump for example, and process a dissimilar redundancy monitoring on account of its weakness in the operation and its great importance on the electrohydraulic compound integrated actuator (EHCA).

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II. EXACT LINEARIZATION MODEL OF EHCA

Here we will adopt three channels for swashplate angle measure of variable displacement pump. One is directly reading value from the angle sensor; the other two are respectively computing the angle by detecting the control cylinder displacement of pump and by EHCA health model. So the EHCA system model should be built first. Due to the load flow nonlinear problem caused by the product of pump displacement and motor rotary speed, we will use exact linearization method to handle it.

A. System Sructure of EHCA

First, the EHCA system construction will be introduced.

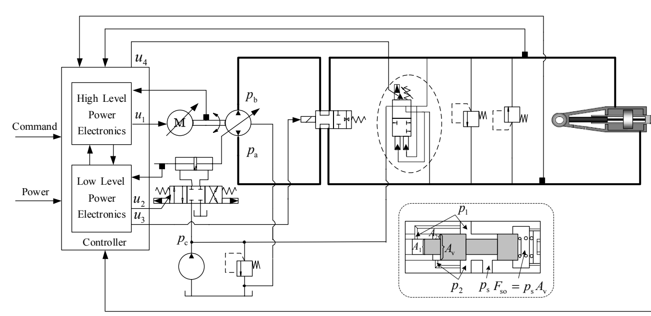

Figure 1. Schematic diagram of EHCA

Fig.1 in [4] is the schematic diagram of EHCA. The common volume system only has one control chamber, because it can not make the pressures sum of two chambers be a constant. The total pressure control valve in [4] takes the sum of pressures in both cylinder chambers as a controlled object, and finally implements the two chambers of pump controlled system both are controlled, through maintaining the pressures sum always be the double of slippage pump outlet pressure.

This can make EHCA have same dynamic response with valve controlled actuator. The high pressure slippage pump mainly provides a reference pressure which is used to compensate the flow losses due to volume efficiency.

B. *Exact Linearization Model*

Three parts will be analyzed in the following.

1) *Dual-winding Brushless DC Motor(BLDCM):* The BLDCM with double sets of windings not only can enhance the motor reliability, but also could reduce the elctromagnetic torque ripple, increase efficiency and enlarge the rotary speed regulating range. The double sets windings circuits equations can be written as:

$$\mathrm{d}i_{s1}/\mathrm{d}t=\left[u_{s1}-15R_0\left(1+\alpha K\right)i_{s1}+13R_0\left(1+\alpha K\right)i_{s2}-e_1\right]/5.6L \quad (1)$$

$$\mathrm{d}i_{s2}/\mathrm{d}t=\left[u_{s2}-15R_0\left(1+\alpha K\right)i_{s2}+13R_0\left(1+\alpha K\right)i_{s1}-e_2\right]/5.6L \quad (2)$$

Where, i_{s1} and i_{s2} are separately the phase current of two sets windings, u_{s1} and u_{s2} are respectively the control voltages, e_1 and e_2 are the electromotive force of two sets windings, R_0 is initial phase resistance, L is phase reluctance, K is the temperature rising, α is the variance ratio of R_0. Considering the phase resistance varying as working temperature, the heat equation is described as:

$$R_0\left(1+\alpha K\right)\left(i_{s1}+i_{s2}\right)^2+k_{ir}\omega_m^2=k_0\left(1+k_T\omega_m\right)K+H\,\mathrm{d}K/\mathrm{d}t \quad (3)$$

The left side of (3) is the copper and iron loss, its right side shows the heat generated by losses. k_0 is thermal conduction coefficient, k_T is the variant of k_0 as motor speed varying. Due to the same structure of two sets windings, we can suppose that $e_1=e_2=2e=2k\omega=k_e\omega$, k and k_e are the electromotive force factor of phase and total set windings respectively, e is the phase electromotive force, ω_m is motor speed, k_{ir} is factor about iron loss, H is the enthalpy. The output torque of BLDCM is:

$$T_{em}=\left(e_1 i_{s1}+e_2 i_{s2}\right)/\omega_m=k_e\left(i_{s1}+i_{s2}\right) \quad (4)$$

2) *Stroking* M*echanism of Pump:* Due to the stroking mechanism of variable pump consists of electrohtdraulic servo valve and control cylinder, the first-order differential equation models of these two parts are:

$$\mathrm{d}\gamma_p/\mathrm{d}t=\left(-\gamma_p+k_r x_v\right)/T_p \quad (5)$$

$$\mathrm{d}x_v/\mathrm{d}t=\left(-x_v+k_v u_p\right)/T_v \quad (6)$$

Where, γ_p is swashplate angle, k_r and k_v are the regulating gains of swashplate angle and spool displacement of servo valve separately, T_p and T_v are time constants of pump and servo valve respectively, x_v is displacement of servo valve spool, u_p is the control voltage of servo of stroking mechanism.

