Supervisory Chaos Control of a Two-Link Rigid Robot Arm Using OGY Method

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***Abstract*—In this article, we have proposed a new approach for controlling a two-rigid link robot arm which has been chaotic by applying an external periodic input. The whole system includes a plant, chaos controller and a supervisor. Supervisor chooses suitable external input by using some chaotic criteria such as "Lyapunov Exponent" and "Bifurcation Diagrams". Chaotic control of the plant involves two steps, finding unstable periodic orbits (UPOs) of the system by "Poincare map", and using a control technique in order to stabilize a desirable orbit. OGY method has been utilized as chaos controller. Moreover, the supervisor selects suitable UPOs as intermediate goals in order to guide the plant and OGY controller in the chaotic attractor to reach to the target orbit. By using this method, the chaotic behavior of the system turns to a certain periodic behavior and speed of control enhances in contrast to OGY method.**

***Keywords*— Supervisor, Chaotic Control, OGY, Lyapunov Exponent, Poincare Map, Rigid Robot Arm, Unstable Period Orbits**

I. INTRODUCTION

A. Motivation

The study of chaos has presented new conceptual and theoretical tools allowing us to classify and understand complex behavior that had confused previous theories. It has been almost three decades that this behavior of nonlinear dynamics, chaos, has fascinated the consideration of scientists in different fields such as biology, ecology, sociology, engineering, etc. Therefore, understanding, studying and controlling this phenomenon in complex systems is extremely significant.

Primary and one of the important characteristics of chaotic system is sensitivity to initial conditions or "Near by Divergence". This means that future of the system is totally different in the presence of even a small perturbation. However, this behavior can not be observed in deterministic systems. States of chaotic system have local instability but global stability behavior. Moreover, flexibility in chaotic systems allows system to adapt to its environment. In the other hand, sensitivity to initial condition in the chaotic system causes awareness of changes in environment.

Beside chaotic systems, supervisory and hierarchical control structures have been considered by engineers, nowadays. Moreover, motor control and learning in human body is hierarchical and supervisory. Therefore, this is an excellent clue for using these structures in controlling man-made nonlinear dynamic and complex systems.

Control and coordination between layers in these structures are top-down. Consequently, each unit can control unites of lower layer, and is controlled by the layer above. By the concept of using hierarchical structure, control task would be faster and more robust compared to one layer control structure. Furthermore, hierarchical control structures can help us to overcome problem of high degrees of freedom in complex systems. Additionally, they can facilitate decreasing linguistic rules in fuzzy logic and knowledge base control systems. They can also guide us to achieve great intuition about how human's "Central Nervous System" works.

B. Previous Works

The first scientist who reported chaotic behavior in 1982 was Poincare. Afterwards, published papers in this field have been extremely increased. In fact, from 1990 till 2000, the number of articles about chaos control achieved approximately 2700 [1], [2], [3]. Otte, Grebogi and Yorke introduced OGY method in 1990 [4]. After this proposed method, several papers had been published for developing OGY controller on different models [5], [6], [7], [8]. This method has two main problems. First of all the time which one has to wait before starting and achieving control might be too long [11].

The second problem is sensitivity to disturbance and perturbation. Although there have been tremendous attempts to resolve these problems in several published papers, still OGY method should be improved.

In 1997, Vincent proposed a real two link rigid robot arm model which was chaotic via periodic external input [9]. He used Lyapunov control method for controlling the chaotic system. We have used this model for our plant.

Beside chaos theory, hierarchical control structures have been used recently for various models such as robot arms and legs [10], [11],[12],[13],[14],[15],[16]. Although huge number of papers has been published, necessity of using hierarchical and supervisory method for controlling chaos phenomenon should be considered. Therefore, proposed approach is a connection between chaos theory and hierarchical control structure.

C. Defining Target

In this article, the plant is a robot arm model which performs under gravity force. It has two rigid links, two revolute joints and no end- effecter (Fig. 1.).

