Delay-Dependent Guaranteed Cost Control for T-S Fuzzy Descriptor Systems with Time Delays

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***Abstract*—This paper concerns on the problem of delay-dependent guaranteed cost control for a class of nolinear descriptor delay systems which can be represented by the Takagi–Sugeno (T–S) fuzzy descriptor models with time-varying delays. A quadratic cost function is used as a guaranteed performance index. Based on the delay-dependent Lyapunov-Krasovskii functional approach, sufficient conditions for the existence of guaranteed cost controllers via state feedback are given in terms of linear matrix inequalities (LMIs). The upper bound of time-delays can be obtained by solving a convex optimization problem such that the system can be stabilized for all time-delays whose sizes are not larger than the bound. A numerical example is provided to illustrate the effectiveness of the proposed method.**

***Keywords*—Delay-dependent; T-S fuzzy descriptor systems; guaranteed cost control; time-varying delay; linear matrix inequalities (LMIs)**

I. Introduction

The well-known Takagi-Sugeno (T-S) fuzzy model is a popular and convenient tool to approximate nonlinear systems[1,2]. Once the T-S fuzzy models are obtained, the control design can be carried out by the so-called parallel distributed compensation (PDC) scheme, and linear control methodology can be used to design local state feedback controllers for each linear model. Recently, the descriptor system, which can describe a wider class of systems, including physical models and non-dynamic constraints, is paid a lot of attention. Therefore, it is meaningful to employ fuzzy descriptor model in control systems design. In [3,4], the fuzzy descriptor model is stated and the stability and stabilization problems of the systems are addressed. It is shown that the main feature of the fuzzy descriptor systems is it can reduce the number of LMI conditions for controller design. And this rule reduction is an important issue for LMI-based control synthesis. Thereafter, many further contributions have been made to the study of fuzzy descriptor systems[5,6]. On the other hand, control of delay systems has been a topic of recurring interest over the past decades since time-delays are often the main causes for instability and poor performance of systems and encountered frequently in various engineering systems. It is of great importance to study the stability and control synthesis of time delay systems. Very recently, some authors have paid their attention to the control of nonlinear systems with time-delays by using T-S fuzzy descriptor models [7]. Through that the controllers contain or not any delay information, the stabilization problems for time-delay systems can be classified into two types: delay-independent stabilization and delay-dependent stabilization. The delay-independent stabilization is considered to be robust to time-delay, but may be conservative especially when the size of time-delay is small. And the delay-dependent method is thought to be less conservative in this case than the delay-independent method. Although the delay-dependent stabilization problems for normal T-S fuzzy systems have been extensively studied by many researchers for the past several years[8,9,10], there is still little results on fuzzy descriptor systems because of the control complexity of descriptor systems. Furthermore, there are few people go beyond the stability, and have considered the performance. It is known that the guaranteed cost control aims at stabilizing the systems while maintaining an adequate level of performance represented by quadratic cost function [11,12,13]. To the best of the authors' knowledge, it seems that there is no results on delay-dependent guaranteed cost control for T-S fuzzy descriptor systems. In this paper, we mainly study the problem of delay-dependent guaranteed cost control for a class of T-S fuzzy descriptor systems with time-varying delay. A quadratic cost function is used as a guaranteed performance index. The descriptor type of delay-dependent Lyapunov-Krasovskii functional is employed to analyze the stability and design the guaranteed cost controller. The sufficient conditions for the existence of guaranteed cost controller are given in terms of LMIs. The upper bound of time-delay is obtained by solving a convex optimization problem such that the system can be stabilized for all time-delays whose sizes are not larger than the bound. A simulation example is provided to show the effectiveness of the proposed method.