3) *Hydraulic actuating cylinder:* The output displacement equation of EHCA actuating cylinder is written as:

$$\mathrm{d}x_c/\mathrm{d}t=\left(k_p\gamma_p\omega_m-C_L p_L\right)/A_c \quad (7)$$

Where, x_c is actuating cylinder displacement, p_L is the load pressure, C_L is leakage coefficient of cylinder, A_c is piston area of cylinder, k_p is regulating gain of pump displacement. When load pressure keeping constant, the acceleration of cylinder is only related to swashplate angle and motor speed and it can be shown in (8):

$$\frac{\mathrm{d}^2x_c}{\mathrm{d}t^2}=\frac{k_p}{A_c}\left[\frac{\gamma_p}{J}\left(k_e\left(i_{s1}+i_{s2}\right)-B_m\omega_m-k_p\gamma_p p_L\right)+\frac{\omega_m}{T_p}\left(-\gamma_p+k_r x_v\right)\right] \quad (8)$$

For p_L is shown as (9):

$$p_L A_c=m_c\frac{\mathrm{d}^2x_c}{\mathrm{d}t^2}+B_c\frac{\mathrm{d}x_c}{\mathrm{d}t}+k_L x_c+F_L \quad (9)$$

Where, k_L is load elastic coefficient, m_c is equivalent mass of load, B_c is damping coefficient, F_L is load force. If taking (8) into (9), and let $N=A_c+B_cC_L/A_c, N_0=m_ck_p^2/(A_cJ)$, so (9) can be further described as:

$$p_L=\frac{\dfrac{m_ck_pk_e}{A_cJ}\gamma_p\left(i_{s1}+i_{s2}\right)+\dfrac{m_ck_pk_r}{A_cT_p}\omega_m x_v+k_Lx_c+F_L}{N+N_0\gamma_p^2}+\frac{\left(\dfrac{B_ck_p}{A_c}-\dfrac{m_ck_pB_m}{A_cJ}-\dfrac{m_ck_p}{A_cT_p}\right)\omega_m\gamma_p}{N+N_0\gamma_p^2} \quad (10)$$

Setting six state variables: $x_1=i_{s1}+i_{s2}$, $x_2=\omega_m$, $x_3=K$, $x_4=\gamma_p$, $x_5=x_v$, $x_6=x_c$, so the EHCA model equations can be expressed as (11).

$$\begin{cases}\dot{x}_1=\left[u_s-R_0\left(1+\alpha x_3\right)x_1-k_ex_2\right]/2.8L\\[4pt]
\dot{x}_2=\dfrac{k_e}{J}x_1-\dfrac{B_m}{J}x_2-\dfrac{\left(B_c-\dfrac{m_cB_m}{J}-\dfrac{m_c}{T_p}\right)\dfrac{k_p^2}{A_c}x_2x_4^2}{J(N+N_0x_4^2)}\\[4pt]
\qquad-\dfrac{\dfrac{m_ck_p^2k_e}{A_cJ}x_1x_4^2+\dfrac{m_ck_p^2k_r}{A_cT_p}x_2x_4x_5+k_Lk_px_4x_6+k_pF_Lx_4}{J(N+N_0x_4^2)}\\[4pt]
\dot{x}_3=\left[R_0\left(1+\alpha x_3\right)x_1^2+k_{ir}x_2^2-k_0\left(1+k_Tx_2\right)x_3\right]/H\\[4pt]
\dot{x}_4=\left(-x_4+k_rx_5\right)/T_p\\[4pt]
\dot{x}_5=\left(-x_5+k_vu_p\right)/T_v\\[4pt]
\dot{x}_6=\dfrac{k_p}{A_c}x_2x_4-\dfrac{C_L}{A_c}\dfrac{\dfrac{m_ck_pk_e}{A_cJ}x_1x_4+\dfrac{m_ck_pk_r}{A_cT_p}x_2x_5+k_Lx_6+F_L}{(N+N_0x_4^2)}\\[4pt]
\qquad-\dfrac{C_L}{A_c}\dfrac{\left(\dfrac{B_ck_p}{A_c}-\dfrac{m_ck_pB_m}{A_cJ}-\dfrac{m_ck_p}{A_cT_p}\right)x_2x_4}{(N+N_0x_4^2)}\end{cases} \quad (11)$$

Let the system output are:

$$\begin{cases}y_1=h_1\left(\mathbf{x}\right)=x_2=\omega_m\\ y_2=h_2\left(\mathbf{x}\right)=x_4=\gamma_p\\ y_3=h_3\left(\mathbf{x}\right)=x_6=x_c\end{cases} \quad (12)$$

The following will verify whether (12) satisfies the conditions that can be exact linearized. Ignoring the details, the relationship degrees of y_1, y_2 and y_3 respectively are

$r_1 = 2$, $r_2 = 2$ and $r_3 = 2$. Thus, the system relationship degree is $r = r_1 + r_2 + r_3 = 6$. Defining $\mathbf{A}_1(\mathbf{x})$ as:

$$\mathbf{A}_1(\mathbf{x}) = \begin{bmatrix} L_{g1}L_f h_1(\mathbf{x}) & L_{g2}L_f h_1(\mathbf{x}) \\ L_{g1}L_f h_2(\mathbf{x}) & L_{g2}L_f h_2(\mathbf{x}) \\ L_{g1}L_f h_3(\mathbf{x}) & L_{g2}L_f h_3(\mathbf{x}) \end{bmatrix} \tag{13}$$

In account of $\operatorname{rank}\mathbf{A}_1 = 2$, $\mathbf{A}_1(\mathbf{x})$ has a left inverse $\mathbf{A}_{1L}^{-1}(\mathbf{x})$. So model (11)-(12) satisfy the conditions of exact linearization.

Choosing the nonlinear transform as: $z_1 = h_1(\mathbf{x}) = x_2$, $z_2 = L_f h_1(\mathbf{x})$, $z_3 = h_2(\mathbf{x}) = x_4$, $z_4 = L_f h_2(\mathbf{x})$, $z_5 = h_3(\mathbf{x}) = x_6$, $z_6 = L_f h_3(\mathbf{x})$. So the EHCA model (11) can be transferred into controllable canonical form (14).

