The Research of the Improved 3D L- System and Its Application in Plant Modeling

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***Abstract*—Transportation simulation includes vehicles simulation and traffic environment simulation, this paper carries out the plant modeling research as the preparation of the latter. It first describes the original 3D L-system and how to implement it, then brings forward the improved modeling method of 3D L-system by specifying the growth vector and azimuth vector and applies the quaternion based computer algorithm in it, in the end by using this improved method and algorithm generates some 3D plant models of sympodia and uniaxiality.**

***Keywords*—L-system, virtual plant, computer modeling, virtual reality**

I. INTRODUCTION

Intelligent vehicle function test and performance analysis based on virtual reality and virtual prototype will help improve design quality, shorten design period and reduce design cost. Real-time 3D vision traffic environment simulation take an important role in intelligent vehicle virtual simulation system. From fig.1. and fig.2. which are from the website(http:// Driving simulation software SILAB.htm), it can be seen that no matter highway environment simulation or lane environment simulation, the plants modeling and animating is inevitable.

Figure.1. Highway simulation

Figure.2. Lane simulation

At present, there are many methods to generate botanical models, and among them L-system is most effective and frequent in application. L-system was first introduced by the biologist Aristid Lindenmayer in 1968 as a foundation for an axiomatic theory of biological development. An L-system is a scheme for rewriting strings of symbol words. The rewriting takes place by replacing certain strings of characters in a word by other strings of characters [1]. These replacements often utilize repetitions of the string of characters being replaced, and so complex strings are built up from simple strings, the complex strings containing replications of the basic strings. The idea of repetition is inherent in L-systems, and so the geometric objects that result from application of an L-system have fractal properties. While biologists originally used L-systems to create biological models, scientists in computer graphics have garnered recent attention by rendering attractive and sophisticated graphics based upon L-systems formalisms [2], [3] .

II. THREE-DIMENSIONAL L-SYSTEM

An L-system consists of a set of textual rules called productions that describe the development of plant branches, leaves, flowers, and other components. In a generation phase these productions are applied in a sequence of derivation steps to the initial string, called the axiom. The state of the L-system model after any number of steps is encoded in a string of symbols, called the L-string. In a subsequent interpretation phase the L-string is converted to a geometric representation of a plant. L-strings encode form using turtle geometry. A turtle, starting at a specified location and orientation in world-space,

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interprets an L-string as a series of position and orientation-changing instructions.

To specify a particular L-system, we include:

1) an *axiom*, or initial string.

2) the angle θ by which the turtle may turn.

3) one or more *production rules*, which indicate what string replacements are to be made.

Beginning with the *axiom*, the process of applying the production rules is repeated, to form successively longer strings. The string-rewriting stops after a specified number of steps, and the resulting string is then interpreted by a turtle graphics processor [4].

At present, L-system and its application is emphasized in the plane, and the model produced by it is 2D, but in fact, the plants in nature are 3D. The L-system with 3D topology information is the only way to produce the virtual plant which can effectively reflect the reality in the world.

Contrasted with 2D L-system, the 3D L-system has many in common. For example, in the simplified system, the alphabet still consists of the symbols ***F***, ***P***, [and], with the following geometric interpretations:

TABLE I 2D L-STRING SYMBOLS AND THEIR TURTLE INTERPRETATIONS.