II. Delay-Dependent State Feedback Control with Guaranteed Cost Performance

In this section, we consider a class of T-S fuzzy descriptor systems with time delays described by the following fuzzy If-Then rules:

If $\xi_1(t)$ is M_{i1} and $\cdots$ and $\xi_p(t)$ is M_{ip}, Then

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$$\begin{cases} E\dot{x}(t) = A_i x(t) + A_{1i} x(t-\tau(t)) + B_i u(t), \\ y(t) = C_i x(t), \\ x(t) = \phi(t), -\tau_0 \le t \le 0, \end{cases} \quad (1)$$

where $i = 1,2,\cdots,k$, k is the number of If-Then rules. $x(t) \in R^n$ denotes the state vector. $u(t) \in R^{n_u}$, $y(t) \in R^{n_y}$ are the control input, measurement output, respectively. The matrix $E \in R^{n \times n}$ is singular. We shall assume that $rankE = r < n$. $\tau(t)$ is the time-varying delay in the state, and it is assumed to be $0 \le \tau(t) \le \tau_0$, and $0 \le \dot{\tau}(t) \le d < 1$. A_i , A_{1i} and B_i are known real constant matrices. M_{ij} is the fuzzy set, and $\xi_1(t), \xi_2(t), \cdots, \xi_p(t)$ are the premise variables. It is necessary to define the initial condition $\varphi(t)$ for $-\tau_0 \le t \le 0$ as a constant scalar or differentiable function in order to obtain the upper bound of guaranteed cost performance in the following analysis.

Taking the weighted average of $E\dot{x}(t)$, $i = 1,2,\cdots,k$ as a defuzzification strategy, the final defuzzified output of the fuzzy model is derived as follows

$$\begin{cases} E\dot{x}(t) = A(t)x(t) + A_1(t)x(t-\tau) + B(t)u(t), \\ y(t) = C(t)x(t), \\ x(t) = \phi(t), -\tau_0 \le t \le 0, \end{cases} \quad (2)$$

where

$$A(t) = \sum_{i=1}^{k} \lambda_i(\xi(t)) A_i,$$

$$A_1(t) = \sum_{i=1}^{k} \lambda_i(\xi(t)) A_{1i},$$

$$B(t) = \sum_{i=1}^{k} \lambda_i(\xi(t)) B_i,$$

$$C(t) = \sum_{i=1}^{k} \lambda_i(\xi(t)) C_i,$$

$$\lambda_i(\xi(t)) = \frac{\prod_{j=1}^{p} M_{ij}(\xi_j(t))}{\sum_{i=1}^{k}\prod_{j=1}^{p} M_{ij}(\xi_j(t))},$$

and $M_{ij}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{ij} . $\lambda_i(\xi(t)) \ge 0,\ i = 1,2,\cdots,k$ and $\sum_{i=1}^{k} \lambda_i(\xi(t)) = 1\ \forall t$.

Consider the unforced form of system (2), which is written as

$$E\dot{x}(t) = \sum_{i=1}^{k} \lambda_i(\xi(t)) \left[A_i x(t) + A_{1i} x(t-\tau(t)) \right]. \quad (3)$$

Definition 2.1 The fuzzy descriptor system (3) is regular if there exists $s \in C$ satisfying

$$\det\left[sE - \sum_{i=1}^{k} \lambda_i(\xi(t))(A_i + A_{1i}e^{-s\tau(s)} \right] \ne 0,\ \forall t \ge 0.$$

Definition 2.2 The regular fuzzy descriptor system (3) is impulse free if

$$\deg\det\left[sE - \sum_{i=1}^{k} \lambda_i(\xi(t))(A_i + A_{1i}e^{-s\tau(s)} \right] = rankE.$$

Definition 2.3 The regular and impulse free fuzzy descriptor system (3) is asymptotically stable if $\frac{dV(x(t))}{dt} < 0$.

Lemma 1 [9]: Assume that $a(\cdot) \in R^{n_a}$, $b(\cdot) \in R^{n_b}$ and $N(\cdot) \in R^{n_a \times n_b}$ are defined on the interval Ω. Then, for any matrice $X \in R^{n_a \times n_b}$ and $Z \in R^{n_a \times n_b}$, the following inequality holds:

$$-2\int_{\Omega} a^T(s) N b(s) ds \le \int_{\Omega} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix} ds,$$

where $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$.

Lemma 2 [10]:For any real matrices X_i , Y_i for $1 \le i \le k$, and $S > 0$ with appropriate dimensions, we have

$$2\sum_{i=1}^{k}\sum_{j=1}^{k} \lambda_i \lambda_j X_i^T S Y_j \le \sum_{i=1}^{k} \lambda_i (X_i^T S X_i + Y_i^T S Y_i),$$

$$2\sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{l=1}^{k}\sum_{m=1}^{k} \lambda_i \lambda_j \lambda_l \lambda_m X_{ij}^T S Y_{lm} \le \sum_{i=1}^{k}\sum_{j=1}^{k} (X_{ij}^T S X_{ij} + Y_{ij}^T S Y_{ij}),$$

where $\lambda_i (1 \le i \le k)$ are defined as $\lambda_i(\xi(t)) \ge 0$, $\sum_{i=1}^{k} \lambda_i(\xi(t)) = 1$.