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = L_f^2 h_1(\mathbf{x}) + L_{g1}L_f h_1(\mathbf{x}) u_s + L_{g2}L_f h_1(\mathbf{x}) u_p = v_1 \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = L_f^2 h_2(\mathbf{x}) + L_{g1}L_f h_2(\mathbf{x}) u_s + L_{g2}L_f h_2(\mathbf{x}) u_p = v_2 \\ \dot{z}_5 = z_6 \\ \dot{z}_6 = L_f^2 h_3(\mathbf{x}) + L_{g1}L_f h_3(\mathbf{x}) u_s + L_{g2}L_f h_3(\mathbf{x}) u_p = v_3 \end{cases} \tag{14}$$

Due to the varying uncertainty of outer load force F_L, and it is difficult to measure accurately, we regard F_L as the outer disturbance. Thus the exact linearization model of EHCA can be divided into three controllable linear subsystems (15)-(17). S_1 is about the BLDCM rotary speed ω_m, S_2 is about the swashplate angle of pump γ_p, and S_3 is about the actuating cylinder displacement x_c. So this is very easy for designers to respectively construct the effective and pertinence controllers for three measures.

$$S_1: \begin{cases} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi_1 \\ y_1 = z_1 \end{cases} \tag{15}$$

$$S_2: \begin{cases} \begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_2 \\ y_2 = z_3 \end{cases} \tag{16}$$

$$S_3: \begin{cases} \begin{bmatrix} \dot{z}_5 \\ \dot{z}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_5 \\ z_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_3 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi_1 \\ y_3 = z_5 \end{cases} \tag{17}$$

By the linear method, we can design the H_∞ control signals v_1, v_2 and v_3, then the original control voltages u_s and u_p can be induced through (18).

$$\begin{bmatrix} u_s \\ u_p \end{bmatrix} = \mathbf{A}_{1L}^{-1}(\mathbf{x}) \begin{bmatrix} v_1 - L_f^2 h_1(\mathbf{x}) \\ v_2 - L_f^2 h_2(\mathbf{x}) \\ v_3 - L_f^2 h_3(\mathbf{x}) \end{bmatrix} \tag{18}$$

III. EHCA Control Based On H_∞ Norm

Here we mainly consider four design tasks of EHCA model in the following: (1) Tracking problem. The first design goal of system is to make the output follow the given value and maintain all the state bounded. In the EHCA, the actuating cylinder output displacement x_c belongs to it. (2) H_∞ output feedback problem. The EHCA output (12) should make the whole closed loop system to be steady and minimize the influence of disturbance on the state variables in the meaning of H_∞ norm. (3) H_∞ filter problem. It is demanded to evaluate the state variables values in the disturbance and noise situation. (4) Faults detection problem. System should check out the failures when some sensor is inactivation and rapidly use the software rebuilding signals to keep system normally.

A. Swashplate Angle Software Rebuilding Model

Here we have two software ways to reconstruct the swashplate angle γ_p.

1) *Computing γ_p by Control Cylinder Displacement x_{ps} of Pump Stroking Mechanism:* Due to the relationship of x_{ps} and γ_p is

$$x_{ps} = \gamma_p r \tag{19}$$

Where, r is the distance between axis of control cylinder piston and rotary axis of swashplate. So we can compute γ_p easily by detecting the x_{ps} through displacement sensor.

2) *Computing γ_p By System Flow Q_L, Load Pressure Difference p_L and Motor Speed ω_m: Through t*he load flow equation of pump, we can obtain γ_p from:

$$\gamma_p = \frac{Q_L + C_{pL} p_L}{K_p \omega_m} \tag{20}$$

Where, C_{pL} is the total leakage coefficient of pump.

B. H_∞ Control Index of EHCA System

Considering the generalized dynamic model of linear time-invariance system including actuator and sensor faults information can be written as the following form:

$$\begin{cases} \dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}_1\xi_1 + \mathbf{B}_2\mathbf{v} + \mathbf{R}_1\mathbf{f}_1 \\ \mathbf{q} = \mathbf{C}_1\mathbf{z} + \mathbf{D}_{11}\xi_2 + \mathbf{D}_{12}\mathbf{v} \\ \mathbf{y} = \mathbf{C}_2\mathbf{z} + \mathbf{D}_{21}\xi_2 + \mathbf{D}_{22}\mathbf{v} + \mathbf{R}_2\mathbf{f}_2 \end{cases} \tag{21}$$

Where, $\mathbf{z} \in \mathbf{R}^{n\times1}$; $\xi_1 \in \mathbf{R}^{r\times1}$; $\xi_2 \in \mathbf{R}^{k\times1}$; $\mathbf{q} \in \mathbf{R}^{m\times1}$; $\mathbf{v} \in \mathbf{R}^{p\times1}$; $\mathbf{y} \in \mathbf{R}^{q\times1}$; $\mathbf{f}_1 \in \mathbf{R}^{n\times1}$; $\mathbf{f}_2 \in \mathbf{R}^{q\times1}$. Constant matrix $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{D}_{11}$, $\mathbf{D}_{12}$, $\mathbf{D}_{21}$, $\mathbf{D}_{22}$, $\mathbf{R}_1$, $\mathbf{R}_2$ have the relative dimensions. $\mathbf{R}_1$ and $\mathbf{R}_2$ are respectively the failure direction vectors of actuators and sensors, $\mathbf{f}_1$ and $\mathbf{f}_2$ are separately the failure types matrix of actuators and sensors. Just consider the sensor faults, so $\mathbf{R}_1 = 0$ and $\mathbf{f}_1 = 0$. ξ_1 and ξ_2 are the process noise and measure noise. $\mathbf{z}$, $\mathbf{q}$ and $\mathbf{y}$ are respectively the vectors of state, controllable output and measure output.