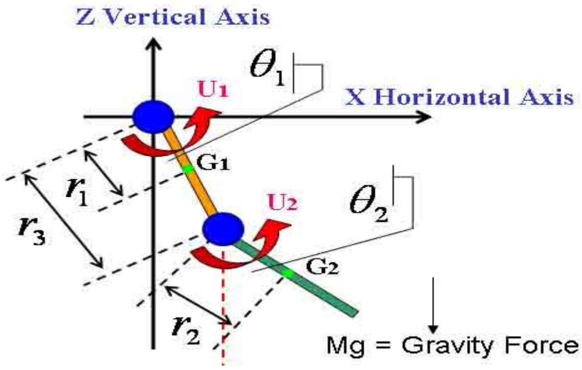


Figure 1. Two link rigid robot arm model which performs under gravity in (X-Z) plane with two revolute joints and no end-effecter. The torque inputs are applied to joints.

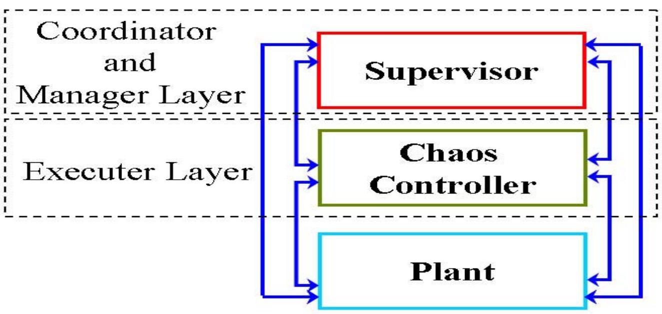


Figure. 2. This hierarchical structure of control has two layers. The supervisor layer which coordinates and manages the whole system performance. The chaos controller is OGY controller which controls chaotic behavior in order to stabilize period behavior.

After making the plant behavior chaotic by applying appropriate external input, a hierarchical control structure which contains OGY chaos controller and supervisor (Fig. 2.), has been utilized to gain desired periodic behavior out of chaotic behavior.

The rest of paper is organized as follow: In section II we propose a mathematical model of robot arm and in section III finding appropriate external input has been discussed. Section IV indicates Poincare map, Poincare section and unstable period orbits of the system. In section V, controlling system with two methods, OGY and hierarchical control structure, has been argued. In section VI, the conclusion is stated and references can be found in section VII.

II. Robot Arm Model

In order to find model of two-link rigid robot arm under gravity (Fig.1), we used Lagrangian motion equation. We could write nonlinear model of this robot arm in terms of its angles and torque inputs as written in (1). List of robot arm parameters can be found in Table1 where θ_1 and θ_2 indicate first and second link angles vs. vertical line, respectively.

TABLE I. LIST OF ROBOT AEM PARAMETERS

Link number	Parameters
Link1	$r_1 = 0.292$ m , $r_3 = 0.413$ m , $I_1 = 0.068$ Kg m^2 , $R_1 = 7.7$ v/A , $m_1 = 0.602$ Kg , $K_{\gamma 1} = 0.008$ Kg m^2/A s^2 , $K_{\beta 1} = 5.2$ vs^2/rad
Link2	$r_2 = 0.198$ m , $I_2 = 0.00474$ Kg m^2 , $m_2 = 0.076$ Kg , $K_{\gamma 2} = 0.001$ Kg m^2/A s^2

$$\begin{cases} \left(m_1 r_1^2 + m_2 r_3^2\right)\ddot{\theta}_1 \\ \quad + m_2 r_2 r_3 \cos(\theta_2 - \theta_1)\ddot{\theta}_2 \\ \quad - m_2 r_2 r_3 \sin(\theta_2 - \theta_1)\dot{\theta}_2^2 \\ \quad + m_1 g r_1 sain\,\theta_1 \\ \quad + m_2 g r_3 \sin\theta_1 = \tau_1 - \tau_2 \\ m_2 r_2^2 \ddot{\theta}_2 + m_2 r_2 r_3 \cos(\theta_2 - \theta_1)\ddot{\theta}_1 \\ \quad + m_2 r_2 r_3 \sin(\theta_2 - \theta_1)\dot{\theta}_1^2 \\ \quad + m_2 g r_2 \sin\theta_2 = \tau_2 \end{cases}, \qquad (1)$$

$$\begin{cases} \tau_1 = \dfrac{K_{\gamma 1}}{R_1} U_1 - \dfrac{K_{\gamma 1} K_{B1}}{R_1} X_2 \\ \tau_2 = K_{\gamma 2} U_2 \end{cases}.$$

III. Appropriate External Input

There are three useful ways for creating and abolishing chaos in nonlinear dynamical systems:

- Changing system's parameters such as mass, friction coefficient, length of links,
- Changing initial conditions of the system's states,
- Applying external input like torque to joints.