Lemma 3 [14]: For any matrices K_1 , K_2 and K_3 of appropriate dimensions with $K_2 > 0$, we have

$$K_1^T K_3 + K_3^T K_1 \le K_1^T K_2 K_1 + K_3^T K_2^{-1} K_3.$$

Lemma 4 [15]: Given matrices Y , D , G of appropriate dimensions and with Y symmetric, then the following inequality

$$Y + DFG + G^T F^T D^T < 0$$

holds for all F satisfying $F^T F \le I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon D D^T + \varepsilon^{-1} G^T G < 0.$$

Given symmetric positive-definite matrices Q and R, we consider the following cost function

$$J = \int_0^\infty \left\{ x^T(t) Q x(t) + u^T(t) R u(t) \right\} dt. \quad (4)$$

Our purpose is to develop a delay-dependent stabilization method which provides state feedback controller parameters as well as the upper bound of the delay such that the closed-loop system is asymptotically stable with guaranteed cost performance $J < J_0$ for any τ satisfying $0 < \tau \le \tau_0$. In the following, we present the stabilization condition via state feedback guaranteed cost control.

III. Main Results

Considering the fuzzy descriptor time delay system (2), we design the parallel distributed compensation (PDC) controller. The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts and has local linear controllers in the consequent parts. The i th rule of the fuzzy controller is of the following form:

If $\xi_1(t)$ is M_{i1} and $\cdots$ and $\xi_p(t)$ is M_{ip}, Then
$u(t) = K_i x(t)$, $i = 1,2,\cdots,k$.

Hence, the overall fuzzy control law is represented by

$$u(t) = \sum_{i=1}^{k} \lambda_i(\xi(t)) K_i x(t) \tag{5}$$

where $K_i (i = 1,2,\cdots,k)$ are the local control gains. With the control law (5), the overall closed-loop system can be written as

$$\begin{cases} E\dot{x}(t) = \overline{A}(t)x(t) + A_1(t)x(t-\tau) \\ y(t) = C(t)x(t) \\ x(t) = \phi(t), -\tau_0 \le t \le 0 \end{cases} \tag{6}$$

where

$$\overline{A}(t) = \sum_{i=1}^{k}\sum_{j=1}^{k} \lambda_i(t)\lambda_j(t)\overline{A}_{ij},$$

$$\overline{A}_{ij} = A_i + B_i K_j.$$

Theorem 1: For a given delay upper bound τ_0, the state feedback closed-loop system (6) is asymptotically stable with guaranteed cost performance J_0 if there exist positive definite matrices X, U, $\overline{H} > 0$, $\overline{R}_1 > 0$, $\overline{R}_2$ and Y_i satisfying the following LMIs for $1 \le i \le j \le k$:

$$X^T E^T = EX \ge 0 \tag{7}$$

$$\begin{bmatrix} \Lambda_{11} + 2\tau_0 \overline{R}_1 & \Lambda_{12} & \tau_0 \Lambda_{13} & (Y_i + Y_j)^T \\ * & -2\overline{H} & \tau_0 X^T (A_{1i} + A_{1j})^T & 0 \\ * & * & -2\tau_0 U & 0 \\ * & * & * & -2R^{-1} \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} \overline{R}_1 & \overline{R}_2 \\ \overline{R}_2^T & X^T E^T + EX - U \end{bmatrix} \ge 0 \tag{9}$$

where

$$\Lambda_{11} = X^T (A_i + A_j)^T + (A_i + A_j)X + B_i Y_j + B_j Y_i + Y_j^T B_i^T + Y_i^T B_j^T + 2(\overline{R}_2 + \overline{R}_2^T + \overline{Q} + \frac{\overline{H}}{1-d}),$$