F	Move forward one step, drawing the path of motion
P	Move forward one step, but do not draw
[	save state, start new branch
]	end branch, restore state

But at the same time the alphabet is enriched with some terms, the aim of which is to increase more space information[5]. No matter 2D L-system and 3D L-system, the orientation definition of the branch is critical. In the general 3D L-system, the orientation of the turtle is represented by three vectors ***H***, ***L***, and ***U***, indicating in the turtle' s local frame of reference which directions are forward, left, and up. Three above vectors satisfy $\boldsymbol{H}\times\boldsymbol{L}=\boldsymbol{U}$, let $c=\cos\theta$ and $s=\sin\theta$, the matrixes rotating angle θ separately about them counterclockwise are indicated as:

$$\boldsymbol{R}_u(\theta)=\begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix},\quad \boldsymbol{R}_l(\theta)=\begin{bmatrix} c & 0 & -s \\ 0 & 1 & 0 \\ s & 0 & c \end{bmatrix},\quad \boldsymbol{R}_h(\theta)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix} \tag{1}$$

The symbols with parameter and their interpretations are showed as below, which are depicted in Figure.3.:

\(δ), rotate angle δ counterclockwise around axle ***H***.,

/(δ), rotate angle δ clockwise around axle ***H***.

^ (δ), rotate angle δ clockwise around axle ***L***.

+(δ), rotate angle δ counterclockwise around axle ***U***.

&(δ), rotate angle δ counterclockwise around axle ***L***.

-(δ), rotate angle δ clockwise around axle ***U***.

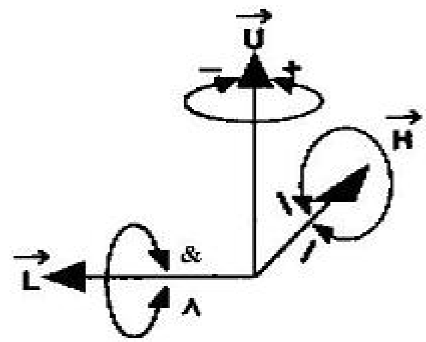

Figure.3. Turtle orientation and rotation symbols

III. IMPROVED 3D L-SYSTEM

L-system is the useful tool to depict the plant topology and also a form language. To form the 3D virtual botanical model accurately and conveniently, the improved 3D L-system is introduced after the full study of the botanical morphology model, according to branch orientation character, the improved modeling method and computer algorithm being proposed and forming the new mathematics model.

A. Modeling Method Research

In 3D L-system, the determination of the branch direction is the key step, so the modeling improvement is first to decide it. Before describing the modeling method and computer algorithm, first define the generation rule, namely the superior and junior in the same plane.

As to botanical morphology, the branch growth orientation depend on the bifurcate angle α(the angle between trunk and branch) and the azimuth β(the angle between two adjacent branches around the trunk). The example is depicted in figure.4.

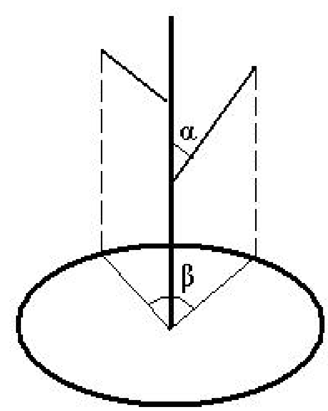

Figure.4. Bifurcate angle and azimuth

Owing that the bifurcate angle and azimuth determine the branch direction, the growth vector expressed as ***I*** is first introduced as a feature vector or reference vector which represents the specific growth orientation. For the figure.5, there is a shoot and three branches, the growth vectors on them are remarked as $\boldsymbol{I}$, $\boldsymbol{I}_1$, $\boldsymbol{I}_2$ and $\boldsymbol{I}_3$ separately [6]. Suppose that the bifurcate angles of three branches are separately α_1, α_2 and α_3, then

$$0°\leq\alpha_1\leq 180°,\ 0°\leq\alpha_2\leq 180°,\ 0°\leq\alpha_3\leq 180°,$$

$$\text{and}\quad \cos\alpha_1=\frac{\boldsymbol{I}\cdot\boldsymbol{I}_1}{|\boldsymbol{I}||\boldsymbol{I}_1|},\ \cos\alpha_2=\frac{\boldsymbol{I}\cdot\boldsymbol{I}_2}{|\boldsymbol{I}||\boldsymbol{I}_2|},\ \cos\alpha_3=\frac{\boldsymbol{I}\cdot\boldsymbol{I}_3}{|\boldsymbol{I}||\boldsymbol{I}_3|} \qquad (2)$$

