Sliding Mode Control Based on Delay Estimation Online for Networked Control System with Model Uncertainties and Stochastic Less Delay

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Abstract—A delay-dependent sliding mode control algorithm for the Networked Control System (NCS) with mismatched uncertainties and stochastic less delay is studied. Considering stochastic less delay of the NCS, delay estimation online is present. The delay iterative measuring and pretest parameters-based estimation methods are adopted. Loop delay estimation online is designed to overcome the adverse influence caused by time-varying delay. Considering mismatched uncertainties, an asymptotically stable sliding mode surface is designed by means of linear matrix inequality (LMI). The sufficient condition for the existence of sliding surface if gained by using LMI with delay depending. A discrete sliding mode control strategy based on the loop delay of estimation and the asymptotically stable sliding mode surface is present. The simulation experiment results show the effectiveness of the algorithm.

Keywords—networked control system, sliding mode control, time delay, delay estimation, discrete-time system; linear matrix inequality (LMI)

I. INTRODUCTION

Modern control systems often use a communication network, such as CAN, Profibus, Ethernet, ATM and even Internet to send data and control signals among sensors, controllers, actuators and plants. The feedback control systems wherein the control loops are closed through a real-time communication network are called Networked Control Systems (NCS)[1-3]. The primary advantages of an NCS are reduced system wiring, ease of system diagnosis and maintenance, and increased system agility. NCS are now widely used in manufacture automation factories, electric factories, robots, advanced aircraft and electrified transportation[4].

However, the insertion of the communication networked in feedback control loop makes the analysis and design of the NCS complex and some new problems are issued. For example, the network-induced delay, data packet dropout, multiple-transmission, universal clock and so on. The network-induced delay (sensor-to-controller delay and controller-to-actuator delay), either constant (up to jitter) or time varying, can degrade the performance of spacecraft attitude control system designed without considering the delay and can even destabilize the system. Conventional control theories with many ideal assumptions, such as synchronized control and nondelayed sensing and actuation, must be reevaluated before they can be applied to NCS[5].

In existing analyzing and designing methods about NCS, system models are mostly assumed as known. Uncertainties of modeling which exist usually in practical systems are not considered.

Sliding mode control is an efficient method for realizing robust control of system. How to deal with state error and input error induced by time-varying delay is the critical problem of using sliding mode control in NCS. Especially, while the time delay and model errors occur together, system design will become more complicated.

Due to sliding mode’s ‘complete self-adaptability’, this method can resolve the stabilizing and tracking problems of uncertain systems with parameter perturbation, non-modeling dynamic, exterior disturbance and cross disturbance, and also has advantages of clear concepts and easy realization [6].

Aiming at NCS with less delay and Uncertainties of modeling, a disturbance-estimated sliding mode control strategy based on augmented state model is put forward. This paper is organized as follows. Section 2 discusses delay on this paper. In section 3, an asymptotically stable sliding mode surface with mismatched uncertainties model is designed, and a sliding mode controller with a delay on-line estimator is designed. The stability is analyzed. In section 4, the above-mentioned design methods are verified by simulation study. Finally the concluding remark is giver.

II. ONLINE DELAY OBSERVING AND SYSTEM MODELING

A. Network Interface Setting

Control network based on Field-bus is usually adopted in NCS with stochastic less delay which the field equipments such as sensors, controllers and actuators are distributed in a certain area.

Some rational assumptions [7] for the system are given:
1) The sensor node is time-driven and the controller node and actuator node are event-driven. The plant output is sampled with the sampling period $T$.

2) The network-delay from sensor to controller $d_{sc}$ and that of from controller to actuator $d_{ca}$ period both less than one sampling, as well as the controller’s CPU computing delay $d_c$ is relative small. The time delays $d_{sc}, d_{ca}$ and $d_c$ are randomly varying and the total delay of the loop is satisfied $d_{sc} + d_{ca} + d_c < T$.

3) Tasks’ average delay and super as well as inferior boundary are known. There is no dropout of data package.

B. Online Delay Observing and Estimating Methods

The loop delay is considered as compensative parameter in designing control law and it is relative to network-delay, interface setting, and node clock. The TEE working mode is adopted here, supposing there is no data package losing and all new data is effective date. The loop delay can be expressed as $\tau_{sc} = d_{sc} + \tau_{ca} = d_{sc} + d_c$, where $d_c$ is determinate for periodic control tasks. And in the closed loop, control algorithm is carried out at controller node so that the compensation can be progressed only when feedback and forward loop-delays are both known. Since $d_{ca}$ cannot be obtained for control signal transmission which has not happened, we have to forecast it.