$$\Lambda_{12} = (A_{1i} + A_{1j})X - 2\overline{R}_2,$$

$$\Lambda_{13} = X^T (A_i + A_j)^T + Y_j^T B_i^T + Y_i^T B_j^T.$$

The state feedback gains are constructed as $K_i = Y_i X^{-1}$, $i = 1,2,\cdots,k$. The upper bound of the guaranteed cost is

$$J_0(\tau_0) = \varphi^T(0)E^T X^{-1}\varphi(0) + \int_{-\tau_0}^{0}\int_{\theta}^{0} \dot{\varphi}^T(s)E^T U^{-1} E\dot{\varphi}(s)dsd\theta + \frac{1}{1-d}\int_{-\tau_0}^{0} \varphi^T(s)X^{-T}\overline{H}X^{-1}\varphi(s)ds. \tag{10}$$

Proof: Take consideration of the relation as follows:

$$x(t-\tau) = x(t) - \int_{t-\tau}^{t} \dot{x}(s)ds.$$

The closed-loop system (6) can be written as

$$\begin{cases} E\dot{x}(t) = (\overline{A}(t) + A_1(t))x(t) - A_1(t)\int_{t-\tau}^{t} \dot{x}(s)ds, \\ y(t) = C(t)x(t), \\ x(t) = \phi(t), -\tau_0 \le t \le 0. \end{cases} \tag{11}$$

Choose the delay-dependent Lyapunov-Krasovskii functional in the following form:

$$\begin{cases} V(t) = V_1(t) + V_2(t) + V_3(t) \\ V_1(t) = x^T(t)E^T P x(t) \\ V_2(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^T S E\dot{x}(s)dsd\theta \\ V_3(t) = \frac{1}{1-d}\int_{t-\tau}^{t} x^T(s)Hx(s)ds \end{cases} \tag{12}$$

where, $E^T P = P^T E \ge 0$ and $H>0$, $S>0$. Then,

$$\dot{V}_1(t) = (E\dot{x}(t))^T Px(t) + x^T(t)P^T E\dot{x}(t) = x^T(t)\left[(\overline{A}(t) + A_1(t))^T P + P^T(\overline{A}(t) + A_1(t))\right]x(t) - 2\int_{t-\tau}^{t} x^T(t)P^T A_1(t)\dot{x}(s)ds.$$

Define $a(\cdot)$, $b(\cdot)$ and N in Lemma 1 as

$a(\cdot) = x(t)$, $N = P^T A_1(t)$,
$b(s) = \dot{x}(s)$.

And applying Lemma1, we have

$$\dot{V}_1(t) \le x^T(t)\left[(\overline{A}(t) + A_1(t))^T P + P^T(\overline{A}(t) + A_1(t))\right]x(t) + \int_{t-\tau}^{t} \begin{bmatrix} x(t) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 - P^T A_1(t) \\ * & E^T S E \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(s) \end{bmatrix} ds$$

$$\le x^T(t)\left[\overline{A}^T(t)P+P^T\overline{A}(t)+\tau_0R_1+R_2+R_2^T\right]x(t)+$$
$$2x^T(t)(P^TA_1(t)-R_2)x(t-\tau)+\int_{t-\tau}^{t}\dot{x}(s)E^TSE\dot{x}(s),$$
where $\begin{bmatrix} R_1 & R_2 \\ R_2^T & E^TSE \end{bmatrix}\ge 0$.

As for the derivative of $V_2(t)$, it yields

$$\dot{V}_2(t)=\tau\left[\overline{A}(t)x(t)+A_1(t)x(t-\tau)\right]^T S\times$$
$$\left[\overline{A}(t)x(t)+A_1(t)x(t-\tau)\right]-\int_{-\tau}^{0}\dot{x}^T(t+\theta)E^TSE\dot{x}(t+\theta)d\theta$$
$$=\tau\sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{l=1}^{k}\sum_{m=1}^{k}\lambda_i\lambda_j\lambda_l\lambda_m\zeta^T(t)\left[\overline{A}_{ij}\quad A_{1i}\right]^T S\times$$
$$\left[\overline{A}_{lm}\quad A_{1l}\right]\zeta(t)-\int_{-\tau}^{0}\dot{x}^T(t+\theta)E^TSE\dot{x}(t+\theta)d\theta,$$
where $\zeta(t)=\begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}$.