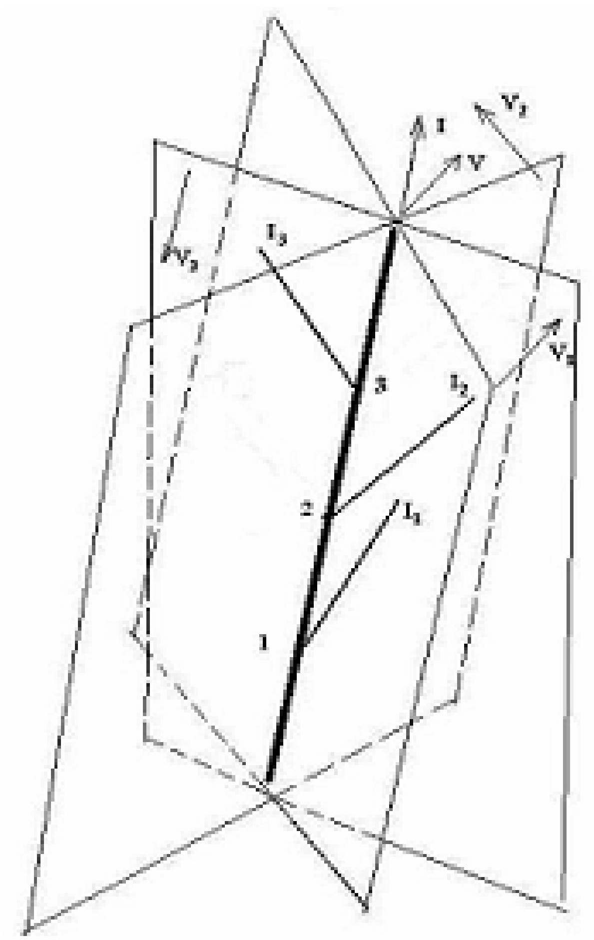

Figure.5. Growth vector and azimuth vector

Besides growth vector taken as feature vector, azimuth vector is defined as the reference accordance of the correct branch orientation. The azimuth vectors of three branches are determined as the following:

$$\boldsymbol{V}_1=\boldsymbol{I}\times\boldsymbol{I}_1,\quad \boldsymbol{V}_2=\boldsymbol{I}\times\boldsymbol{I}_2,\quad \boldsymbol{V}_3=\boldsymbol{I}\times\boldsymbol{I}_3 \qquad (3)$$

At the meantime, specify the azimuth vector of the trunk as $\boldsymbol{V}$, and $\boldsymbol{V}\perp\boldsymbol{I}$. In accordance with the relationship between shoot azimuth vector and those of branches, there are reference azimuths β_1, β_2 and β_3, which are interpreted as the following:

$$-180°\leq\beta_1\leq 180°,\quad -180°\leq\beta_2\leq 180°,\quad -180°\leq\beta_3\leq 180°,$$

$$\text{and}\quad \cos\beta_1=\frac{\boldsymbol{V}\cdot\boldsymbol{V}_1}{|\boldsymbol{V}||\boldsymbol{V}_1|},\quad \cos\beta_2=\frac{\boldsymbol{V}\cdot\boldsymbol{V}_2}{|\boldsymbol{V}||\boldsymbol{V}_2|},\quad \cos\beta_3=\frac{\boldsymbol{V}\cdot\boldsymbol{V}_3}{|\boldsymbol{V}||\boldsymbol{V}_3|} \qquad (4)$$

Because the feature vectors of the improved system are different with those of original system, so the relevant symbols and their interpretations need redefinition:

+(γ), growth vector of shoot rotate angle γ counterclockwise around azimuth vector of branch,

-(γ), growth vector of shoot rotate angle γ clockwise around azimuth vector of branch,

/(γ), azimuth vector of shoot rotate angle γ counterclockwise around growth vector of it,

\(γ), azimuth vector of shoot rotate angle γ clockwise around growth vector of it.