A kind of online delay estimating method called as ADW(Average Delay Window) based on pretest parameters is presented in reference [8], whose basic idea is using current ADW(Average Delay Window) based on pretest parameters is measurable with the sampling period and actuator node are event-driven. The plant output is sampled controller () (4) according to (1).

Detailed delay estimation method is

1) Observe online the network-delay from sensor to controller $d_{sc}(k)$, and make $\tau_{sc}(k) = d_{sc}(k)$.

2) Push $d_{sc}(k)$ to delay windows, which are shown in Fig.1, and calculate $\hat{d}_{sc}(k)$ according to (1).

$$\hat{d}_{sc}(k) = \left( \sum_{i=k-l+1}^{k} d_{sc}(i) / l \right) + \Delta, \Delta = d_{sc}^{avg} - d_{sc}^{avg}$$

3) Calculate forward loop-delay $\tau_{ca}(k) = \hat{d}_{ca}(k) + d_c$.

4) Combine feedback and forward loop-delay, and get $\tau(k) = \tau_{sc}(k) + \tau_{ca}(k)$.

The iterative measuring online method is adopted when observing network-delay $\tau_{sc}(k)$. It can be expressed as

$$d_{sc}(k) = d_{sc}(k-1) + t_k - t_{k-1} - T, d_{sc}(0) = t_0 + \delta_0$$

Where $t_k$, $t_{k-1}$ and $d_{sc}(k-1)$ respectively denote data arrived moments of this time and last time recorded on control node, and last time delay, $\delta_0$ is the clock error of system running initial state. This method can reduce network load and request no clock synchronization due to sensing data without time stamp. The initial error estimation of node clock can be obtained refer to the clock synchronization arithmetic of reference [9].

According to assumption 1), 2), 3), online estimated loop-delay $\tau(k) = d_{sc}(k) + d_{ca}(k) + d_c$ obtained by above method is still less than one sampling period.

III. DESIGN OF NCS SLIDING MODE CONTROLLER BASED ON THE DELAY ONLINE

A. Modeling of NCS with Stochastic Loop Delay

Under these main assumptions, we will discuss the model of the NCS. The model consists a continuous state model of the networked control system can be described as

$$\begin{align} \dot{x}(t) &= (A_p + \Delta A_p)x(t) + B_p u(t - \tau_k) \\ y(t) &= C_p x(t) + H_p w(t) \end{align} \tag{3}$$

Where $x \in R^n, u \in R^n, A_p, \Delta A_p, B_p, C_p, H_p$ are of compatible dimensions.

According to the assumption and considering the time delay, the continuous state model of the networked control system can be described as

$$x(k+1) = Ax(k) + \Delta Ax(k) + B_0(\tau_k)u(k) + B_1(\tau_k)u(k-1) \tag{4}$$

Where

$$A = e^{\Delta_0 T}, \Delta A = e^{\Delta_0 T}$$

$$B_0(\tau_k) = \int_{\tau_k}^{T} e^{\Delta_0 t} dt \cdot B_p, B_1(\tau_k) = \int_{\tau_k}^{T} e^{\Delta_0 t} dt \cdot B_p$$

The next work is to design a new sliding mode controller to control the systems and they are stable.

B. Design Sliding Surface Using LMI

When $B_p$ is nonsingular transformation, $B_0(\tau(k))$ and $B_1(\tau(k))$ are nonsingular. Then nonsingular transformation $T_w$ can be given by the bellow.

$$T_w = \begin{bmatrix} 1 & -b_1 b_2^{-1} \\ 0 & b_2 \end{bmatrix} \tag{5}$$
\( \tilde{x}(k) = T x(k) = [x_1(k) \quad x_2(k)] , \quad x_i(k) \in \mathbb{R}^{n_i} , \quad x_2(k) \in \mathbb{R}^n \)

Transform the original system into reduced type

\[ \tilde{x}(k+1) = \tilde{A} x(k) + \Delta \tilde{A} x(k) + \tilde{B}_i(\tau) u(k) + \tilde{B}_r(\tau) u(k-1) \quad (6) \]

Where

\[
\tilde{A} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \Delta \tilde{A} = \begin{bmatrix} \Delta A_1 & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix},
\]

\[
\tilde{B}_i(\tau(k)) = \begin{bmatrix} 0 \\ b_i(\tau(k)) \end{bmatrix}, \quad \tilde{B}_r(\tau(k)) = \begin{bmatrix} 0 \\ b_r(\tau(k)) \end{bmatrix}
\]