According to Lemma 2, and replacing $t+\theta$ with *s*, we obtain

$$\dot{V}_2(t)\le\tau_0\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\zeta^T(t)\begin{bmatrix}\overline{A}_{ij}^TS\overline{A}_{ij} & \overline{A}_{ij}^TS\overline{A}_{1i} \\ * & \overline{A}_{1i}^TS\overline{A}_{1i}\end{bmatrix}\zeta(t)-$$
$$\int_{t-\tau}^{t}\dot{x}^T(s)E^TSE\dot{x}(s)ds$$

Now consider the derivative of $V_3(t)$.

$$\dot{V}_3(t)=\frac{1}{1-d}x^T(t)Hx(t)-\frac{1-\dot{\tau}(t)}{1-d}x^T(t-\tau)Hx(t-\tau)$$
$$\le\frac{1}{1-d}x^T(t)Hx(t)-x^T(t-\tau)Hx(t-\tau).$$

Then the derivative of $V(t)$ can be presented totally as follows:

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)$$
$$\le\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\zeta^T(t)\Theta_{ij}\zeta(t)-\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_jx^T(t)(Q+K_i^TRK_j)x(t)$$
$$=\sum_{i=1}^{k}\lambda_i^2\zeta^T(t)\Theta_{ii}\zeta(t)+\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\zeta^T(t)(\Theta_{ij}+\Theta_{ji})\zeta(t)-$$
$$\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_jx^T(t)(Q+K_i^TRK_j)x(t),$$
where

$$\Theta_{ij}=\begin{bmatrix}\Pi_{ij}+\tau_0\overline{A}_{ij}^TS\overline{A}_{ij}+Q+K_i^TRK_j & P^TA_{1i}-R_2+\tau_0\overline{A}_{ij}^TSA_{1i} \\ * & -H+\tau_0A_{1i}^TSA_{1i}\end{bmatrix},$$
$$\Pi_{ij}=\overline{A}_{ij}^TP+P^T\overline{A}_{ij}+\tau_0R_1+R_2+R_2^T+\frac{H}{1-d}.$$

If $\Theta_{ii}<0$ and $\Theta_{ij}+\Theta_{ji}<0$ hold for any $1\le i\le j\le k$, we know $\dot{V}(t)<0$. It implies the system (6) to be asymptotically stable.

From Lemma 3, we have

$$K_i^TRK_j+K_j^TRK_i\le K_i^TRK_i+K_j^TRK_j$$
$$2(K_i^TRK_j+K_j^TRK_i)\le(K_i+K_j)^TR(K_i+K_j).$$

Then, using the Schur complement, we know there are $\Theta_{ii}<0$ and $\Theta_{ij}+\Theta_{ji}<0$, if the following inequality

$$\begin{bmatrix}(1,1)+2\tau_0R_1 & (1,2) & \tau_0(\overline{A}_{ij}+\overline{A}_{ji})^TS & (K_i+K_j)^T \\ * & -2H & \tau_0(\overline{A}_{1i}+\overline{A}_{1j})^TS & 0 \\ * & * & -2\tau_0S & 0 \\ * & * & * & -2R^{-1}\end{bmatrix}<0 \tag{13}$$

holds for $1\le i\le j\le k$, where

$$(1,1)=(\overline{A}_{ij}+\overline{A}_{ji})^TP+P^T(\overline{A}_{ij}+\overline{A}_{ji})+2(R_2+\overline{R}_2+Q+\frac{H}{1-d})$$
$$(1,2)=P^T(A_{1i}+A_{1j})-2R_2.$$

Pre- and- post multiply (13) by diag$\{P^{-T},P^{-T},S^{-1},I\}$ and diag$\{P^{-1},P^{-1},S^{-1},I\}$, and note that $\overline{R}_1=P^{-T}R_1P^{-1}$, $\overline{R}_2=P^{-T}R_2P^{-1}$, $\overline{H}=P^{-T}HP^{-1}$, $\overline{Q}=P^{-T}QP^{-1}$, $X=P^{-1}$ and $Y_i=K_iX$, then we obtain condition (8).

Next, we consider the constraint of $\begin{bmatrix} R_1 & R_2 \\ R_2^T & E^TSE \end{bmatrix}\ge 0$.