With generally, we just consider the situation of one sensor fault. If y_i is the practical output of ith sensor, y_{i0} is the output without failure, $i = 1, 2, \ldots, q$, then the possible fault output of sensor with constant deviation can be written as $y_i = y_{i0} + \tau$, τ is a constant. So we can specify the vector of $\mathbf{R}_2$ and $\mathbf{f}_2$ as $\mathbf{R}_2 = [0 \quad 1 \quad 0]$, $\mathbf{f}_2 = [0 \quad \tau \quad 0]^T$.

For system (21), designing the below assumptions: (1) $\mathbf{D}_{11}=\mathbf{D}_{22}=\mathbf{0}$; (2) $\mathbf{D}_{12}^{\mathrm{T}}\left[\mathbf{C}_1 \quad \mathbf{D}_{12}\right]=\left[\mathbf{0} \quad \mathbf{I}\right]$; (3) $\mathbf{D}_{21}\left[\mathbf{B}_1^{\mathrm{T}} \quad \mathbf{D}_{21}^{\mathrm{T}}\right]=\left[\mathbf{0} \quad \mathbf{I}\right]$; (4) $\left(\mathbf{A},\mathbf{B}_1,\mathbf{C}_1\right)$ is controllable and observable; (5) $\left(\mathbf{A},\mathbf{B}_2,\mathbf{C}_2\right)$ is stable and detectable. The details can be found in [2].

Firstly, the output feedback H_∞ control problem is to find a output feedback controller $\mathbf{K}(s)$ in the premises of satisfying hypothesis (1)-(5) above and a given positive value $\chi>0$, to maintain the closed loop system states steady and meet the performance object:

$$J_1=\left\|\mathbf{G}_{\mathbf{q}\boldsymbol{\xi}}\right\|_\infty=\sup_{\omega}\bar{\sigma}\left[\mathbf{G}_{\mathbf{q}\boldsymbol{\xi}}(j\omega)\right]<\chi \tag{22}$$

$\mathbf{G}_{\mathbf{q}\boldsymbol{\xi}}$ is the transfer function matrix from outer disturbance $\boldsymbol{\xi}$ to controllable output $\mathbf{q}$. Thereinto, $\bar{\sigma}\left(\mathbf{G}_{\mathbf{q}\boldsymbol{\xi}}(s)\right)$ describes the maximum eigenvalue of $\mathbf{G}_{\mathbf{q}\boldsymbol{\xi}}(s)$.

Secondly, to construct the dissimilar redundancy measure of γ_{p}, it must use other measurable variables values to estimate γ_{p}. So we should use H_∞ filter to evaluate the clean values of variables in noises. H_∞ filter demands on the estimation unbiasedness, and it designs the controller to satisfy the index (22) in the system with limited power disturbance $\boldsymbol{\xi}$:

$$J_2=\left\|\mathbf{G}_{\mathbf{r}\boldsymbol{\xi}}\right\|_\infty=\sup_{\omega}\bar{\sigma}\left[\mathbf{G}_{\mathbf{r}\boldsymbol{\xi}}(j\omega)\right]<\delta \tag{23}$$

Thus, if the H_∞ norm of closed loop transfer function matrix $\mathbf{G}_{\mathbf{r}\boldsymbol{\xi}}(s)$ which from disturbance $\boldsymbol{\xi}$ to estimate error $\mathbf{r}=\mathbf{q}-\hat{\mathbf{q}}$ is less than a given positive value $\delta>0$, the power of $\mathbf{r}$ will be $1/\delta$ of noise in order to maintain the observer system matrix steady.

Thirdly, in account of the faults detection usually are implement through identifying the range of estimation error $\mathbf{r}$, the system observer should meet a basic condition that the influence of failure on the $\mathbf{r}$ must larger than the influence of disturbance on the $\mathbf{r}$, namely the following index:

$$J_3=\left\|\mathbf{G}_{\mathbf{rf}}\right\|_\infty=\inf_{\omega}\underline{\sigma}\left[\mathbf{G}_{\mathbf{rf}}(j\omega)\right]\geq\eta\,,\ \eta>\delta \tag{24}$$

Thereinto, $\underline{\sigma}\left(\mathbf{G}_{\mathbf{rf}}(s)\right)$ is the minimum eigenvalue of $\mathbf{G}_{\mathbf{rf}}(s)$.

C. H_∞ Controller Design

EHCA is mainly a tracking problem of actuating cylinder displacement x_{c} should always follow the specified input x_{cg}, it means that v_3 is a tracking controller in S_3. Fig.2 shows the diagram. $K_1(s)$ and $K_2(s)$ are controllers, $G_0(s)$ is controlled object, x_{cg} is the specified input signal belongs to a set of limit power. The task is to design $K_1(s)$ and $K_2(s)$ to keep x_{c} following x_{cg}, and let the $\left\|x_{\mathrm{cg}}-x_{\mathrm{c}}\right\|_2$ is minimum. Meantime, to limit control signal v_3 scale, we can rewrite the index as:

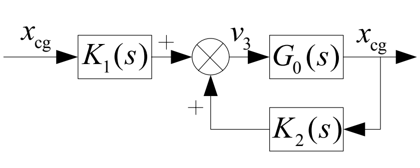

Figure 2. Schematic diagram of S_3 subsystem.