That is

\[
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = A_1 x_1(k) + A_{12} x_2(k) + \Delta A_1 x_1(k) + \Delta A_{12} x_2(k) + b_i(\tau(k)) u(k) + b_r(\tau(k)) u(k-1)
\]

Select switch function as

\[ \tilde{s}(k) = \tilde{G} \tilde{x}(k) = [-K \quad I] \begin{bmatrix} \tilde{x}_1^T(k) \\ \tilde{x}_2^T(k) \end{bmatrix}^T \quad (8) \]

In sliding mode, \( \tilde{s}(k) = 0 \), then

\[ x_i(k+1) = (A_i + A_{ij} K + \Delta A_i + \Delta A_{ij} K) x_i(k) \quad (9) \]

Assumption 4) Uncertainties \( \Delta A \) is uniformly bounded and their upper bound and lower bound are known. Assume that

\[ \Delta A_1 = D_i F(k) E_1, \quad \Delta A_2 = D_2 F(k) E_2. \]

Where \( D_i, E_1 \) and \( E_2 \) are known real constant matrices of appropriate dimensions, and \( F(k) \) is an unknown matrix function and satisfies \( F^T(k) F(k) \leq I \), where \( I \) is the identity matrix.

Under the above condition, the sliding surface can be designed in term of LMIs.

Recall the following lemmas.

**Lemma 1**: Given constant matrices \( D \), \( E \) and a symmetric constant matrix \( Y \) of appropriate dimensions, the following inequality holds:

\[ Y + D F E + E^T F^T D^T < 0 \quad (10) \]

Where \( F \) satisfies \( F^T F < R \), if and only if for \( \gamma > 0 \),

\[ Y + \gamma^{-1} E \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix} \gamma^{-1} E^T < 0 \quad (11) \]

Holds[10].

The main result on the asymptotic stability of the reduced-order system with mismatched uncertainties is summarized in the following theorem.

**Theorem 1**: If there exists a symmetric and positive definite matrix \( P \), some matrix \( W \) and some scalar \( \gamma \) such that the following LMIs (12) are satisfied, then the reduced-order discrete-time system (9) is asymptotically stabilization via the sliding mode surface (8):

\[
\begin{bmatrix}
-\gamma I & * & * \\
A_1^T X + A_1 W & -\gamma I & * \\
E_i X + E_i W & 0 & -\gamma I \\
0 & D^T & 0 -\gamma^{-1} I
\end{bmatrix} < 0
\quad (12)
\]

**Proof**: Consider Lyapunov function candidate

\[ V(k) = x_i^T(k) P x_i(k) \quad (13) \]

Where \( P \) is a positive definite symmetrical matrix. The difference of \( V(k) \) is

\[ \Delta V(k) = V(k+1) - V(k) = x_i^T(k+1) P x_i(k+1) - x_i^T(k) P x_i(k) \quad (14) \]

Substituting (9) into (14) yields,

\[ \Delta V(k) = x_i^T(k) [A_1 + A_{ij} K + \Delta A_1 + \Delta A_{ij} K] x_i(k) - x_i^T(k) P x_i(k) \]

If the right hand of (15) is uniformly negative definite for all \( z_i(k) \) and for all \( k \geq 0 \) except at \( z_i(k) = 0 \) then the reduced-order dynamics (9) is asymptotically stable about its zero equilibrium. Therefore, the following inequality is valid.

\[ [A_1 + A_{ij} K + \Delta A_1 + \Delta A_{ij} K] x_i(k) - x_i^T(k) P x_i(k) \]

By applying Schur complement and Assumption (4) to (16), (16) is equivalent to

\[
\begin{bmatrix}
-P & * \\
A_1 + A_{ij} K + \Delta A_1 + \Delta A_{ij} K & -P^{-1}
\end{bmatrix} < 0
\quad (17)
\]

Where \( \Phi = \begin{bmatrix} -P & * \\ A_1 + A_{ij} K & -P^{-1} \end{bmatrix} \).