The first two ways are not practical and do not create chaotic behavior in this article's plant. Therefore, as written in (2), we use a periodic input which is able to force the whole system to act chaotically. For having chaotic behavior, the ratio of natural frequencies and input frequency should be incommensurate. In addition, coupling between these two frequencies should be high [17].

$$\begin{cases} U_1 = A\cos(Ft) \ \ (v) \\ U_2 = 0 \ \ (v) \end{cases}. \qquad (2)$$

where U_1 and U_2 are first and second links' torques, respectively and (v) indicates Volt unit. In addition, A and F indicate amplitude (v) and frequency (s^{-1}), respectively.

For finding appropriate external input of this model, we would use some criteria which explained in the next subsections.

A. Bifurcation Diagram

In order to find appropriate frequency and amplitude of the input for controlling the system feed forward, we would like to utilize bifurcation diagrams. These diagrams can demonstrate that which region of the defined parameter can make system's behavior chaotic, periodic or unstable. In addition, route to chaos in our system is "crisis". It means that chaos occurs and disappears suddenly (Fig.3 and Fig.4). For plotting these diagrams, we used local maximums of two angles' trajectories. Moreover, initial conditions in all the diagrams have been set to zero.

B. Lyapunov Exponent Diagram

Lyapunov exponent can assist us to distinct periodic, a-periodic and unstable behaviors from chaotic behavior. In chaotic behavior, at least one of the Lyapunov exponents should be positive and one should be negative. In addition, the summation of all exponents should be more than zero [18].

Definition: Consider a d-dimensional system defined by a set of ordinary differential equations:

$$\frac{d}{dt}\mathrm{x}(t) = \mathrm{F}(\mathrm{x};\varphi). \tag{3}$$

If a small initial perturbation, $\varepsilon(0) = \varepsilon_0$, is applied to that, trajectory becomes $\mathbf{y}(t) = \mathbf{x_0} + \varepsilon(t)$. For an appropriately chosen ε_0, the exponential rate of expansion or contraction in the direction of ε_0 on the trajectory passing through $\mathbf{x_0}$ defines the Lyapunov exponent along that direction [18]:

$$\lambda_i = \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} \ln\left(\frac{\|\varepsilon_k\|}{\|\varepsilon_0\|}\right). \tag{4}$$

where $\|.\|$ denotes the vector norm. By using results of bifurcation diagrams and Lyapunov exponents, input parameters have been chosen as $A = 5.3$ (v) and $F = 5$ ($^{rad}/_{s}$).

IV. Poincare Section and Unstable Perriod Orbits

In order to find unstable period orbits (UPOs) of the chaotic system, we use Poincare map method. The UPOs are the characteristic of the chaotic attractor. OGY controller stabilizes these UPOs. Consequently, chaotic behavior can be converted to periodic behavior if period orbits can be stabilized.

The first step in finding Poincare map is choosing suitable Poincare section. After numerous experiments, our Poincare section is the plane which angular velocity of each link is zero.

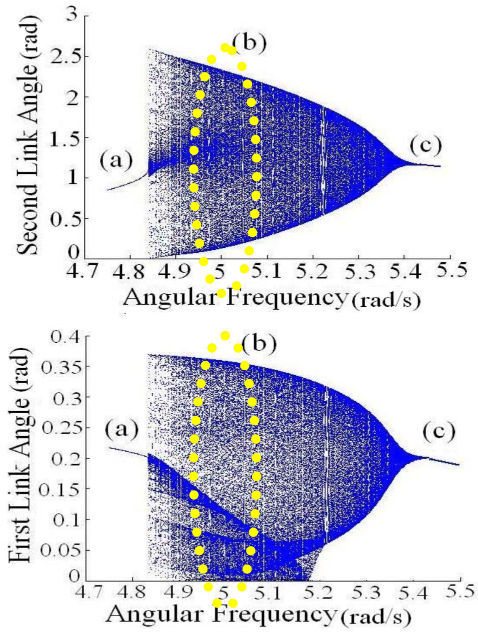


Figure 3. Bifurcation diagrams vs. angular frequency of the external periodic input: For plotting them we have set $A = 5.3$ (v). There are three parts in these diagrams. (a) $F \approx< 4.85$ (rad/s) : Periodic behavior, (b) 4.85 (rad/s) $\approx< F \approx< 5.4$ (rad/s) : Chaotic behavior and (c) $F >\approx 5.4$ (rad/s) : periodic behavior. Selected yellow region is the selected input angular frequency for the system.