$\left\|x_{\mathrm{cg}}-x_{\mathrm{c}}\right\|_2^2+\left\|v_3\right\|_2^2$. If we design $\mathbf{q}_3=\left[x_{\mathrm{cg}}-x_{\mathrm{c}} \quad v_3\right]^{\mathrm{T}}$, then the system S_3 should minimize $\sup\left[\left\|\mathbf{q}_3\right\|_2 \middle| v_3\in H_2,\left\|v_3\right\|\leq 1\right]$. Fig.2 shows that $v_3=\left[-K_2(s) \quad K_1(s)\right]\left[x_{\mathrm{c}} \quad x_{\mathrm{cg}}\right]^{\mathrm{T}}$, and we can take $x_{\mathrm{cg}}-x_{\mathrm{c}}$ as the new state variables in subsystem S_3. Thus the tracking problem also can be regarded as H_∞ control problem. So we can write the three subsystems of EHCA in form of (21).

Let $\mathbf{A}=\begin{bmatrix}0&1\\0&0\end{bmatrix}$, $\mathbf{B}_1=\begin{bmatrix}0&0\\1&0\end{bmatrix}$, $\mathbf{B}_2=\begin{bmatrix}0\\1\end{bmatrix}$, $\mathbf{C}_2=\begin{bmatrix}1&0\end{bmatrix}$, $\mathbf{D}_{12}=\begin{bmatrix}0\\1\end{bmatrix}$, $\mathbf{D}_{21}=\begin{bmatrix}0&1\end{bmatrix}$. S_1 and S_2 have $\mathbf{C}_1=\begin{bmatrix}1&0\\0&0\end{bmatrix}$, and $\mathbf{C}_1=\begin{bmatrix}-1&0\\0&0\end{bmatrix}$ in the S_3. And through verification, S_1, S_2 and S_3 are all satisfied the five assumptions.

$$S_1:\begin{cases}\begin{bmatrix}\dot{z}_1\\\dot{z}_2\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}z_1\\z_2\end{bmatrix}+\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}\xi_1\\\xi_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}v_1\\ \mathbf{q}_1=\begin{bmatrix}z_1\\v_1\end{bmatrix}=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}z_1\\z_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}v_1\\ y_1=\begin{bmatrix}1&0\end{bmatrix}\begin{bmatrix}z_1\\z_2\end{bmatrix}+\begin{bmatrix}0&1\end{bmatrix}\begin{bmatrix}\xi_1\\\xi_2\end{bmatrix}\end{cases} \tag{25}$$

$$S_2:\begin{cases}\begin{bmatrix}\dot{z}_3\\\dot{z}_4\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}z_3\\z_4\end{bmatrix}+\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}\xi_1\\\xi_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}v_2\\ \mathbf{q}_2=\begin{bmatrix}z_3\\v_2\end{bmatrix}=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}z_3\\z_4\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}v_2\\ y_2=\begin{bmatrix}1&0\end{bmatrix}\begin{bmatrix}z_3\\z_4\end{bmatrix}+\begin{bmatrix}0&1\end{bmatrix}\begin{bmatrix}\xi_1\\\xi_2\end{bmatrix}+\tau\end{cases} \tag{26}$$

$$S_3:\begin{cases}\begin{bmatrix}\dot{z}_5\\\dot{z}_6\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}x_{\mathrm{td}}-z_5\\z_6\end{bmatrix}+\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}\xi_1\\\xi_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}v_3\\ \mathbf{q}_3=\begin{bmatrix}x_{\mathrm{td}}-z_5\\v_3\end{bmatrix}=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}x_{\mathrm{td}}-z_5\\z_6\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}v_3\\ y_3=\begin{bmatrix}1&0\end{bmatrix}\begin{bmatrix}x_{\mathrm{td}}-z_5\\z_6\end{bmatrix}+\begin{bmatrix}0&1\end{bmatrix}\begin{bmatrix}\xi_1\\\xi_2\end{bmatrix}\end{cases} \tag{27}$$

1) *Output Feedback H_∞ Controller:* Accordding to the reletive theorem in [2], to obtain the H_∞ controller satisfied (22), the matrix $\mathbf{P}$ and $\tilde{\mathbf{P}}$ should exist and respectively satisfy:

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}+\mathbf{P}\left(\chi^{-2}\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}}-\mathbf{B}_2\mathbf{B}_2^{\mathrm{T}}\right)\mathbf{P}+\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1=0 \tag{28}$$

$$\mathbf{A}\tilde{\mathbf{P}}+\tilde{\mathbf{P}}\mathbf{A}^{\mathrm{T}}+\tilde{\mathbf{P}}\left(\chi^{-2}\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1-\mathbf{C}_2^{\mathrm{T}}\mathbf{C}_2\right)\tilde{\mathbf{P}}+\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}}=0 \tag{29}$$

Known after computing, $\chi=1$ or 2 is not agree demand, but $\chi=3$ meets the condition and $\lambda_{\max}(\mathbf{P}\tilde{\mathbf{P}})=6.5572<3^2=9$. Now we can specify the matrix $\mathbf{P}$ and $\tilde{\mathbf{P}}$ as:

$$\mathbf{P}=\begin{bmatrix}1.4565&1.0607\\1.0607&1.5448\end{bmatrix},\ \tilde{\mathbf{P}}=\begin{bmatrix}1.5448&1.0607\\1.0607&1.4565\end{bmatrix} \tag{30}$$