According to Lemma 1, the matrix inequality (17) holds for all \( F(k) \) which satisfies \( F^T(k) F(k) \leq I \) if and only if there exists a constant \( \gamma > 0 \) such that


\[ \Phi = \begin{bmatrix} (E_i + E_K)^\dagger & 0 \\ -\gamma^{-1}I & 0 \end{bmatrix} \begin{bmatrix} \gamma^{-1}I & 0 \\ 0 & D^\dagger \end{bmatrix} \begin{bmatrix} (E_i + E_K)^\dagger & 0 \end{bmatrix} < 0 \]

(18)

Applying Schur complement to (18) and taking the congruence transformation with \( P^{-1} I I I \) result in

\[ \begin{bmatrix} -P^{-1} & * & * & * \\ A_1 P^{-1} + A_2 K P^{-1} & -P^{-1} & * & * \\ E_i P^{-1} + E_2 K P^{-1} & 0 & -\gamma I & * \\ 0 & D^\dagger & 0 & -\gamma^{-1}I \end{bmatrix} < 0 \]

(19)

Denoting \( X = P^{-1} W = K P^{-1} \) and taking the congruence transformation with \( \text{diag}[I I I] \) yield (12).

C. Design of sliding mode controller

Design of the reaching law as follows

\[ s(k+1) = (1-qT)s(k) - \eta(s(k))\sgn(s(k)) \]

\[ \eta(s(k)) = \begin{cases} \varepsilon_T, & |s_i(k)| > \Delta_i, \\ 2(1-qT)|s_i(k)|, & |s_i(k)| \leq \Delta_i, \end{cases} \quad \Delta_i = \frac{\varepsilon_T + \phi}{qT} \]

(20)

Where \( q = \text{diag}(q_1,\ldots,q_n) \), \( \eta = \text{diag}(\eta,...,\eta) \), \( 0 < qT < 1 \), \( \varepsilon_i > 0 \), and \( \phi \) is a lesser positive integer and related to the variance of equivalent disturbance. Combining (4) to get

\[ u(k) = -(GB_i(T(k)))(GAx(k) + GB_i(T(k))u(k-1) - (I-qT)s(k) + \eta(s(k))\sgn(s(k))) \]

(21)

In above equation, \( u(k) \) is the function of \( x(k) \) and \( \tau(k) \), which is equivalent to compensating delay in control algorithm. Adopt the online delay estimation mentioned in above section, and obtain the feedback loop-delay and forward loop-delay of this time which can be combined as parameter variable \( \tau(k) \) of (21). In this way, \( \tau(k) \) contains approximate value \( \hat{\tau}_s(k) \) that has deviation comparing with actual delay. As a result, computing online \( B_i(\tau(k)) \) and \( B_i(\tau(k)) \) can approach the model in a certain extent. Here, we consider segmenting the variant section \( \tau(k) \) and taking a constant \( \bar{\tau}(k) \) to close to \( \tau(k) \) in each segment. Accordingly, \( B_i(\tau(k)) \) and \( B_i(\tau(k)) \) can be computed offline. Divide \( [\tau_{\text{min}}, \tau_{\text{max}}] \) into average \( n \) segments. If \( \tau(k) \) in a segments then use the middle time of the segment as the value of \( \bar{\tau}(k) \). Base on the value of segment function, \( B_i(\bar{\tau}(k)) \) and \( B_i(\bar{\tau}(k)) \) can be calculated offline in advance to get a set of parameter list.

So the equation (21) translate into (22)

\[ u(k) = -(GB_i(\bar{\tau}(k)))(GAx(k) + GB_i(\bar{\tau}(k))u(k-1) - (I-qT)s(k) + \eta(s(k))\sgn(s(k))) \]

(22)

**Theorem 2:** For the time-varying system described by (4) under the condition of assumption 1),2),3),4), the sliding mode control law (22) based on delay estimating online is adopted to compose a closed-loop feedback system, this system is gradual stable and the state track will reach and stabilize at a neighborhood of switch surface and this neighborhood is expressed \( S:\|s\| \leq \Delta_i', i = 1,2,\cdots,m \), where \( \Delta_i' \) is the width of boundary layer and \( \Delta_i' = \phi/(\delta T) \).

**Proof:** As a matter of convenience we express the ‘actual value’ of \( \tau(k) \) as

\[ \tau(k) = \bar{\tau}(k) + \Delta \]

(23)

According the character of definite integral the equation (4) be transform into (24)

\[ B_i(\tau_i) = \int_0^{\bar{\tau} - \tau_i - \Delta} e^{\lambda t} dt \bullet B_p \]

\[ = \int_0^{\bar{\tau} - \tau_i - \Delta} e^{\lambda t} dt \bullet B_p + e^{\lambda(\bar{\tau} - \tau_i - \Delta)} \int_0^{\Delta} e^{-\lambda t} dt \bullet B_p \]

(24)

Where \( D_2 = e^{\lambda(\bar{\tau} - \tau_i)} \), \( F(k) = \int_0^{\Delta} e^{\lambda t} dt \), \( E = B_p \). Like above we have

\[ B_i(\tau_i) = B_i(\bar{\tau}_i) - D_2 F(k) E \]

(25)

Then

\[ x(k+1) = Ax(k) + \Delta Ax(k) + (B_i(\bar{\tau}_i) + D_2 F(k) E) u(k) \]

\[ + (B_i(\bar{\tau}_i) - D_2 F(k) E) u(k-1) = Ax(k) + \Delta Ax(k) \]

\[ + B_i(\bar{\tau}_i) u(k) + B_i(\bar{\tau}_i) u(k-1) + D_2 F(k) E \]

(26)

Where \( D_3(k) = D_2 F(k) E (u(k) - u(k-1)) \).