B. Computer Algorithm of 3D L-system

From the above modeling method, an 3D model can be constructed by setting the *axiom* and *production rules*, but the model is just the concept description. Though L-system is the mathematical model to depict the botanic configuration and growth law, and yet in itself is a nonobjective rule that without the computer experiment, we can't feel it concretely.

For the original 3D L-system, the branch determination from the trunk need three rotation transformations in different planes, which are based on the heading, left and up vectors. In the improved L-system, the feature vectors are defined as azimuth vector and growth vector, so the transformations can be simplified. Euler theorem is explained as if the origins of two right Descartes coordinate systems are at superposition; the two systems can be also at superposition by rotation around some axle. According to the theme, when the rotation angle and axle is given, the transformation can be achieved only once. that's say the branch can be directly determined by the trunk, and all of this need the application of the quaternion.

The quaternion is denoted as $\boldsymbol{q}=a+b\mathbf{i}+c\mathbf{j}+d\mathbf{k}$, a is real part, recorded as $\mathrm{Re}(\boldsymbol{q})=a$, $b\mathbf{i}+c\mathbf{j}+d\mathbf{k}$ is imaginary part, recorded as $\mathrm{Im}(\boldsymbol{q})=b\mathbf{i}+c\mathbf{j}+d\mathbf{k}$.

Suppose the unit vector of the rotation axle is $\boldsymbol{u}$, rotation angle is θ, let [7]:

$$s=\cos\frac{\theta}{2},\ \boldsymbol{v}=\boldsymbol{u}\sin\frac{\theta}{2} \qquad (5)$$

The quaternion composed of scalar part and vector part is recorded as $\boldsymbol{Q}=(s,\ \boldsymbol{v})$, if the rotation vector is $\boldsymbol{p}$, define the relevant quaternion is $\boldsymbol{P}=(0,\boldsymbol{p})$, the operation

$$\boldsymbol{P}'=\boldsymbol{Q}\boldsymbol{P}\boldsymbol{Q}^{-1}$$

will certainly make a new quaternion whose scalar is 0 recorded as $\boldsymbol{P}'=(0,\ \boldsymbol{p}')$, $\boldsymbol{p}'$ being regarded as the new spatial vector. $\boldsymbol{p}'$ is also regarded as the vector obtained by $\boldsymbol{p}$ rotating angle θ around $\boldsymbol{u}$.

For the two different quaternion $\boldsymbol{Q}_1=(s_1,\ \boldsymbol{v}_1)$ and $\boldsymbol{Q}_2=(s_2,\ \boldsymbol{v}_2)$, we have

$$Q_1Q_2=s_1s_2-v_1\cdot v_2+s_1v_2+s_2v_1+v_1\times v_2 \quad (6)$$

The reciprocal of $\boldsymbol{Q}$ is indicated as $\boldsymbol{Q}^{-1}=(s, -\boldsymbol{v})$, so $\boldsymbol{p}'$ can be presented as the format composed of scalar product and vector product, i.e.

$$p'=s^2p+v(p\cdot v)+2s(v\times p)+v(v\times p)=R_u(\theta)p \quad (7)$$

If $\boldsymbol{u}$ is expressed as $\boldsymbol{u}=(x, y, z)$, then $x^2+y^2+z^2=1$. Let $e_0=\cos(\theta/2)$, $e_1=x\sin(\theta/2)$, $e_2=y\sin(\theta/2)$, $e_3=z\sin(\theta/2)$, the transformation matrix can be expressed as:

$$R_u(\theta)=\begin{bmatrix} 2e_0^2+2e_1^2-1 & 2e_1e_2-2e_0e_3 & 2e_1e_3+2e_0e_2 \\ 2e_1e_2+2e_0e_3 & 2e_0^2+2e_2^2-1 & 2e_2e_3-2e_0e_1 \\ 2e_1e_3-2e_0e_2 & 2e_2e_3+2e_0e_1 & 2e_0^2+2e_3^2-1 \end{bmatrix} \quad (8)$$