So we get $J_1 = \left\|\mathbf{G}_{\mathbf{q}\xi}\right\|_\infty < 3$. For S_1 and S_2, the state space description of feedback controller is

$$\begin{cases} \begin{bmatrix} \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \end{bmatrix} = \begin{bmatrix} -5.0887 & 1 \\ -4.3743 & 0.1716 \end{bmatrix} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1 + \begin{bmatrix} 5.0887 \\ 4.4922 \end{bmatrix} y_1 \\ v_1 = \begin{bmatrix} -1.0607 & -1.5448 \end{bmatrix} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} \\ \begin{bmatrix} \hat{z}_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1 \\ y_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \end{cases} \tag{31}$$

Where, $\hat{z}_1$ and $\hat{z}_2$ are the unbiased estimations of z_1 and z_2.

For S_3, the state space description of feedback controller is:

$$\begin{cases} \begin{bmatrix} \dot{\hat{z}}_5 \\ \dot{\hat{z}}_6 \end{bmatrix} = \begin{bmatrix} -5.0887 & 1 \\ -4.3743 & 0.1716 \end{bmatrix} \begin{bmatrix} x_{\mathrm{td}} - \hat{z}_5 \\ \hat{z}_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_3 + \begin{bmatrix} 5.0887 \\ 4.4922 \end{bmatrix} y_3 \\ v_3 = \begin{bmatrix} -1.0607 & -1.5448 \end{bmatrix} \begin{bmatrix} x_{\mathrm{td}} - \hat{z}_5 \\ \hat{z}_6 \end{bmatrix} \\ \begin{bmatrix} x_{\mathrm{td}} - \hat{z}_5 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{\mathrm{td}} - \hat{z}_5 \\ \hat{z}_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_3 \\ y_3 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{\mathrm{td}} - z_5 \\ z_6 \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \end{cases} \tag{32}$$

Now we have already computed v_1, v_2 and v_3, so the u_{s} and u_{p} can be obtained by (18).

2) *H_∞ Filter design:*

Let $\mathbf{e} = \mathbf{z} - \hat{\mathbf{z}}$, we will get the dynamic equation of state variables error as:

$$\begin{cases} \dot{\mathbf{e}} = \left(\mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2\right)\mathbf{e} + \left(\mathbf{B}_1 - \mathbf{Z}^{-1}\mathbf{L}\mathbf{D}_{21}\right)\xi - \mathbf{B}_1\hat{\xi}_{\mathrm{worst}} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\mathbf{f}_2 \\ \mathbf{r} = \mathbf{q} - \hat{\mathbf{q}} = \mathbf{C}_1\mathbf{e} \end{cases} \tag{33}$$

Then taking the Laplace transform of (33), we have

$$\begin{aligned} \mathbf{r}(s) &= \begin{bmatrix} \mathbf{G}_{z1}(s) & \mathbf{G}_{z2}(s) & \mathbf{G}_{z3}(s) \end{bmatrix} \begin{bmatrix} \xi(s) & \hat{\xi}_{\mathrm{worst}}(s) & \mathbf{f}_2(s) \end{bmatrix}^{\mathrm{T}} \\ &= \mathbf{G}_z(s) \begin{bmatrix} \xi(s) & \hat{\xi}_{\mathrm{worst}}(s) & \mathbf{f}_2(s) \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{34}$$

$$\mathbf{G}_z(s) = \left[\begin{array}{c|ccc} \mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2 & \mathbf{B}_1 - \mathbf{Z}^{-1}\mathbf{L}\mathbf{D}_{21} & \mathbf{B}_1 & -\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2 \\ \hline \mathbf{C}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right] \tag{35}$$

So the state space implement of transfer function matrix which is from disturbance ξ to error $\mathbf{r}$ can be described as:

$$\mathbf{G}_{\mathbf{r}\xi}(s) = \left[\begin{array}{c|cc} \mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2 & \mathbf{B}_1 - \mathbf{Z}^{-1}\mathbf{L}\mathbf{D}_{21} & \mathbf{B}_1 \\ \hline \mathbf{C}_1 & \mathbf{0} & \mathbf{0} \end{array}\right] \tag{36}$$

And we can prove that $\mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2$ is steady. The object index of H_∞ filter is (23), the task here is to fix the proper of δ.

According the relative theorem of H_∞ filtering problem, for the system (21) with disturbance and satisfying the five assumptions, given a positive number $\delta > 0$, the necessary and sufficient condition to make system have a unbiased estimation (31) and can make transfer function $\left\|\mathbf{G}_{\mathbf{r}\xi}\right\|_\infty < \delta$ is, existing a $\bar{\mathbf{P}} = \bar{\mathbf{P}}^{\mathrm{T}} > 0$ which is the solution of (37):

$$\mathbf{A}\bar{\mathbf{P}} + \bar{\mathbf{P}}\mathbf{A}^{\mathrm{T}} + \bar{\mathbf{P}}\left(\delta^{-2}\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 - \mathbf{C}_2^{\mathrm{T}}\mathbf{C}_2\right)\bar{\mathbf{P}} + \mathbf{B}_1\mathbf{B}_1^{\mathrm{T}} = 0 \tag{37}$$