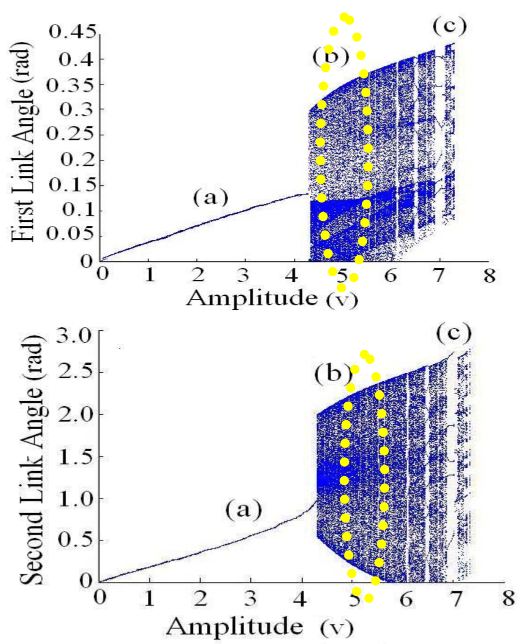


Figure 4. Bifurcation diagrams vs. amplitude of external periodic input: For plotting them we have set $F = 5$ (rad/s). There are three parts in these diagrams. (a) $A \approx< 4.25$ (v) : Periodic behavior, (b) 4.25 (v) $\approx< A \approx< 7$ (v) : Chaotic behavior and (c) $A >\approx 7$ (v) : Unstable behavior. Selected yellow region is the selected input amplitude for the system.

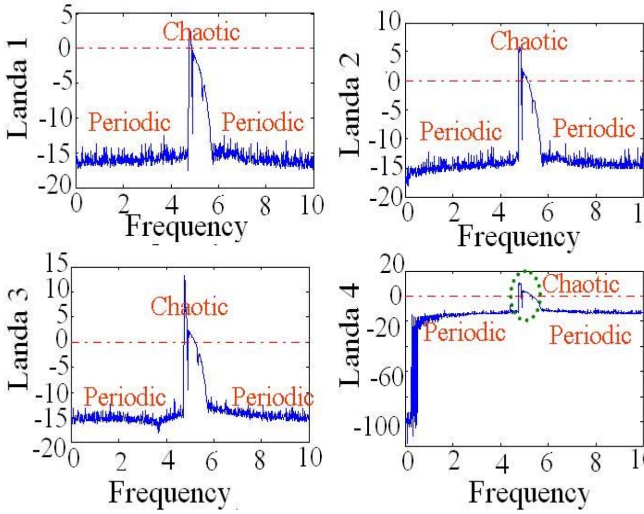
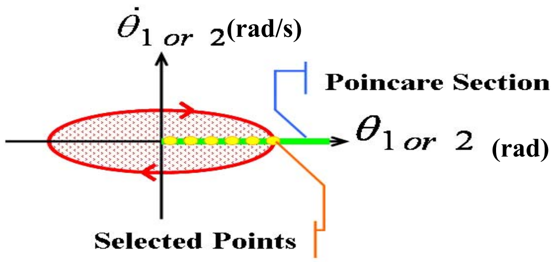
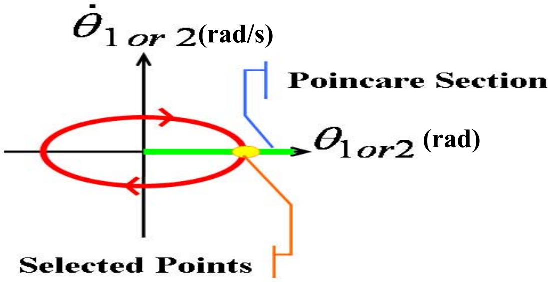


Figure 5. Lyapunov exponents vs. angular frequency of external periodic input: These diagrams are coincident with bifurcation diagrams. Green region is showing the region for selecting input frequency of the system.