Pre-and-post multiplying it by diag $\{P^{-T},P^{-T}\}$ and $\{P^{-1},P^{-1}\}$, we get

$$\begin{bmatrix}\overline{R}_1 & \overline{R}_2 \\ \overline{R}_2^T & X^TE^TU^{-1}EX\end{bmatrix}\ge 0. \tag{14}$$

In view of the following inequality:

$$(EX)^TU^{-1}EX>(EX)^T+EX-U,$$

which is resulting from

$$(EX-U)^TU^{-1}(EX-U)=$$
$$(EX)^TU^{-1}EX-(EX)^T-EX+U>0,$$

we know (9) can guarantee that (14) holds. Then by the conditions (7) , (8) and (9), we obtain

$$\dot{V}(t)<-\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_jx^T(t)(Q+K_i^TRK_j)x(t)$$
$$=-(x^T(t)Qx(t)+u^T(t)Ru(t))<0, \tag{15}$$

which implies that the system (6) is asymptotically stable.

Integrating (15) from 0 to *T* produces

$$\int_0^T(x^T(t)Qx(t)+u^T(t)Ru(t))dt<-V(T)+V(0).$$

Because $V(t) \geq 0$ and $\dot{V}(t) < 0$, $V(t) \to 0$ when $t \to \infty$. Therefore, we can get the following inequality

$$\int_0^{\infty} (x^T(t)Qx(t) + u^T(t)Ru(t))dt < \varphi^T(0)E^T X^{-1}\varphi(0) + \int_{-\tau_0}^{0}\int_{\theta}^{0} \dot{\varphi}^T(s)E^T U^{-1} E\dot{\varphi}(s)dsd\theta + \frac{1}{1-d}\int_{-\tau_0}^{0} \varphi^T(s)X^{-T}\bar{H}X^{-1}\varphi(s)ds.$$

This completes the proof.

In general, the guaranteed cost depends on the given upper bound $\tau_{0\max}$ to some extent. If $\tau_{0\max}$ is too large, it must result in very conservative guaranteed cost bound of J_0. So, we desire the delay upper bound of τ_0, noted as $\tau_{0\max}$, can be estimated. On the other hand, we also desire to get the closed-loop value of the upper bound of guaranteed cost function with respect to $\tau_{0\max}$.

Theorem 2 For the state feedback closed-loop system (6), there exists an upper bound of delay $\tau_{0\max} = 1/\varsigma$ such that for any $0 < \tau(t) \leq \tau_{0\max}$ the control law in form of (5) can stabilize system (6) with guaranteed cost performance $J_0(\tau_{0\max})$, if the following GEVP is feasible for

$$\text{Minimize}\, \varsigma = \frac{1}{\tau_0} > 0 \tag{16}$$

$$s.t.\ \begin{cases} \text{LMIs (7), (9),} \\ M < \begin{bmatrix} N & 0 \\ 0 & 0 \end{bmatrix}, \\ N < \varsigma Z, \end{cases}$$

for $1 \leq i \leq j \leq k$, where

$$M = \begin{bmatrix} 2\bar{R}_1 & 0 & 0 & \Lambda_{13} \\ 0 & 0 & 0 & X^T(A_{1i} + A_{1j})^T \\ 0 & 0 & 0 & 0 \\ * & * & * & -2U \end{bmatrix},$$

$$Z = \begin{bmatrix} -\Lambda_{11} & -\Lambda_{12} & -(Y_i + Y_j)^T \\ * & 2\bar{H} & 0 \\ * & * & 2R^{-1} \end{bmatrix}.$$

Proof: Considering condition (8), we exchange Column 3 with Column 4, and Row 3 with Row 4 of the matrix on the left side. The new inequality

$$\begin{bmatrix} \Lambda_{11} + 2\tau_0\bar{R}_1 & \Lambda_{12} & (Y_i + Y_j)^T & \tau_0\Lambda_{13} \\ * & -2\bar{H} & 0 & \tau_0 X^T(A_{1i} + A_{1j})^T \\ * & * & -2R^{-1} & 0 \\ * & * & 0 & -2\tau_0 U \end{bmatrix} < 0$$

still holds. Then, we have

$$M < \varsigma\bar{Z},$$

where $\bar{Z} = \begin{bmatrix} Z & 0 \\ 0 & 0 \end{bmatrix}$.