The feedback gain $\mathbf{K}(s)$ that can make (31) stable is $\mathbf{K}(s) = \bar{\mathbf{P}}\mathbf{C}_2^{\mathrm{T}}$. From the output feedback H_∞ control, we know that $\mathbf{Z}^{-1}\mathbf{L} = \left(\mathbf{I} - \chi^{-2}\tilde{\mathbf{P}}\mathbf{P}\right)^{-1}\tilde{\mathbf{P}}\mathbf{C}_2^{\mathrm{T}}$, and the H_∞ filter design also shows $\bar{\mathbf{P}} = \left(\mathbf{I} - \chi^{-2}\tilde{\mathbf{P}}\mathbf{P}\right)^{-1}\tilde{\mathbf{P}} = \begin{bmatrix} 5.0887 & 4.4921 \\ 4.4922 & 4.7978 \end{bmatrix}$. Taking specified $\bar{\mathbf{P}}$ into (37), we can obtain three values of δ are especially 0.81, 0.89 and 0.97. Thus choosing a maximum value of the three, we have $\delta = 1$, i.e. (31) can satisfy the object $J_2 = \left\|\mathbf{G}_{\mathbf{r}\xi}\right\|_\infty < 1$.

3) *Robust Failure Detectionr:* As above analyzed, in order to meet the demand that the influence of failure on the $\mathbf{r}$ is larger than that of disturbance on the $\mathbf{r}$, i.e. $\left\|\mathbf{G}_{\mathbf{r}\mathbf{f}_2}\right\|_{-\infty} > \left\|\mathbf{G}_{\mathbf{r}\xi}\right\|_\infty$, for subsystem S_2, its error dynamic equation of state variables can be written in (38):

$$\begin{cases} \begin{bmatrix} \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} -5.0887 & 1 \\ -4.4922 & 0 \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \end{bmatrix} + \left[\begin{pmatrix} 0 & -5.0887 \\ 1 & -4.4922 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}\right] \begin{bmatrix} \xi \\ \hat{\xi}_{\mathrm{worst}} \end{bmatrix} \\ \quad - \begin{bmatrix} 5.0887\vartriangle \\ 4.4922\vartriangle \end{bmatrix} \\ \begin{bmatrix} r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} q_3 - \hat{q}_3 \\ q_4 - \hat{q}_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \end{bmatrix} \end{cases} \tag{38}$$

So the state space implement of transfer function matrix which is from fault $\mathbf{f}_2$ to error $\mathbf{r}$ can be described as:

$$\mathbf{G}_{\mathbf{r}\mathbf{f}_2} = \left[\begin{array}{c|c} \mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2 & -\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2 \\ \hline \mathbf{C}_1 & \mathbf{0} \end{array}\right] \tag{39}$$

We can define the Hamiltonia matrix as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2 & \eta^{-2}\left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)\left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)^{\mathrm{T}} \\ -\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 & -\left(\mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2\right)^{\mathrm{T}} \end{bmatrix} \tag{40}$$

To satisfy the index (24), the following method is effective.

Theorem 1: If the system matrix $\left(\mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2\right)$ of (33) is stable, then the necessary and sufficient condition of

$$\left\|\mathbf{G}_{\mathbf{r}\mathbf{f}_2}\right\|_\infty = \sup_\omega \bar{\sigma}\left[\mathbf{G}_{\mathbf{r}\mathbf{f}_2}(j\omega)\right] < \eta$$

is that the eigenvalue of Hamiltonia matrix (40) being not on the imaginary axis.

Let $\eta = 1$. If $\eta \neq 1$, then taking $\eta^{-1}\left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)$ as $\left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)$ and regarding $\eta^{-1}\left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)^{\mathrm{T}}$ as $\left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)^{\mathrm{T}}$. Due to $\mathbf{G}_{\mathbf{r}\mathbf{f}_2}^*(s) = \mathbf{G}_{\mathbf{r}\mathbf{f}_2}^{\mathrm{T}}(-s)$, so we have

$$\left[\mathbf{I} - \mathbf{G}_{\mathbf{r}\mathbf{f}_2}^*(s)\mathbf{G}_{\mathbf{r}\mathbf{f}_2}(s)\right]^{-1} = \begin{bmatrix} \mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2 & \left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)\left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)^{\mathrm{T}} & -\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2 \\ -\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 & -\left(\mathbf{A} - \mathbf{Z}^{-1}\mathbf{L}\mathbf{C}_2\right)^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \left(-\mathbf{Z}^{-1}\mathbf{L}\mathbf{R}_2\right)^{\mathrm{T}} & \mathbf{I} \end{bmatrix}$$

It is proved that system (33) is complete controllable and observable on the imaginary axis, namely the equivalence of matrix $\mathbf{H}(\eta=1)$ in (40) has no pole on imaginary axis means reversible matrix $\left[\mathbf{I}-\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)\right]^{-1}$ has no pole on imaginary axis. So theorem 1 means that if existing $\left\|\mathbf{G}_{\mathbf{rf}_2}\right\|_\infty<1$, then it must has $\mathbf{I}-\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)>0$, $\forall\omega\in\mathbf{R}$. So $\forall\omega\in\mathbf{R}$, if $\left[\mathbf{I}-\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)\right]^{-1}$ exists, then $\left[\mathbf{I}-\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)\right]^{-1}$ has no pole on imaginary axis.