(a) Chaotic behavior

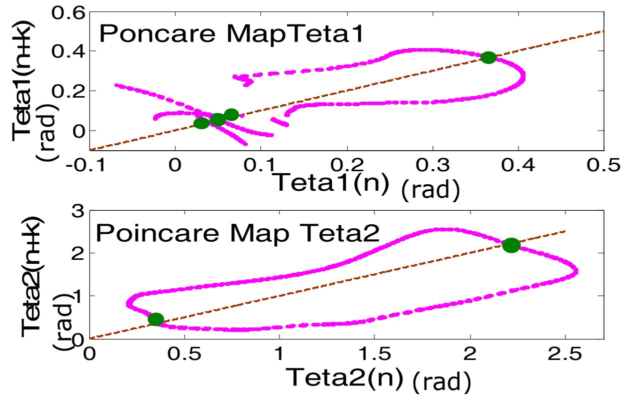


(b) Periodic behavior

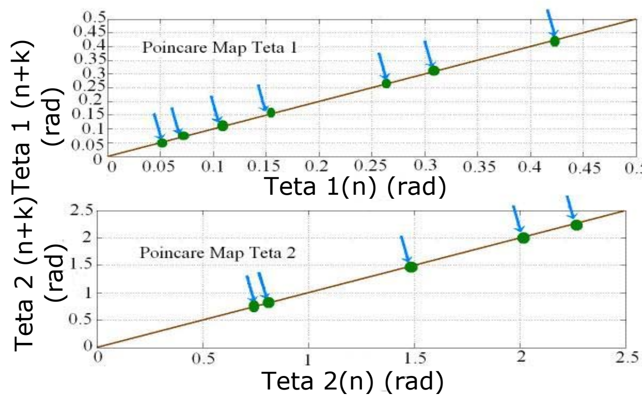
Figure 6. Schematic Poincare section: The selected points are points of links' angle trajectories which have intersected with zero angular velocity planes.

Fixed points of the system are places where Poincare map intersects with bisector. These points show where UPOs located in state space (Fig. 7.). One of the fixed points of the system at $A = 5.3\,(v)$ can be found in (5):

$$p^* = \begin{bmatrix} 0.05110782 & 14224897 \\ 0.00470770 & 054162622 \\ 0.20483940 & 683527 \\ 0.03940941 & 32346755 \end{bmatrix}_{Number\ \ of\ \ States\ \times 1} . \quad (5)$$



(a) Chaotic behavior



(b) Periodic behavior

Figure 7. Poincare map in different input amplitude: (a) Chaotic behavior, $A = 5.3\,(v)$: Structure of the map is complex and UPOs are places where Poincare map intersects with bisector. Fixed points are demonstrated via green points (b) Periodic behavior, $A = 4.5\,(v)$: Poincare map of the system consists several dots which indicate period of the cycle. These green dots occur on bisector. K here is 9 for Teta 1 and 7 for Teta 2. The (n) indicates number of iterations.

V. CONTROLLING CHAOS METHODS

A. OGY Control Method

OGY Algorithm Definition: We applied this method in order to stabilize selected unstable period orbits of the chaotic system by using method proposed in [6]. Moreover, amplitude of the external input has been selected as control parameter $b = \bar{b} + \delta b$ where δb is a small correction to the standard value of $\bar{b} = 5.3$. The control parameter δb is adjusted at each switching point. Consequently, this task leads dependence of the Poincare map of the system which we indicate it via ϕ :

$$p_{i+1} = \phi(p_i, b_i)\,. \quad (6)$$

Let p^* be an unstable fixed point of the Poincare map for the nominal value $\bar{b} = 5.3$. This fixed point match with one of the unstable period orbits of the system.

$$p^* = \phi(p^*, \bar{b})\,. \tag{7}$$

For values of p_i close to p^*, and values of b_i close to $\bar{b}$ the Poincare map in (6) can be approximated by the linear map in (8):

$$\delta p_{i+1} = A_{LS}\,\delta p_i + B_{LS}\delta b_i\,. \tag{8}$$

where $\delta p = p_i - p^*$ and $\delta b_i = b_i - \bar{b}$ are the deviations from the nominal values.