Apparently \( \|D_2\|, \|F(k)\|, \|u(k) - u(k-1)\| \) are bounded, \( \|E\| \) are constant matrixes, so \( D_1(k) \) is bounded.

\[ s(k+1) = (1-qT)s(k) - \eta(s(k))\sgn(s(k)) + GD_1(k) \]

\[ = (1-qT)s(k) - \eta(s(k))\sgn(s(k)) + D_3(k) \]

(27)

Appropriately \( \|D_3(k)\| \leq D_3 \), \( D_3 \) is a positive constant.

According the boundary of the sliding mode reaching law, it can be proved that there is a \( k_0 \) for \( k \geq k_0 + 1 \) to make \( s_n \) reach and stabilize at the boundary layer \((-\Delta_i,\Delta_i)\) as well as \( \Delta_i' = \phi/(\delta T) \), then the theorem 2 is proved.
IV. SIMULATION STUDY

To verify the design methods above mentioned, simulation study targeting a DC position control system is carried out.

Select state vector \( X = [x_1 \ x_2]^T = [\theta \ \dot{\theta}] \), the state equation of the controlled plant be express as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -20
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
640
\end{bmatrix} u(t)
\]

(28)

In network, sensor is triggered by time; controller and actuator are triggered by event. Setting sampling period \( T = 10 \text{ms} \), the loop delay \( \tau \in (0, T) \) and average loop delay \( \bar{\tau}_{\text{avg}} = 5 \text{ms} \). According the equality of (4) get the discrete model of (28) as (29).

\[
\begin{bmatrix}
x_{1}(k+1) \\
x_{2}(k+1)
\end{bmatrix} =
\begin{bmatrix}
1 & 0.0114 \\
0 & 0.08189
\end{bmatrix}
\begin{bmatrix}
x_{1}(k) \\
x_{2}(k)
\end{bmatrix} +
\begin{bmatrix}
B_0(\tau(k))u(k) + B_1(\tau(k))u(k-1)
\end{bmatrix}
\]

(29)

Suppose that initial value of state system is \([x_{10} \ x_{20}]^T = [0.05 \ 0.01] \). We select controller parameters as \( G = [8.2356 \ 1] \), \( q = 5 \), \( \varepsilon = 0.5 \).

Simulation studies are carried out in two conditions as follows

1) Setting network condition and considering the random networked-delays which satisfy loop delay \( \tau(k) < T \). Adopt the switching function and control law as in (4), simply set loop delay \( \bar{\tau}_{\text{avg}} = 5 \text{ms} \).

2) Network conditions are same with 1). Set the delay windows at controller node’s receiving port so as to estimate the loop-delay \( \hat{\tau}(k) = d_{\text{on}}(k) + d_{\text{off}}(k) + d_{\text{c}} \) online. Then adopt (28) as object, select \( s(k) = Gx(k) \) , adopt the delay estimating as part 1. Set the length of FIFO windows \( l = 10 \).

Suppose the parameter perturbation

\[
\Delta A =
\begin{bmatrix}
0.05 \sin(0.2k\pi) & 0 \\
0 & 0.05 \sin(0.2k\pi)
\end{bmatrix}
\]

Figure 2. Respond curve of \( x_1 \)

Figure 3. Respond curve of \( x_2 \)

V. CONCLUSION

Aiming at the random less delay in NCS which system model with uncertainties, the delay observing and pretest parameters-based estimating methods are presented, and based on them the sliding mode control strategy is studied in this paper. In design of sliding surface uncertainties is considered in order to improve system robustness. The stable sliding surface is easily designed by solving linear matrix inequalities by means of the LMI toolbox of Matlab. A sliding mode controller with loop-delay estimation online is designed to overcome the adverse influence caused by time-varying delay. Simulation study indicates that the above-mentioned algorithm has good control performance and also can avoid large numbers of on-line mean and variant operations in normal stochastic control.

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