In the other hand, due to (39) is the strictly proper rational fraction matrix, there must exist $\bar{\sigma}\left(\mathbf{G}_{\mathbf{rf}_2}(s)\right)\to 0$, as $\omega\to\infty$. Because, if $\left\|\mathbf{G}_{\mathbf{rf}_2}\right\|_\infty\geq 1$, then it must exist a ω that can make $\bar{\sigma}\left(\mathbf{G}_{\mathbf{rf}_2}(s)\right)=1$, i.e. an eigenvalue of $\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)$ is one. Thus $\left[\mathbf{I}-\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)\right]$ is a singularity matrix and $\left[\mathbf{I}-\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)\right]^{-1}$ has poles on the imaginary axis. So the equivalence of $\left\|\mathbf{G}_{\mathbf{rf}_2}\right\|_\infty\geq 1$ is $\left[\mathbf{I}-\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)\right]^{-1}$. This provides a method about how to compute H_∞ norm of strictly proper rational fraction matrix. First, choosing a $\eta>0$ and constructing the $\mathbf{H}$ matrix; then identifying whether $\mathbf{H}$ does not exist eigenvalues on the imaginary axis. If true, then $\left\|\mathbf{G}_{\mathbf{rf}_2}\right\|_\infty<\eta$; otherwise, $\left\|\mathbf{G}_{\mathbf{rf}_2}\right\|_\infty\geq\eta$, it should choose another $\eta>0$ to compute.

Then we can induce to how to compute the problem of $\left\|\mathbf{G}_{\mathbf{rf}_2}\right\|_{-\infty}=\inf_\omega\underline{\sigma}\left[\mathbf{G}_{\mathbf{rf}_2}(j\omega)\right]\geq\eta$. First, it is no need to specify a η value, instead, to find all the eigenvalues of $\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)$ on every ω point, as the process of ω varying from 0 to ∞. So we can obtain the respective varying range of every eigenvalue in the whole ω varying domain.

Now taking measure mechanism of pump swashplate angle in EHCA as the research object, try to find the possible maximized η. Here the state space expression of $\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)$ and its transfer matrix form can be specified as:

$$\mathbf{G}^*_{\mathbf{rf}_2}(s)\mathbf{G}_{\mathbf{rf}_2}(s)=\left[\begin{array}{cccc:ccc} -5.0887 & 1 & 0 & 0 & 0 & -5.0887 & 0\\ -4.4922 & 0 & 0 & 0 & 0 & -4.4922 & 0\\ 1 & 0 & 5.0887 & 4.4922 & 0 & 0 & 0\\ 0 & 0 & -1 & 0 & 0 & 0 & 0\\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 5.0887 & 4.4922 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Then, we can get:

$$\mathbf{G}^*_{\mathbf{rf}_2}(j\omega)\mathbf{G}_{\mathbf{rf}_2}(j\omega)=\begin{bmatrix} 0 & 0 & 0\\ 0 & \dfrac{20.1799+25.8949\omega^2}{\omega^4+16.9105\omega^2+20.1799} & 0\\ 0 & 0 & 0 \end{bmatrix} \quad (41)$$

Because the main noises in the electrical sensors system involves thermal noise (inner noise) and AC power supply noise (outer noise), so these noises are below about 60Hz. Here, we specify the whole ω varying domain as $\omega\in[0,60]$. By using of MATLAB, all the eigenvalues of every frequency point can be found. So we can obtain $\sup\underline{\sigma}\left[\mathbf{G}_{\mathbf{rf}_2}(j\omega)\right]=1.2096$, and $\inf\underline{\sigma}\left[\mathbf{G}_{\mathbf{rf}_2}(j\omega)\right]=0.0072$. Thus $\eta=0.0072$. Combining the other analysis, finally $\chi=3$, $\delta=1$, $\eta=0.0072$, but $\eta<\delta$.

Through the above verifying, we know that in the situation of no preprocessing on noise and disturbance, η can be found is obviously far less than δ, which may severity influence the fault detection result: increasing misjudging or failing to detect to the failures. So we must handle the system noises and disturbances in advance by using filter to enhance η in the maximum limitation. So the effective method to increase failure detection accuracy is to enlarge η as possible.

$$\min_\omega\left\|\mathbf{G}_{\mathbf{r}\xi}(j\omega)\right\|_\infty=\min\sup\left[\|\mathbf{q}\|_2\,\big|\,\|\xi\|_2=1,\xi\in L_2\right] \quad (42)$$

$$\max_\omega\left\|\mathbf{G}_{\mathbf{rf}_2}(j\omega)\right\|_{-\infty}=\max\inf\left[\|\mathbf{r}\|_2\,\big|\,\|\mathbf{f}_2\|_2=1,\mathbf{f}_2\in L_2\right] \quad (43)$$

From (42)-(43), increasing the fault function amplitude by adding a amplifier or reduce noise through a wavelet filter (for thermal noise is coupling with measure in the whole frequency domain), can make δ smaller and η bigger. Here design an amplifier Π after fault function output. If let $\Pi=140$, then $\Pi\eta=150\times0.0072=1.08>1$, namely $\Pi\eta>\delta$. Thus $\left\|\mathbf{G}_{\mathbf{rf}_2}\right\|_{-\infty}>\left\|\mathbf{G}_{\mathbf{r}\xi}\right\|_\infty$.

IV. CONCLUSIONS

This paper mainly discussed the dissimilar redundancy measure problem of variable pump swashplate angle. The design process involves four tasks: displacement tracking, H_∞ output feedback, H_∞ filtering with noise and faults detection. Proper controller design for covering all the objects needs some skills. Through the complete analysis in the paper, it provides an effective method for EHCA health detection and H_∞ control with noises. The future valuable work maybe focuses on how to use the wavelet filter replacing H_∞ filter in this paper to obtain more good results.

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