How to depict and construct rotation transformation matrix after rotation angle and axle is given is the key. The method mentioned above is called the quaternion method. Quaternion is the most effective mathematical tool to study rigid body movement, and use it can solve the practical problem of motion analysis and control [8], [9]. Compared with the original method, this method is more direct and convenient, which omits many middle steps.

IV. MODEL VALIDATION AND VITUAL PLANT SIMULATION

In nature, there are many species with different appearance character, which improve the diversity of the world. Besides flowers, leaves and fruits, different species of plants have different growth pattern. On the whole, there are two kinds of branch pattern, namely sympodia and uniaxiality. The base shoot of uniaxial plant bears many sub-branches, and grows up itself forming the central leader at last, e.g. pine and metasequoia. The tip bud of sympodia plant grows up to certain height and stops, then some lateral buds grow about, turning to strong side branches, e.g. peach and pear.

No matter sympodia or uniaxiality, the improved 3D L-system is the effective tool to describe the topology and growth process. Figure.6. is the chico model, 'a', 'b'and 'c' separately represents the view from different direction. The *axiom* and *production rule* is

$\boldsymbol{w}$: $F(h_0, d_0)$

$\boldsymbol{P}$: $F(h, d)\rightarrow F(h*h_1, d*d_1)[/(\alpha)F(h*h_2, d*d_2)]F(h*h_1, d*d_1)[+(\beta)/(\alpha)F(h*h_2, d*d_2)]F(h*h_1, d*d_1)[-(\beta)/(\alpha)F(h*h_2, d*d_2)]F(h*h_1, d*d_1)$

Figure.7. is another uniaxiality model based on the parameter L-system and with different views.

Figure.8. is the sympodial model that for every maternal branch, three sub-branches grow up from the top of it. The branches are of full symmetry, so the model is named regular ternary tree. The *axiom* and *production rule* is

$\boldsymbol{w}$: $F(h_0, d_0)$

$\boldsymbol{P}$: $F(h, d)\rightarrow H(h*h_1, d*d_1)[/(\alpha)F(h*h_2, d*d_2)][+(\beta)/(\alpha)F(h*h_2, d*d_2)][-(\beta)/(\alpha)F(h*h_2, d*d_2)]$

Figure.9. is the sympodial model of regular binary tree that each stem has two bifurcations which are symmetrical to each other.

Figure.10. is the stochastic tree model. For the steam of every level, the bifurcation number is randomly determined, 2 or 3, so it reflects better the diversity of the nature. The *production rules* are

$\boldsymbol{P}_1$: $F(h, d)\rightarrow(0.5)H(h*h_1, d*d_1)[/(\alpha)F(h*h_2, d*d_2)][+(\beta_1)/(\alpha)F(h*h_2, d*d_2)]$

$\boldsymbol{P}_2$: $F(h, d)\rightarrow(0.5)H(h*h_1, d*d_1)[/(\alpha)F(h*h_2, d*d_2)][+(\beta_2)/(\alpha)F(h*h_2,d*d_2)][-(\beta_2)/(\alpha)F(h*h_2,d*d_2)]$