Since the positivity of $\bar{Z}$ is not strictly feasible, we replace the constraint $M < \varsigma\bar{Z}$ with $M < \begin{bmatrix} N & 0 \\ 0 & 0 \end{bmatrix}$, $N > 0$ and $N < \varsigma Z$. Then, with the upper bound $\tau_{0\max} = 1/\varsigma_{\min}$ given by the feasible solution of the GEVP (16), and based on theorem 1, we know the system (6) is asymptotically stable with guaranteed cost performance $J_0(\tau_{0\max})$ in the form of (10). The proof is completed.

IV. EXAMPLE

To demonstrate the effectiveness of the design procedure of the guaranteed cost controller, consider the following fuzzy descriptor delay system. It is supposed that x_1 is measurable online.

If x_1 is P, then

$$E\dot{x}(t) = A_1x(t) + A_{11}x(t-\tau(t)) + B_1u(t).$$

If x_1 is N, then

$$E\dot{x}(t) = A_2x(t) + A_{12}x(t-\tau(t)) + B_2u(t).$$

Here the membership functions of P and N are given as follows

$$\lambda_1(x_1) = 1 - \frac{1}{1+e^{-2x_1}},\ \lambda_2(x_1) = \frac{1}{1+e^{-2x_1}}.$$

And $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $A_{11} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.4 \end{bmatrix}$, $\phi(t) = \begin{bmatrix} -70 \\ 25 \end{bmatrix}$, $t \in [-\tau_0, 0]$.

Based on Theorem 2, we can get $\tau_{0\max} = 3.2396$. The state feedback guaranteed cost controller is

$$u = \begin{Bmatrix} (1 - \frac{1}{1+e^{-2x_1}})[-2.8368 \quad -8.1145] + \\ \frac{1}{1+e^{-2x_1}}[-1.8942 \quad -4.4550] \end{Bmatrix} x(t),$$

and the upper bound of the guaranteed cost is

$$J_0 = 321.43.$$

The simulation results of the delay-dependent guaranteed cost control are given in Fig.1.

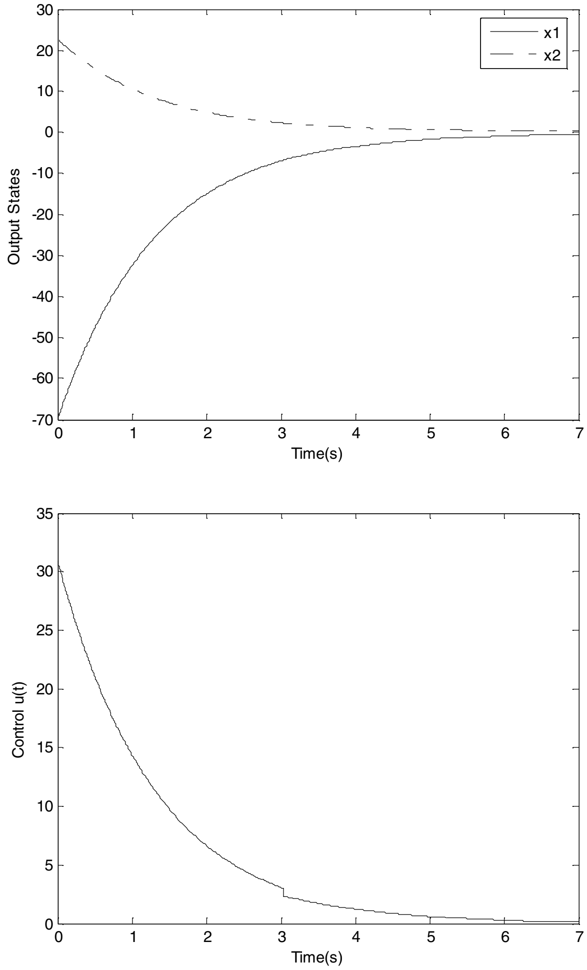

Fig.1. The simulation results of the delay-dependent guaranteed cost control

V. CONCLUSIONS

The delay-dependent guaranteed cost control problem for a class of T-S fuzzy descriptor systems with time-varying delay is investigated. Based on LMIs, the fuzzy guaranteed cost controller is determined via state feedback. The sufficient conditions of the existing of the guaranteed cost controller is given. The upper bound of delay and the closed value of the guaranteed cost are also presented by a GEVP method. The numerical example shows the effectiveness of the proposed approaches.

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