A_{LS} and B_{LS} for the discrete linear system can be found as in (9). We would set these two parameters as in (10) and (11):

$$A_{LS} = \left.\frac{\partial \phi}{\partial p}\right|_{(p^*,\bar{b})} \quad , \quad B_{LS} = \left.\frac{\partial \phi}{\partial b}\right|_{(p^*,\bar{b})}, \tag{9}$$

$$A_{LS} = \begin{bmatrix} -1.587469 & -4.462099 & -0.497808 & -1.639133 \\ 0.010789 & 0.030328 & 0.003383 & 0.011141 \\ -3.412386 & -9.591625 & -1.070077 & -3.523442 \\ -1.202648 & -3.380436 & -0.377133 & -1.241789 \end{bmatrix}, \tag{10}$$

$$B_{LS} = \begin{bmatrix} -39.687696 & -39.687696 \\ 0.269755 & 0.269755 \\ -85.311743 & -85.311743 \\ -30.066950 & -30.066950 \end{bmatrix}. \tag{11}$$

By applying linear state feedback control method (12) to the discrete time system in (8), it can be seen that the closed loop system is stable as long as (13) is correct for a specific K.

$$\delta b_i = -K\,\delta p_i\,. \tag{12}$$

$$\left|\, \mathrm{eig}\,(A_{LS} - B_{LS} K)\,\right| < 1, \tag{13}$$

By using DLQR method, gain K can be found as in (14):

$$K_{2\times 4} = \begin{bmatrix} 0.019998 & 0.056212 & 0.006271 & 0.020649 \\ 0.019998 & 0.056212 & 0.006271 & 0.020649 \end{bmatrix}. \tag{14}$$

By using [19] for analyzing state space, applying OGY method to chaotic system can stabilize UPOs (Fig. 8). This method allows system to behave chaotically till the trajectory reaches to the selected fixed point. At this time, OGY controller switches on and periodic behavior occurs.

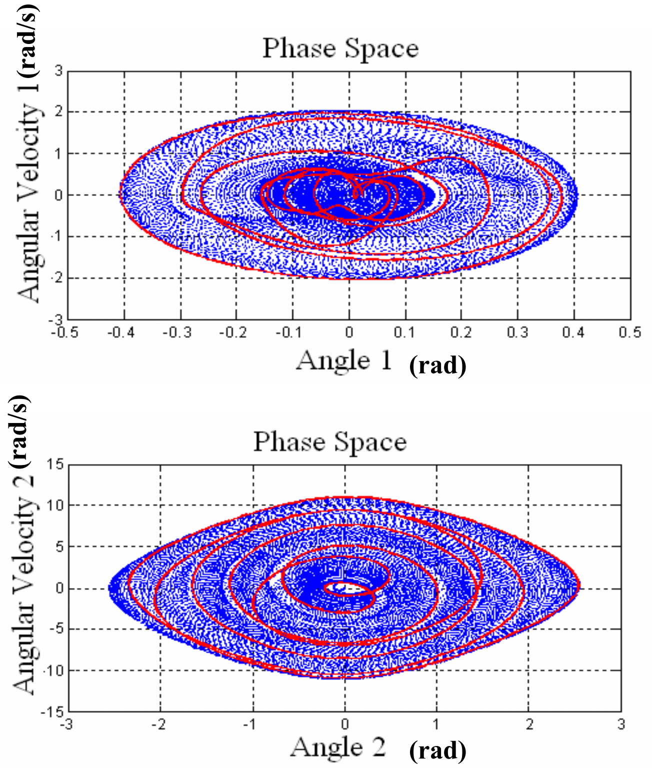


Figure 8. Phase space of the robot arm system sing OGY control method: The blue diagrams are chaotic strange attractors for both links. The red trajectories demonstrate unstable period orbit of the system which has been stabilized by OGY controller. As dense strange attractors demonstrate, a lot of time takes to reach desired fixed point by using OGY method.

B. Supervisory Control Method

As discussed in subsection C of introduction, our proposed method consists 2 layer of control, supervisor and OGY. Supervisor has supervision on OGY and plant performance. Moreover, for reaching desired unstable period orbit which it has been picked by user, supervisor selects intermediate goals in order to guide OGY controller and plant in the strange attractor. In addition, it affects on OGY control targets and plant's dynamic by changing amplitude of external input and choosing appropriate intermediate fixed points.