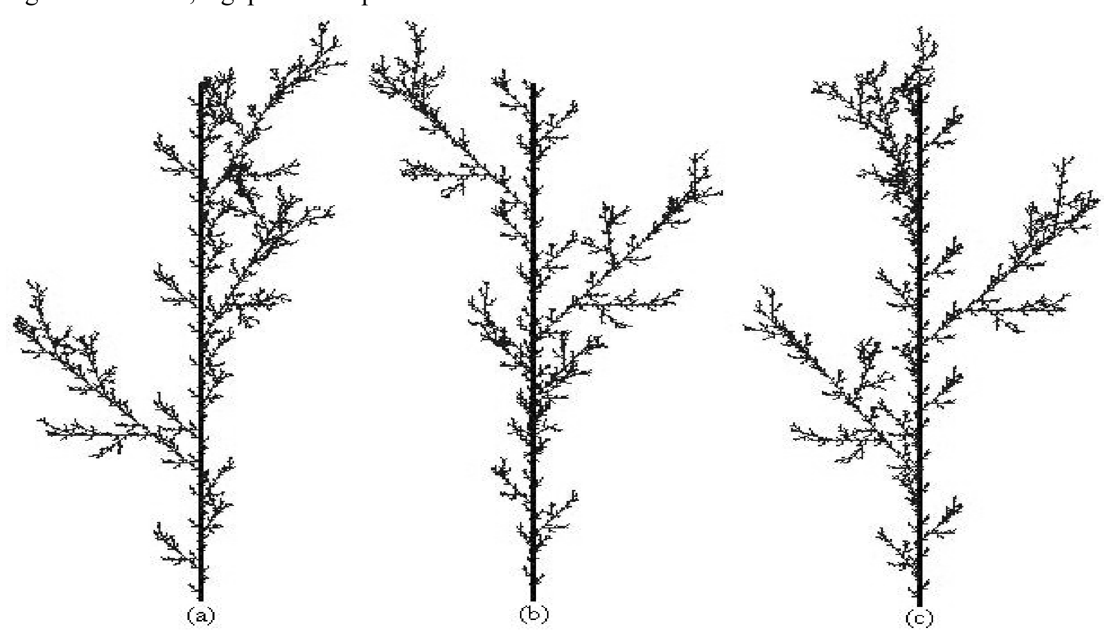

Figure.6. Chico model

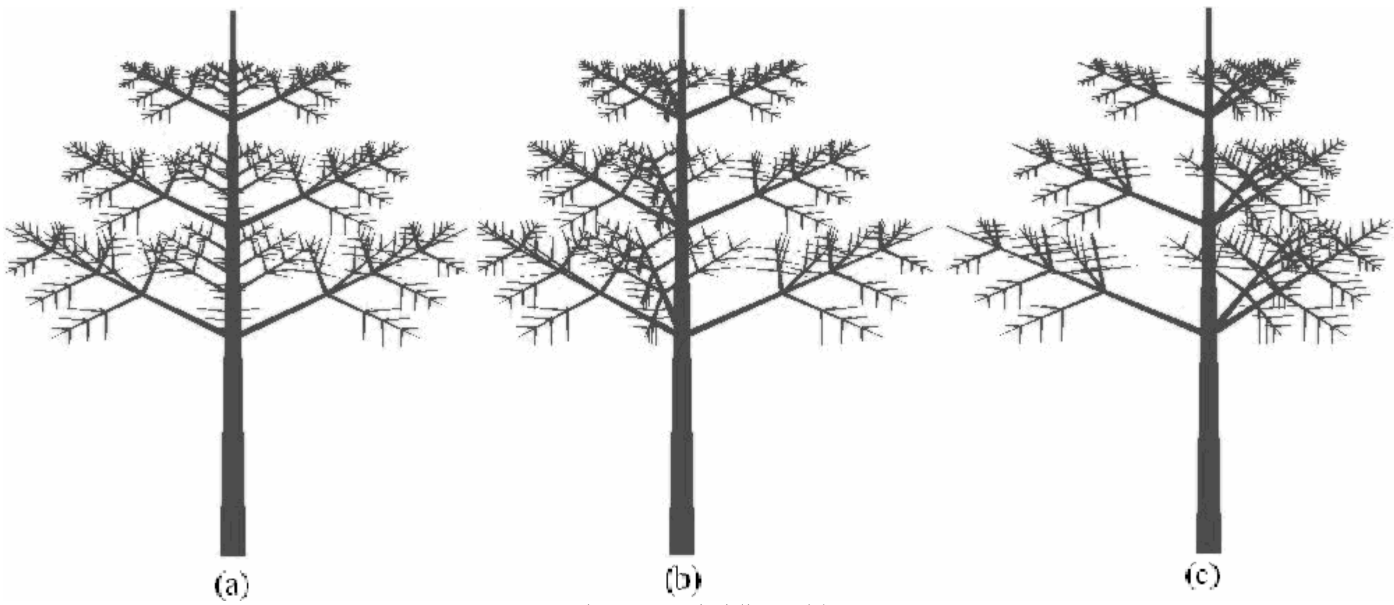

Figure.7. Uniaxiality model

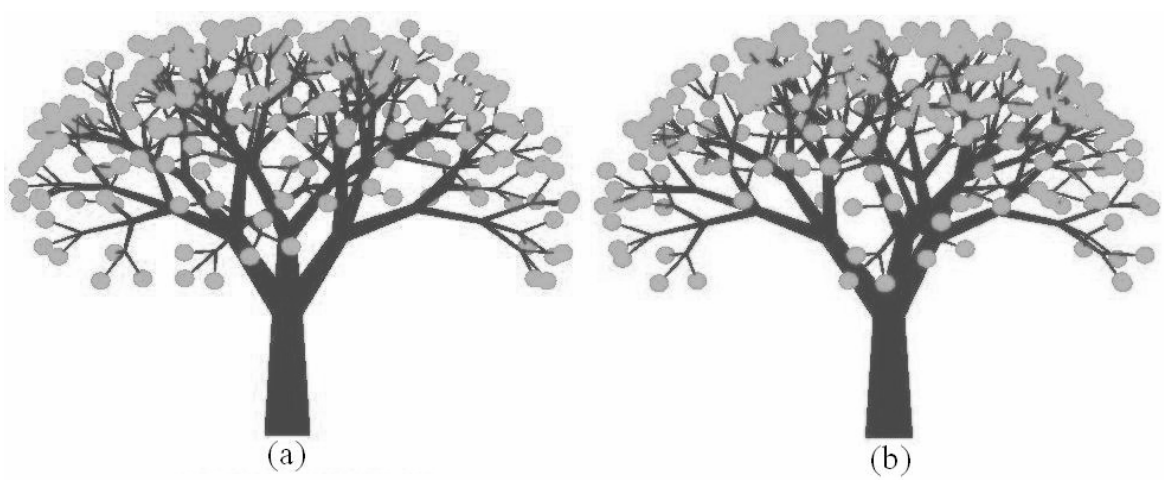

Figure.8. Ternary tree model

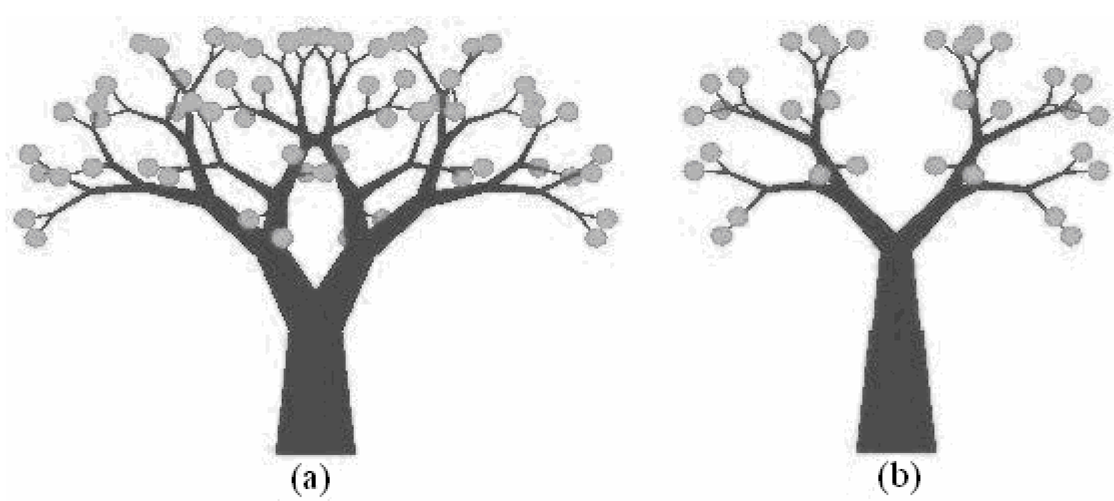

Figure.9. Binary tree model

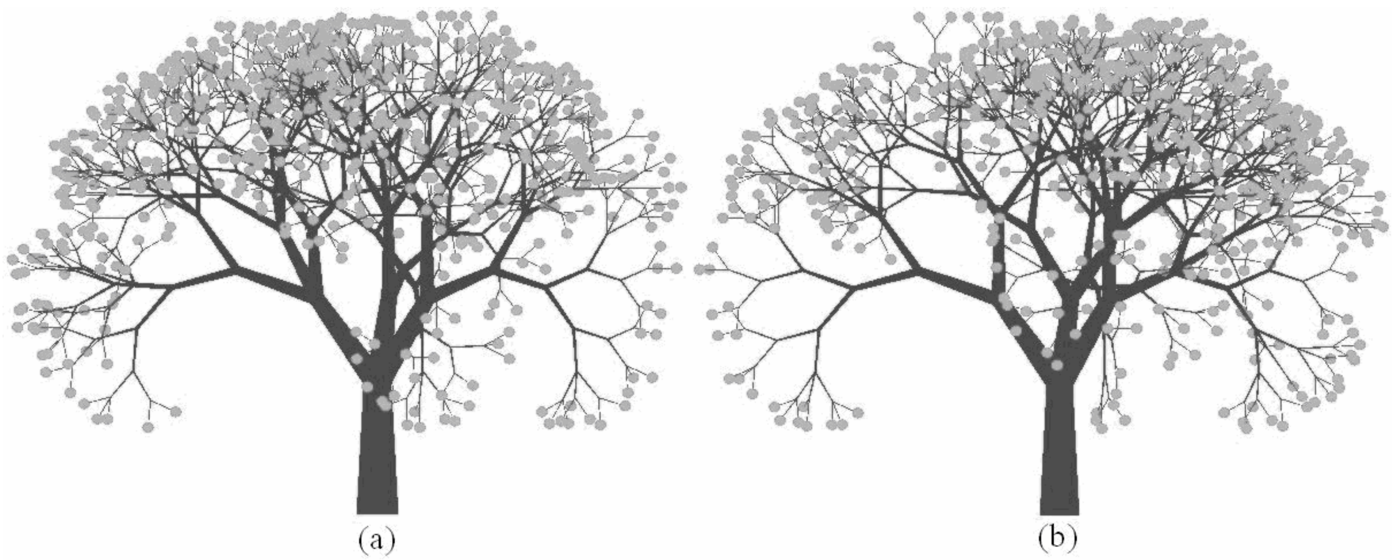

Figure.10. Stochastic tree model

IV. CONCLUSION

The plant modeling is the important part of traffic environment animation. Compared with the road model, the plant model is irregular in shape, so it is hard to build. The improved 3D L-system is brought forward in this paper. As to modeling methodology, according that in botanical morphology the branch growth orientation depend on the bifurcate angle and the azimuth, specify the azimuth vector and growth vector as the feature vectors to determine the branch orientation. In the computer algorithm, the quaternion is introduced to establish the rotation transformation matrix. In the end, some models are generated with different branch pattern.

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