These intermediate goals are other fixed points of the system which are near either the initial states or desired fixed point. Considering how many intermediate goals is needed has been done by user. Supervisor chooses these intermediate targets one by one as temporary goals for OGY controller. Then, when OGY stabilizes chaotic system on one of theses unstable period orbits, another goal for OGY will be picked by supervisor. This will lead turning off OGY and letting trajectory moves in the strange attractor of the system again until it reaches near the other intermediate goal. This process continues until all intermediate goals have been picked by supervisor beside and trajectory reaches desired unstable period orbit. Again, OGY stabilizes system on the desired unstable period orbit in order to achieve desired periodic behavior. The schematic of what happens in the supervisory algorithm can be seen in Fig. 9. With this method, system's response is faster and more robust to noise and perturbation respect to OGY controller (Fig. 10).

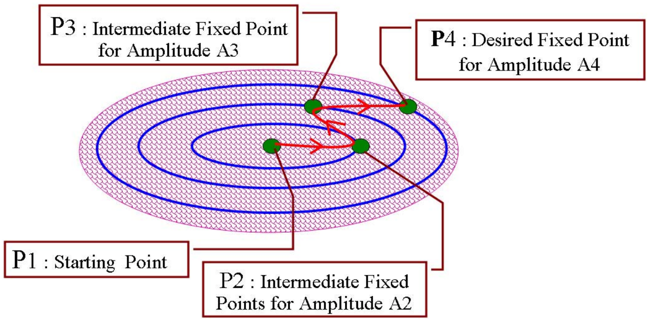
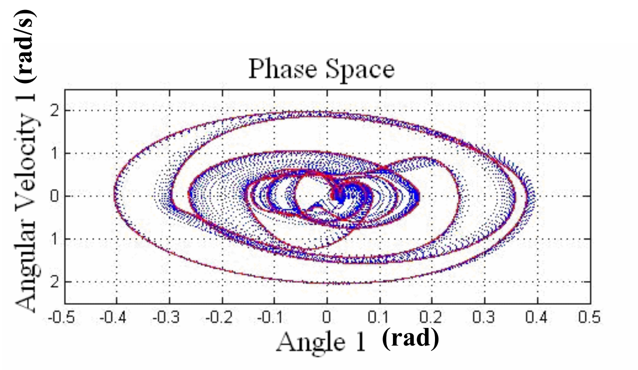
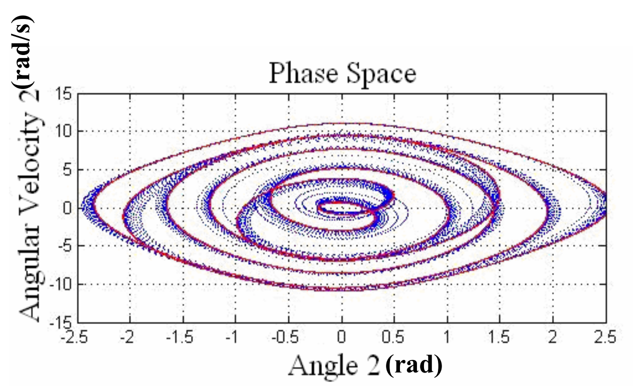


Figure 9. Schematic of the supervisor, OGY and plant performance in the strange attractor: The starting point is zero initial conditions. There two sets of goals for the whole system: (1) Intermediate fixed points (2) Desired fixed point. Supervisor guides the whole system toward attractor in order to settle down system on desired unstable period orbit faster and more reliable.



(a) Phase Space for First Link of the Robot Arm



(b) Phase Space for Second Link of the Robot Arm

Figure 10. Phase space of the robot arm model using hierarchical control method: The blue diagrams are chaotic strange attractors for both links. The red trajectories demonstrate unstable period orbit of the system which has been stabilized by controller. This method can control system faster compared to OGY method. Compared with Fig.8., before attractor becomes dense, controller can stabilized system on desired unstable period orbit.

VI. Conclusion

In this article, we have proposed a new approach for controlling a system which has been chaotic by applying external input. The hierarchical structure of control consisted supervisor and OGY controller in two layers. For finding appropriate external input, bifurcation diagrams and Lyapunov exponents have been utilized. Unstable period orbits had been found by Poincare map. Moreover, guiding whole system in the strange attractor toward desired period orbit was one of the supervisor's responsibilities. By using appropriate fixed points as intermediate goals in a supervisory manner, controlling was faster and more robust compared to OGY technique.

Acknowledgment

Authors would like to dedicate this article to our dear friend, Mohammad Hossien Khosravi, in the memory of him.

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